Data dissemination problem in wireless networks

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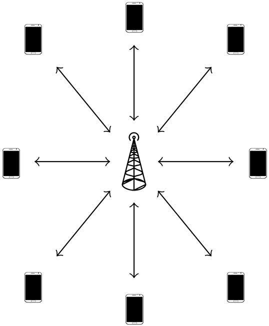
Centralised network



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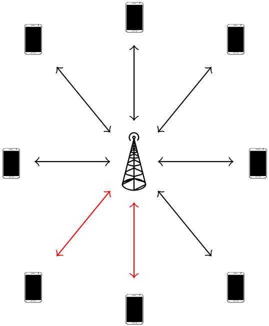

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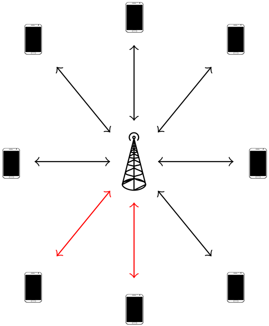
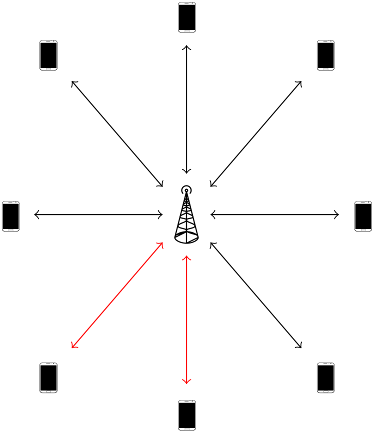


- Easy to set up

Centralised network



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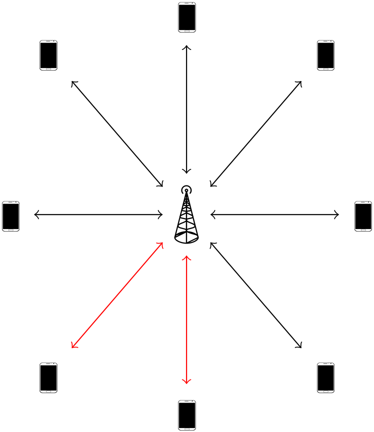
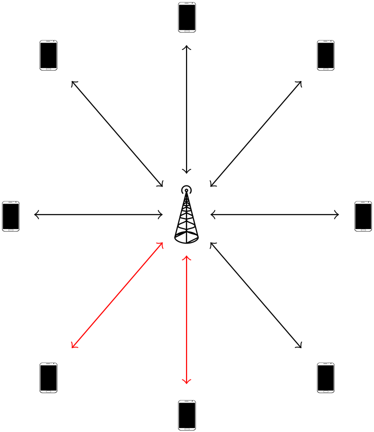


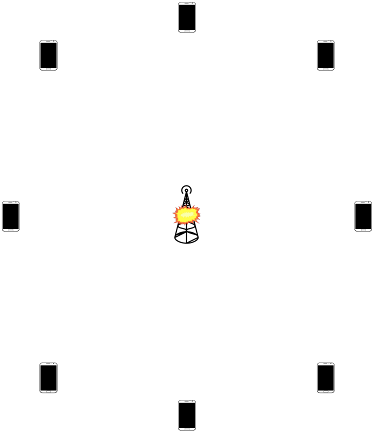
- Easy to set up
- Transmission power grows quadratically with the distance.

Centralised network



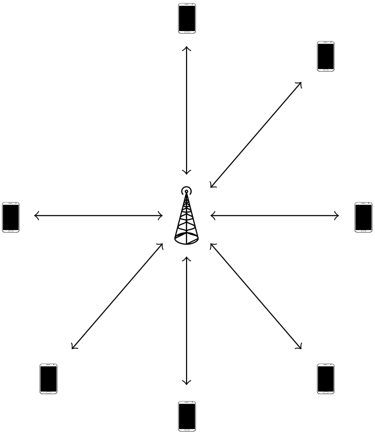
- Easy to set up
- Transmission power grows quadratically with the distance.
- Dependency on the central unit

Centralised network



- Easy to set up
- Transmission power grows quadratically with the distance.
- Dependency on the central unit

