Võrrandite lahendamine unifitseerimisega

- Võrrandeid saab lahendada lihtsustusreeglite korduva rakendamise kaudu.
- Kaks peamist reeglit:
  - asenda võrdus kujul $\tau_1 \to \tau_2 = \tau_3 \to \tau_4$ kahe võrdusega $\tau_1 = \tau_3$ ja $\tau_2 = \tau_4$;
  - olgu võrdus kujul $\alpha = \tau$. Kui $\alpha \in \mathrm{FV}(\tau)$ siis raporteeri viga, vastasel korral asenda kõigis võrdustes α asemel τ.
- Abireeglid:
  - eemalda reeglid kujul $\alpha = \alpha$, $\mathsf{Bool} = \mathsf{Bool}$, jne.;
  - asenda $\tau = \alpha$ võrdusega $\alpha = \tau$;
  - kui leidub võrdus $\tau_1 = \tau_2$, kus peatüübikonstruktorid on erinevad (näit. $\mathsf{Bool} = \alpha_1 \to \alpha_2$), siis raprorteeri viga.

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\frac{}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{}{x^{\alpha_4} \vdash x^{\alpha_5}}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \ \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \ \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6}}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_2\} \\ E_2 &= \{\alpha_4 = \alpha_5\} \end{aligned}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \;\Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \;\Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \;\Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \;\Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_2\} \\ E_2 &= \{\alpha_4 = \alpha_5\} \\ E_3 &= \{\alpha_3 = \alpha_1 \to \alpha_2\} \cup E_1 \\ E_4 &= \{\alpha_6 = \alpha_4 \to \alpha_5\} \cup E_2 \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \;\Rightarrow\; E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \;\Rightarrow\; E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \;\Rightarrow\; E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \;\Rightarrow\; E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \;\Rightarrow\; E_5}
$$

Kitsendused:

$$
\begin{array}{lcl}
E_1 & = & \{\alpha_1 = \alpha_2\} \\
E_2 & = & \{\alpha_4 = \alpha_5\} \\
E_3 & = & \{\alpha_3 = \alpha_1 \rightarrow \alpha_2\} \cup E_1 \\
E_4 & = & \{\alpha_6 = \alpha_4 \rightarrow \alpha_5\} \cup E_2 \\
E_5 & = & \{\alpha_3 = \alpha_6 \rightarrow \alpha_7\} \cup E_3 \cup E_4
\end{array}
$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{&\alpha_1 = \alpha_2,\ \alpha_4 = \alpha_5, \\ &\alpha_3 = \alpha_1 \rightarrow \alpha_2, \\ &\alpha_6 = \alpha_4 \rightarrow \alpha_5, \\ &\alpha_3 = \alpha_6 \rightarrow \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$
\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}
$$

Kitsendused:

$$
\begin{aligned}
E_5 \quad = \quad \{ \quad & \alpha_4 = \alpha_5, \\
& \alpha_3 = \alpha_2 \rightarrow \alpha_2, \\
& \alpha_6 = \alpha_4 \rightarrow \alpha_5, \\
& \alpha_3 = \alpha_6 \rightarrow \alpha_7 \}
\end{aligned}
$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_3 = \alpha_2 \to \alpha_2, \\ & \alpha_6 = \alpha_5 \to \alpha_5, \\ & \alpha_3 = \alpha_6 \to \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_6 = \alpha_5 \to \alpha_5, \\ & \alpha_2 \to \alpha_2 = \alpha_6 \to \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_2 = \alpha_6, \\ & \alpha_6 = \alpha_5 \to \alpha_5, \\ & \alpha_2 = \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$\begin{aligned} E_5 \quad = \quad \{ & \\ & \alpha_2 = \alpha_5 \to \alpha_5, \\ & \alpha_2 = \alpha_7\} \end{aligned}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1}}{\vdash (\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3} \Rightarrow E_3} \qquad \dfrac{\dfrac{x^{\alpha_4} \in \{x^{\alpha_4}\}}{x^{\alpha_4} \vdash x^{\alpha_5} \Rightarrow E_2}}{\vdash (\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6} \Rightarrow E_4}}{\vdash ((\lambda x^{\alpha_1}.x^{\alpha_2})^{\alpha_3}(\lambda x^{\alpha_4}.x^{\alpha_5})^{\alpha_6})^{\alpha_7} \Rightarrow E_5}$$

