Elektriahelad

loeng 5

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V loengu teema

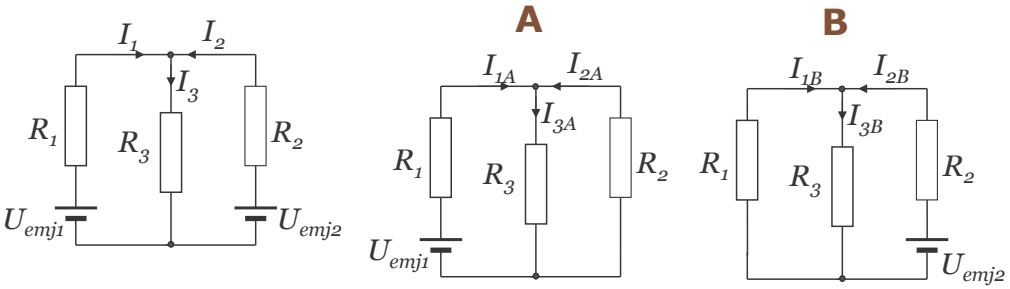
- keerulisemad alalisvooluhelad
  - superpositisiooniprintsiip
  - Kirchoffi reeglid

Superpositsiooni printsiip

Kuidas leida voolud mitme emj. allikaga ahelas?

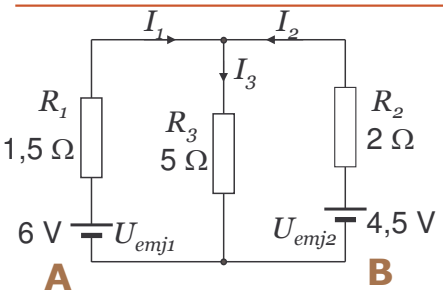
1) leida ahela harudes olevad voolud iga emj. allika jaoks eraldi, lühistades teised emj. allikad
2) liita need voolud kokku

n ahela haru, *m* emj. allikat $I_n = \sum_m I_{nm}$



$I_1 = I_{1A} + I_{1B} \qquad I_2 = I_{2A} + I_{2B} \qquad I_3 = I_{3A} + I_{3B}$

Superpositsiooniprintsiip: näide



A $R = R_1 + R_2 \cdot R_3/(R_2 + R_3)$

$R = 1{,}5\Omega + 10\Omega/7\Omega = 2{,}93\Omega$

$I_{1A} = U_{emj1}/R$ $I_{1A} = 6\text{V}/2{,}93\Omega = 2{,}05\text{A}$

$I_{2A} = I_{1A} \cdot R_3/(R_2 + R_3)$ $I_{2A} = 2{,}05\text{A} \cdot 5/7 = 1{,}46\text{A}$

$I_{3A} = I_{1A} \cdot R_2/(R_2 + R_3)$ $I_{3A} = 2{,}05\text{A} \cdot 2/7 = 0{,}59\text{A}$

B $R = R_2 + R_1 \cdot R_3/(R_1 + R_3)$

$R = 2\Omega + 7{,}5\Omega/6{,}5\Omega = 3{,}15\Omega$

$I_{2B} = U_{emj2}/R$ $I_{2B} = 4{,}5\text{V}/3{,}15\Omega = 1{,}43\text{A}$

$I_{1B} = I_{2B} \cdot R_3/(R_1 + R_3)$ $I_{1B} = 1{,}43\text{A} \cdot 5/6{,}5 = 1{,}1\text{A}$

$I_{3B} = I_{2B} \cdot R_1/(R_1 + R_3)$ $I_{3B} = 1{,}43\text{A} \cdot 1{,}5/6{,}5 = 0{,}33\text{A}$

$I_1 = I_{1A} + I_{1B}$ $I_1 = 2{,}05\text{A} - 1{,}1\text{A} = 0{,}95\text{A}$

$I_2 = I_{2A} + I_{2B}$ $I_2 = -1{,}46\text{A} + 1{,}43\text{A} = 0{,}03\text{A}$

$I_3 = I_{3A} + I_{3B}$ $I_3 = 0{,}59\text{A} + 0{,}33\text{A} = 0{,}92\text{A}$

$I_1 = 0{,}95$ A

$I_2 = 0{,}03$ A

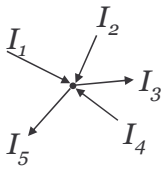
$I_3 = 0{,}92$ A

Kirchoff'i reeglid

Kuidas leida voolud mitme emj. allikaga ahelas?