Completely decentralised network



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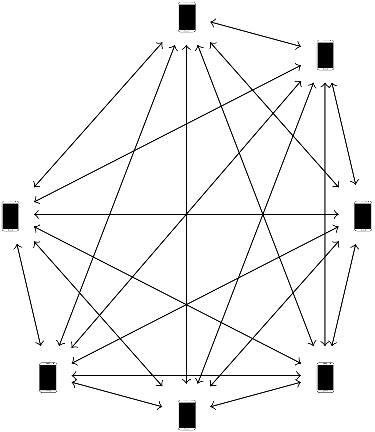
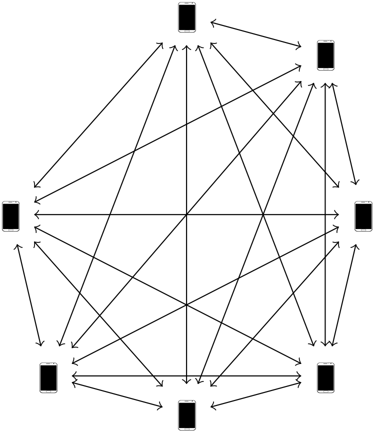
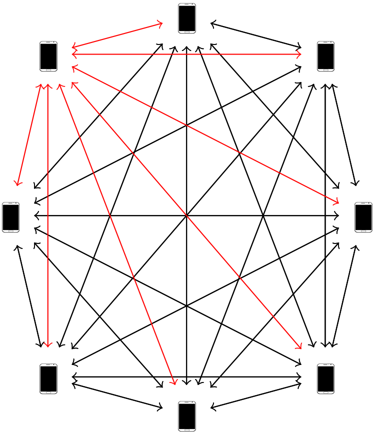


Completely decentralised network



- Requires knowledge of the entire network
- Worst-case distance increases

Completely decentralised network



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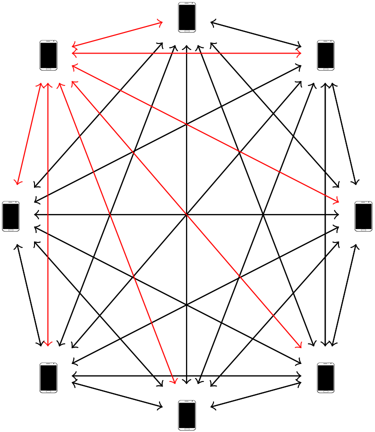
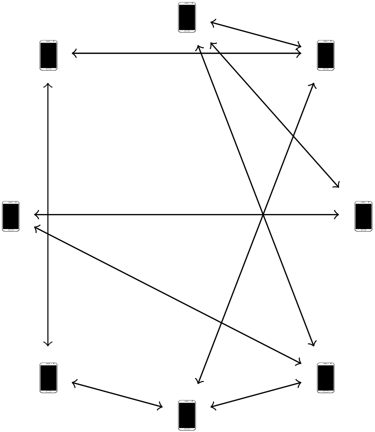


- Requires knowledge of the entire network
- Worst-case distance increases
- Difficult to set up

Partially decentralised network



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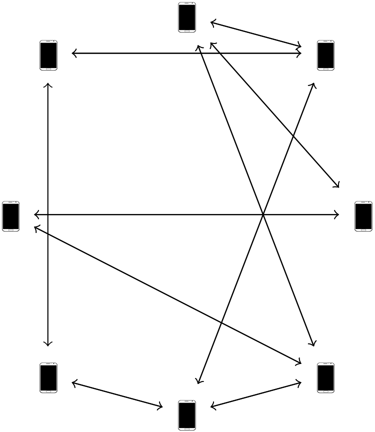
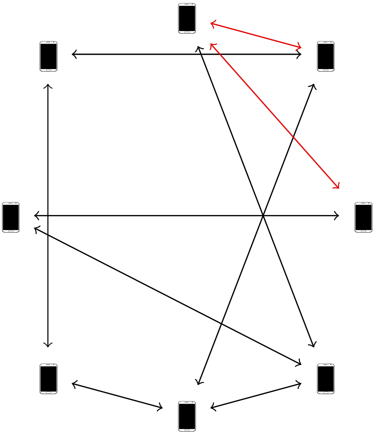