Kitsendused:

$$E_5 = \{ \; \alpha_7 = \alpha_5 \rightarrow \alpha_5 \; \}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\frac{}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Tüübituletus $\lambda{\rightarrow}$ a'la Curry jaoks

$$\dfrac{\dfrac{}{x^{\alpha_1} \vdash x^{\alpha_2}} \qquad \dfrac{}{x^{\alpha_1} \vdash x^{\alpha_3}}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4}}}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_2\} \\ E_2 &= \{\alpha_1 = \alpha_3\} \end{aligned}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5}}}$$

Kitsendused:

$$\begin{aligned} E_1 &= \{\alpha_1 = \alpha_2\} \\ E_2 &= \{\alpha_1 = \alpha_3\} \\ E_3 &= \{\alpha_2 = \alpha_3 \to \alpha_4\} \cup E_1 \cup E_2 \end{aligned}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{array}{lcl} E_1 & = & \{\alpha_1 = \alpha_2\} \\ E_2 & = & \{\alpha_1 = \alpha_3\} \\ E_3 & = & \{\alpha_2 = \alpha_3 \to \alpha_4\} \cup E_1 \cup E_2 \\ E_4 & = & \{\alpha_5 = \alpha_1 \to \alpha_4\} \cup E_3 \end{array}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad & \{\alpha_1 = \alpha_2,\ \alpha_1 = \alpha_3, \\ & \ \alpha_2 = \alpha_3 \to \alpha_4, \\ & \ \alpha_5 = \alpha_1 \to \alpha_4 \\ & \} \end{aligned}$$

Tüübituletus $\lambda\!\to$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$E_4 = \{ \alpha_2 = \alpha_3,\ \alpha_2 = \alpha_3 \to \alpha_4,\ \alpha_5 = \alpha_2 \to \alpha_4 \}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{array}{lcl} E_4 & = & \{ \\ & & \quad \alpha_3 = \alpha_3 \rightarrow \alpha_4, \\ & & \quad \alpha_5 = \alpha_3 \rightarrow \alpha_4 \\ & & \} \end{array}$$

Tüübituletus $\lambda\!\rightarrow$ a'la Curry jaoks

$$\dfrac{\dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_2} \Rightarrow E_1} \qquad \dfrac{x^{\alpha_1} \in \{x^{\alpha_1}\}}{x^{\alpha_1} \vdash x^{\alpha_3} \Rightarrow E_2}}{\dfrac{x^{\alpha_1} \vdash (x^{\alpha_2} x^{\alpha_3})^{\alpha_4} \Rightarrow E_3}{\vdash (\lambda x^{\alpha_1}.(x^{\alpha_2} x^{\alpha_3})^{\alpha_4})^{\alpha_5} \Rightarrow E_4}}$$

Kitsendused:

$$\begin{aligned} E_4 \quad = \quad \{ & \\ & \alpha_3 = \alpha_3 \rightarrow \alpha_4, \\ & \alpha_5 = \alpha_3 \rightarrow \alpha_4 \\ \} & \end{aligned}$$

Error!

Tüübituletus

- Väga suure tüübiga term:

$$
\begin{array}{l}
\text{let } pair = \lambda xyz.z\ x\ y \text{ in} \\
\quad \text{let } x_1 = \lambda y.pair\ y\ y \text{ in} \\
\quad\quad \text{let } x_2 = \lambda y.x_1(x_1\ y) \text{ in} \\
\quad\quad\quad \text{let } x_3 = \lambda y.x_2(x_2\ y) \text{ in} \\
\quad\quad\quad\quad \text{let } x_4 = \lambda y.x_3(x_3\ y) \text{ in} \\
\quad\quad\quad\quad\quad \text{let } x_5 = \lambda y.x_4(x_4\ y) \text{ in} \\
\quad\quad\quad\quad\quad\quad x_5(\lambda y.y)
\end{array}
$$

Taust: Curry-Howard vastavus

- C.-H.: loogikad ja tüübisüsteemid on samasuguse struktuuriga.