reegel 1: sõlmpunkti sisenevate ja väljuvate voolude summa on 0

$$\Sigma I = 0$$

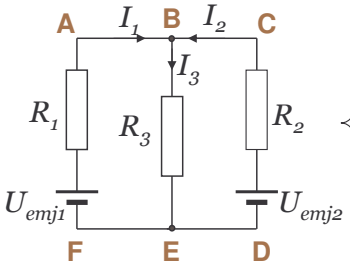


$$I_1 + I_2 + I_3 + I_4 + I_5 = 0$$

$$I_1 + I_2 + I_4 = I_3 + I_5$$

reegel 2: igas suletud vooluringis on pingelangude summa võrdne ringis olevate elektromotoorjõudude summaga

$$\Sigma U_{emj} = \Sigma IR$$



ABEFA:

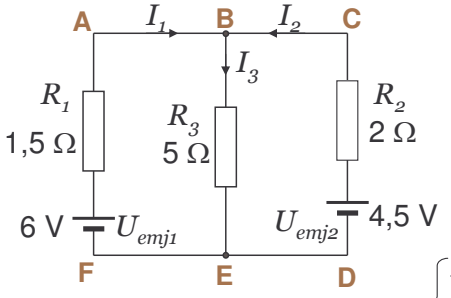
$$U_{emj1} = I_1 \cdot R_1 + I_3 \cdot R_3$$

CBEDC:

$$U_{emj2} = I_2 \cdot R_2 + I_3 \cdot R_3$$

$$I_1 + I_2 = I_3$$

Kirchoff'i reeglid: näide



$I_1 + I_2 = I_3$

ABEFA:

$U_{emj1} = I_1 \cdot R_1 + I_3 \cdot R_3$

$U_{emj1} = I_1 \cdot R_1 + (I_1 + I_2) \cdot R_3$

$6V = 6{,}5\Omega \cdot I_1 + 5\Omega \cdot I_2$ [1]

CBEDC:

$U_{emj2} = I_2 \cdot R_2 + I_3 \cdot R_3$

$U_{emj2} = I_2 \cdot R_2 + (I_1 + I_2) \cdot R_3$

$4{,}5V = 5\Omega \cdot I_1 + 7\Omega \cdot I_2$ [2]

ABCDEFA:

$U_{emj1} - U_{emj2} = I_1 \cdot R_1 - I_2 \cdot R_2$

$1{,}5V = 1{,}5\Omega \cdot I_1 - 2\Omega \cdot I_2$ [3]

$12V = 13\Omega \cdot I_1 + 10\Omega \cdot I_2$ .....[1] x 2

$7{,}5V = 7{,}5\Omega \cdot I_1 - 10\Omega \cdot I_2$ .....[3] x 5

$19{,}5V = 20{,}5\Omega \cdot I_1 \Rightarrow \boxed{I_1 = 0{,}951\ A}$

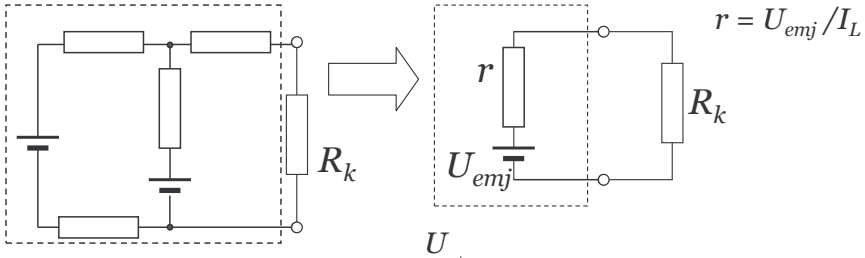
$1{,}5V = 1{,}5\Omega \cdot 0{,}95A - 2\Omega \cdot I_2$

$0{,}075V = -2\Omega \cdot I_2 \Rightarrow \boxed{I_2 = -0{,}037\ A}$

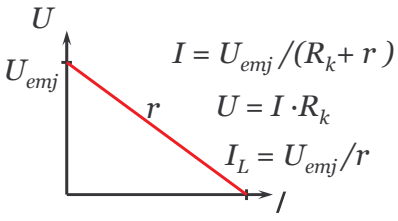
$0{,}951A + 0{,}037A = I_3 \Rightarrow \boxed{I_3 = 0{,}914\ A}$

Thevenin'i teoreem

- Ükskõik millise emj. allikaid ja takisteid sisaldava kahe väljundiga ahela saab esitada ühe lihtsa pingeallikaga emj. U_{emj} ja sisetakistusega r

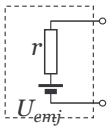


Skeem, kui pingeallikas: pingegeneraator

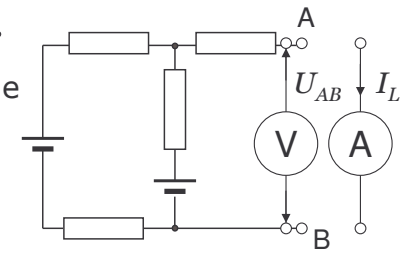


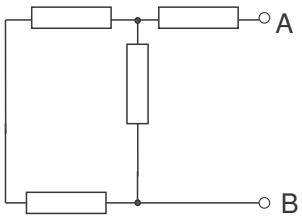
Thevenin'i teoreem: rakendamine

- Kuidas leida U_{emj} ja r?