- Connected network

Partially decentralised network



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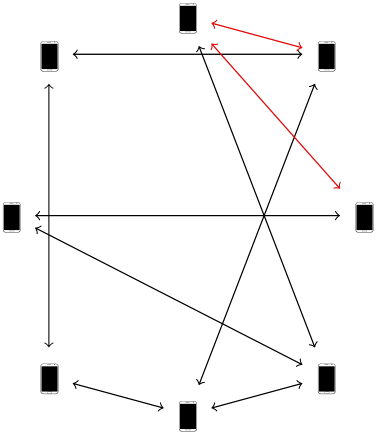
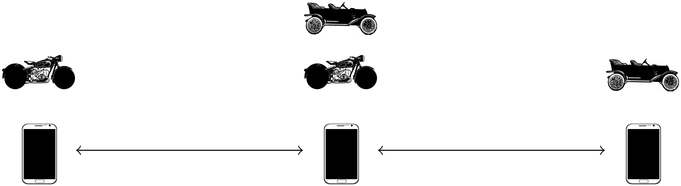


- Connected network
- Use participants as proxies

Data dependency

Data dependency

- Everyone knows **something**



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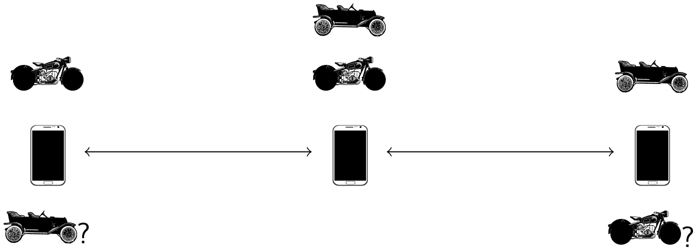
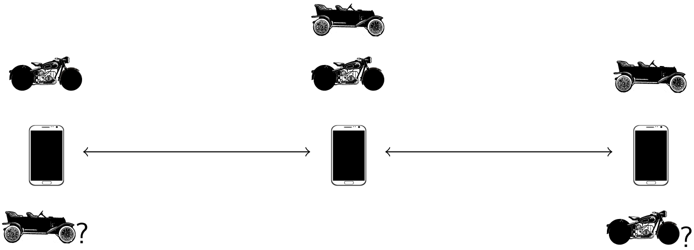


Data dependency

- Everyone knows **something**
- Someone wants **something**
- Broadcast: “car is a motorcycle with two more wheels”



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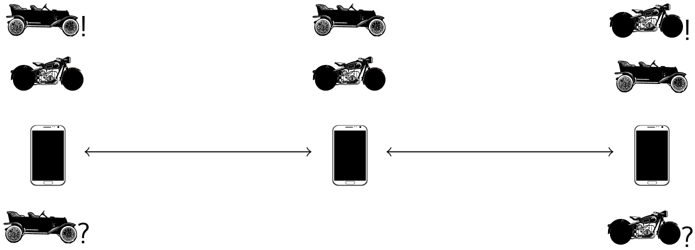

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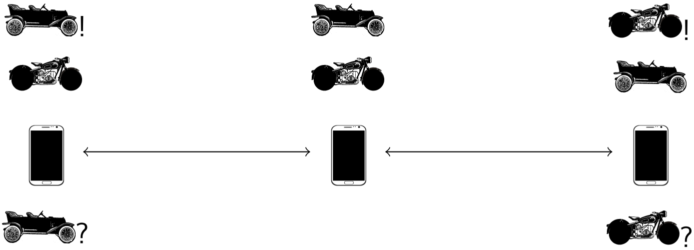
- Linear combination



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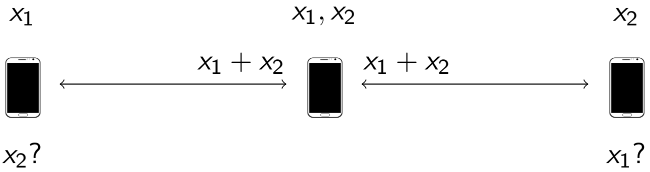
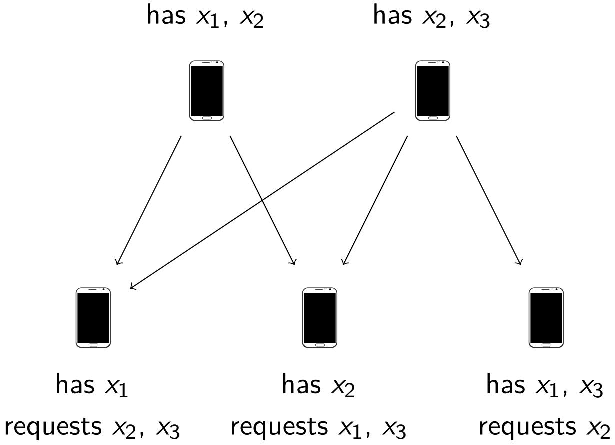