- Nägime: saame teadmisi ja oskusi üle kanda.
- Nägime: saame kasutada samu vahendeid.

- Kordamine: mida arvutab järgneva väite tõestus?

$$\forall x, y \in \mathbb{N}.\ x > y \to \exists z \in \mathbb{N}.\ x = y + z$$

- Täna:
  - ülekandmine teises suunas;
  - lõpetame range tõestuse ja programmeerimise eristamise;
  - ehk, vaatame sõltuvate tüüpidega programmeerimist.

Probleem

Tahame listi indekseerida funktsiooniga: `Nat → List a → a`

Probleem:

- List ei pruugi sisaldada `n`-ndat elementi!
- List võib olla täiesti tühi! Siis ei saa isegi valet elementi tagastada.

Võime teha nii: `Nat → List a → Maybe a`.

Töötab, kuid pole optimaalne juhul, kui listis on piisavalt elemente.

Listi tüüp võiks sisaldada infot tema pikkuse kohta.

Pikkusega listid

- Tavalised listid (uue süntaksi abil)

```
data List : Type → Type where
    Nil : List a
    (::) : a → List a → List a
```

- Paneme pikkused külge

```
data Vec : Nat → Type → Type where
    Nil : Vec 0 a
    (::) : a → Vec n a → Vec (1+n) a

test5 : Vec 3 Int
test5 = [1,2,3]
```

- selekteerimine

```
nth : (n:Nat) → Vec (1+n+m) a → a
nth 0     (x :: xs) = x
nth (S k) (x :: xs) = nth k xs
```

- konkateneerimine

```
concatVec : Vec n a → Vec m a → Vec (n+m) a
concatVec []       ys = ys
concatVec (x :: y) ys = x :: concatVec y ys
```

- Vektorite map

```
Functor (Vec n) where
 -- map : (a → b) → Vec n a → Vec n b
    map f []       = []
    map f (x :: y) = f x :: map f y
```

NB! Definitsioon sama mis listidel!

Funktsiooni `nth n` tüüp `Vec (1+n+m) a → a` sõltub `n`-i väärtusest.

- `nth 0 : Vec (1+m) a → a`
- `nth 2 : Vec (3+m) a → a`

Sõltuvad tüübid võimaldavad arvutamist ja selle tõestamist koos teha.

Aga see ei tööta alati nii lihtsalt ...

- Ümberpööramine listidel

```
revList : List a → List a
revList [] = []
revList (x :: ys) = (revList ys) ++ [x]
```

- Saame tõestada lihtsa teoreemi:

```
revrev :  (xs:List a) → revList (revList xs) = xs
```

- Ümberpööramine vektoritel

```
idVec : {n:Nat} → Vec (n + 1) a → Vec (1 + n) a
idVec {n = 0}      xs        = xs
idVec {n = (S k)} (x :: ys) = x :: idVec ys

revVec : {n:Nat} → Vec n a → Vec n a
revVec {n = 0   } []        = []
revVec {n = S k} (x :: ys) =
    idVec ((revVec ys) `concatVec` [x])
```

- Ilma `idVec`-ta tuleb veateade. Idris ei tea, et `k + 1` on `S k` ehk `1+k`.
- `idVec` asemel võib kasutada **`rewrite`** ... **`in`** ... väite `1+k = k+1` tõestusega.

Mitte ainult Vec!

- `Vec` kasutatakse palju aga selles pole midagi sisuliselt erilist.
- Näiteks, saame teha ülemise rajaga listi.

```
maks : Nat → Nat → Nat
maks 0 m = m
maks n 0 = n
maks (S k) (S j) = S (maks k j)

data BList : Nat → Type where
    Nil : BList 0
    (::) : (n:Nat) → BList m → BList (maks n m)
```

- Funkstiooniga saame ka teha tüüpe, mis sõltuvad väärtusest:

```
Arv : Bool → Type
Arv False = List Int -- False: mittedeterministlik
Arv True  = Int      -- True: deterministlik
```

- … aga siis Idris tea, et kui List Int ≡ Arv ?b siis ?b ≡ False.