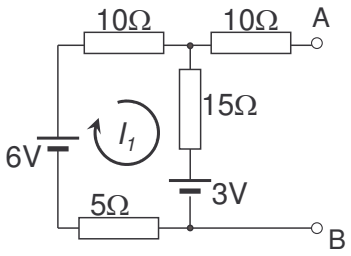


- Eksperimentaalselt
  - mõõta lühisvool ja klemmipinge avatud vooluringi korral
- analüütiliselt
  - arvutada pinge U_{AB} väljundis lahtise vooluringi korral kasutades näit: Kirchoff'i reegleid
  - määrata r asendades kõik sisemised pingeallikad nende sisetakistustega ja leides sellise ahela takistuse





Thevenin'i teoreem: näide

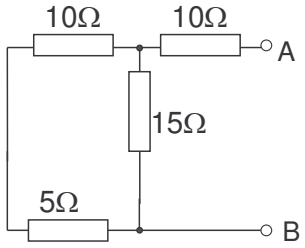


Pinge U_{AB} lahtise vooluringi korral

$6V - 3V = (10\Omega + 15\Omega + 5\Omega)\cdot I_1$

$I_1 = 3V/\ 30\Omega \qquad I_1 = 0{,}1\ A$

$U_{AB} = 3V + 15\Omega\cdot 0{,}1A$ $\boxed{U_{emj} = 4{,}5V}$



sisetakistus r

$r = 10\Omega + 15\Omega\cdot(10\Omega + 5\Omega)/(15\Omega+10\Omega+5\Omega)$

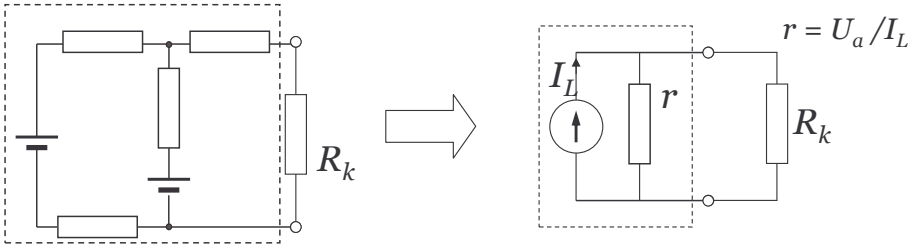
$\boxed{r = 17{,}5\Omega}$

lühisvool I_L

$I_L = 4{,}5V/\ 17{,}5\Omega = 257mA$

Norton'i teoreem

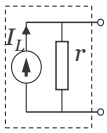
- keerulise ahela saab esitada konstantse voolugeneraatori ja paralleelse sisetakistusega



Vool tekib välisahelast sõltumatult: voolugeneraator

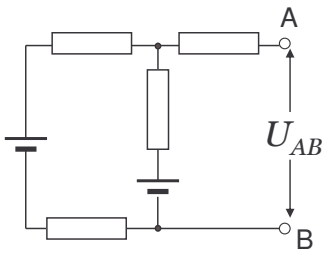
Norton'i teoreem: rakendamine

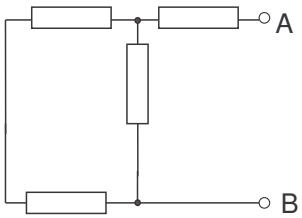
- Kuidas leida I_L ja r?



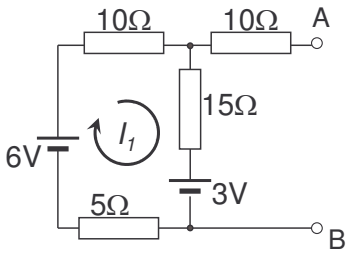
- Eksperimentaalselt
  - nagu Thevenin'i teoreemi korral
- analüütiliselt
  - arvutada pinge U_{AB} väljundis lahtise vooluringi korral
  - määrata r sarnaselt Thevenin'i teoreemiga
  - Arvutada I_L kasutades valemit

$$r = U_{AB} / I_L$$





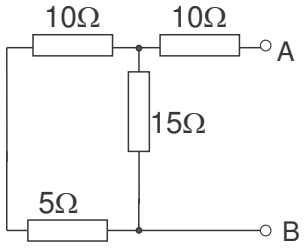
Norton'i teoreem: näide



Pinge U_{AB} lahtise vooluringi korral

I_1 = 0,1 A

U_{AB} = 3V + 15Ω·0,1A U_{AB} = 4,5V



sisetakistus r

r = 10Ω + 15Ω·(10Ω + 5Ω)/(15Ω+10Ω+5Ω)

r = 17,5Ω

lühisvool I_L

I_L = 4,5V/ 17,5Ω I_L = 257mA

praktiliselt samasugune metoodika, kui Thevenini teoreemil