A bit more advanced example



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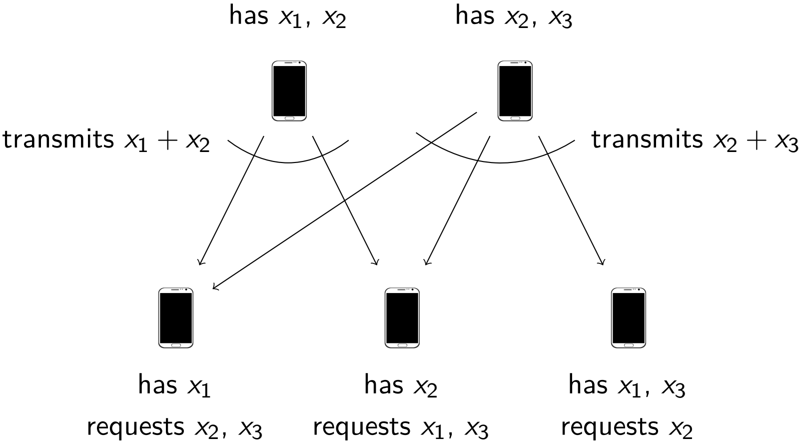

General 1-solvable networks

- Network is said to be 1-solvable if all requests can be fulfilled in a single round.

Theorem

Consider a wireless network defined by the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The minimal number of transmissions needed to satisfy the demands of all nodes ***in one round*** *of communications is*

$$\tau = \min_{\boldsymbol{A} \in \mathbb{A}} \left\{ \sum_{\ell \in \mathcal{V}} \operatorname{rank}(\boldsymbol{A}_\ell) \right\} ,$$

where for all $\ell \in \mathcal{V}$

$$\operatorname{rowspace}\left(\left[\frac{\boldsymbol{A}_{\mathcal{N}_{in}(\ell)}}{\boldsymbol{P}_\ell} \right] \right) \supseteq \operatorname{rowspace}(\boldsymbol{T}_\ell) .$$

If the above matrix $\boldsymbol{A} \in \mathbb{A}$ as above does not exist then there is no algorithm that satisfies all requests in one round.

General case

Theorem

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, $|\mathcal{V}| = k$, be the underlying directed graph of a r_0-solvable network defined by the adjacency matrix $\boldsymbol{D}^T$. Then there exists an iterated data exchange protocol with r rounds, for any $r \geq r_0$, and τ transmissions, where

$$\tau = \sum_{i=1}^{r} \left(\min_{\boldsymbol{A}^{(i)} \in (\boldsymbol{D}^{i-1} \otimes \boldsymbol{E}) \cdot \mathbb{A}} \left\{ \sum_{\ell \in \mathcal{V}} \operatorname{rank} \left(\boldsymbol{A}_\ell^{(i)} \right) \right\} \right)$$

for matrices $\boldsymbol{A}^{(i)}$ which are subject to

$$\forall \ell \in \mathcal{V} \; : \; \operatorname{rank} \left(\left[\frac{\left(\operatorname{diag}\left(\boldsymbol{D}^{[\ell]}\right) \otimes \boldsymbol{I}\right) \cdot \boldsymbol{A}^{(i)}}{\Gamma_\ell((\boldsymbol{D}^{i-1} \otimes \boldsymbol{E}) \cdot \mathbb{A})} \right] \right) = \text{max-rank} \left((\operatorname{diag}(\boldsymbol{e}_\ell) \otimes \boldsymbol{I}) \cdot \left(\boldsymbol{D}^i \otimes \boldsymbol{E} \right) \cdot \mathbb{A} \right) .$$

Thanks!

Questions?

I. Kubjas, and V. Skachek, "Data dissemination problem in wireless networks", *Proc. 53rd Annual Allerton Conference on Communication, Control, and Computing*, Allerton, IL, USA, 2015.