COMPRESSIVE GHOST IMAGING

Andreas Valdmann
Traditional imaging

Object with reflectivity \( O(x, y) \)

Light source

Uniform illumination

Matrix sensor (Camera)

Signal

\( S_{\text{cam}}(x, y) \sim O(x, y) \)
**Ghost imaging**

Object $O(x, y)$

$I(x, y)$ $O(x, y)$

Single pixel detector measures $S = \int I(x, y)O(x, y)dx\,dy$

Image reconstruction

$O(x, y) = \langle (S - \langle S \rangle)(I(x, y) - \langle I(x, y) \rangle) \rangle$

Matrix sensor (Camera) measures $I(x, y)$

$M$ – number of measurements
Computational ghost imaging

Object \( O(x, y) \)

Single pixel detector
measures \( S = \int I(x, y) O(x, y) \, dx \, dy \)

Image reconstruction

\[
O(x, y) = \langle (S - \langle S \rangle)(I(x, y) - \langle I(x, y) \rangle) \rangle
\]

\( I(x, y) \) is determined by user. No camera is needed.

\[
O(x, y) = \frac{1}{M} \sum_{i=1}^{M} S_i I_i(x, y)
\]

\( M \) – number of measurements
Multicolor ghost imaging

Object $O_\mu(x, y)$

$I(x, y)O_\mu(x, y)$

Single pixel detector measures $S_\mu = \int I(x, y)O_\mu(x, y)dx\,dy$

$I(x, y)$ is determined by user. No camera is needed.

$O_\mu(x, y) = \langle (S_\mu - \langle S_\mu \rangle)(I(x, y) - \langle I(x, y) \rangle) \rangle$

$O_\mu(x, y) = \frac{1}{M} \sum_{i=1}^{M} O_{\mu_i} I_i(x, y)$

$M$ – number of measurements
Inversion image reconstruction

M measurements; N pixels

\[ S = [s_1, s_2, ..., s_M] \] array of single pixel detector signals

\[ I = [I_1, I_2, ..., I_M] \] matrix of all light patterns

\( I_1, I_2, ..., I_M \) are flattened light pattern arrays

\( O \) flattened image of the object (unknown)

System of linear equations

\[ IO = S \]
Inversion image reconstruction

M measurements; N pixels

\[ I O = S \]

- **M = N**: perfect image recovery if no noise is present (only in theory).
- **M > N**: can be solved with least squares methods. Larger M gives better signal to noise ratio, but measurements and calculations take more time.
- **M < N**: Theoretically infinite number of solutions. Additional constraints must be given. **Compressive sensing**
Discrete cosine transform

A generic sampled signal

The modulus of its DFT

Its DCT
JPEG compression

DCT of a natural image gives a sparse matrix.
Compressive sensing

Is it necessary to make many measurements if most of the data is omitted during compression anyway?

No, if the measured image is natural, i.e. it is sparse in some basis.

**Idea of compressive sensing algorithm**

- Solve the underdetermined system in some sparse basis.
- Additional constraint: result vector must contain as many zeros as possible: \( L_0 \) norm minimization.
\[ \|x\|_p = \left( \sum_{i=0}^{n} x_i^p \right)^{\frac{1}{p}} \]

\[ \|x\|_0 \] counts non-zero elements in vector

\[ \|x\|_1 \] sum of absolute values of vector components (Manhattan geometry)

\[ \|x\|_2 \] Euclidian norm

Minimizing \(L_0\) norm is NP-hard.
Minimizing \(L_1\) norm also gives a small \(L_0\) norm.

Examples of implementation:
- Matlab L1-MAGIC toolbox
- Python CVXOPT package
Compressive ghost imaging

Initial problem
\[ IO = S \]

Discrete cosine transform
\[ I \rightarrow I_{DCT} \]

\( O_{DCT} \) is in sparse basis
\[ I_{DCT}O_{DCT} = S \]

Minimize
\[ \frac{1}{2} \| S - I_{DCT}O_{DCT} \|^2 + \lambda \sum |O_{DCT}| \]

\( \lambda \) is a small regularization parameter

Inverse discrete cosine transform
\[ O_{DCT} \rightarrow O \]
Experimental results

Fig. 3. Comparison of colored image reconstructions with $128 \times 64$ pixel resolution for increasing sample size using an iterative and a compressive algorithm. For 8000 (approximately Nyquist) measurements the acquisition time is less than 12 seconds.

Potential applications of compressive sensing

- **Energy efficient miniature cameras** [David Schneider (March 2013). "New Camera Chip Captures Only What It Needs". *IEEE Spectrum*.]


Summary

Ghost imaging
• Single pixel detector
• Many measurements
• Replaces matrix sensor

Compressive sensing
• Number of measurements is less than number of pixels
• Uses sparseness of natural images

Thank you