Privacy-preserving Data Mining

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Motivations

- Sophisticated statistical analysis methods require data, a lot of it.
- Often those that need statistical analyses are not the ones that have the data.
- The data owners have nothing against those other guys doing their statistics, if only they wouldn't have to disclose their data completely.
- But the statisticians need the data to run the analsysis! What can they do?

Solutions

1. Magic and wizardry

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2. Armed assault or data theft

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4. PPDM

Two Flavors of PPDM

- Secure multi-party computation: compute $f(D_1, D_2, \ldots, D_n)$ so that D_i are provided by different parties but not disclosed.
- Database randomization: modify the database D to another form D' so that it preserves privacy but still allows to run meaningful analyses.
- The first flavor usually assumes distributed data, the second assumes centralized.

Example: Secure sum

- Problem: parties $P_1, \ldots, P_n$ have *secret values* $u_1, \ldots, u_n$ and wish to compute the sum of these values without disclosing the values themselves.
- Assume: $u_i \in \mathbb{N}$, $\sum_i u_i =: u \in [0, m-1]$
- Solution:
  1. P_1 generates a random r uniformly from $[0, m-1]$.
  2. P_1 passes $r + u_1$ to P_2 (all additions modulo m).
  3. P_2 passes $r + u_1 + u_2$ to P_3
  4. $\ldots$
  5. P_n passes $r + \sum_i u_i$ to P_1.
  6. P_1 subtracts r from the received value getting u.

Example: Secure sum

- The algorithm is secure in the sense that it can be simulated by an *ideal scenario* where every party sends its value to a single trusted server that secretly computes the sum.
- But:
  - The channels between P_i and P_{i+1} must be secure.
  - The parties must be *semi-honest*.

Adversaries

- *Semi-honest* (also *passive*): correctly follow the protocol but might try to use the received messages to crack the protocol.
- *Malicious* (also *active*): might violate the protocol.
- Usually we consider the semi-honest adversaries because it's so much harder to fight the evil guys.
- Usually the adversaries are assumed to have limited computational power (e.g. polynomial computations only).

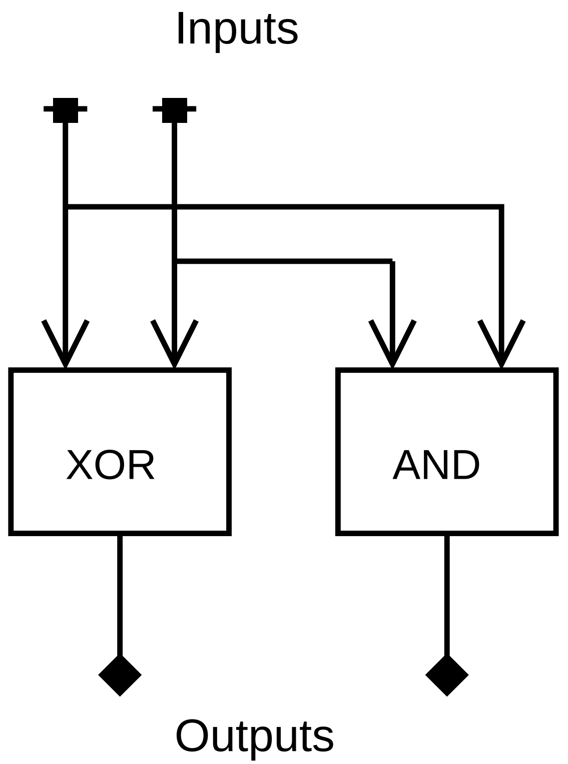
1-out-of-2 Oblivious Transfer

- Bob has two values, y_1 and y_2. Alice wishes to get y_k. However, Alice does not want Bob to know which y_k it wants.
- *1-out-of-2 oblivious transfer (OT) protocol* allows Alice to get y_k from Bob not disclosing which y_k it was that it got.
- Here's a simple example of such a protocol:
  1. Alice sends to Bob two public keys (c_1, c_2) and a proof that she knows the private key for only one of them.
  2. Bob sends to Alice the values y_i encrypted with the corresponding keys: $E_{c_i}(y_i)$.

Secure Function Computation

- Suppose Alice and Bob have pieces of data x and y correspondingly. They want to find out $f(x, y)$ without disclosing their values to each other.
- This can be done using a protocol by Yao (1986).
  - Express f as a combinatorial circuit (e.g. boolean circuit).
  - Bob *garbles* the circuit and his input values, and sends everything to Alice.
  - Alice plugs in her values and computes the function in this "garbled" representation.
  - The results are "ungarbled".

Function as a Circuit



A boolean circuit representation of a function that adds two bits (a half-adder).

Garbling

- For each wire i in the circuit Bob generates random numbers $w_i^{(0)}$ and $w_i^{(1)}$.
- For each gate Bob constructs a table that maps garbled input values to the encryptions of the garbled output values. E.g. for the AND gate:

Wire i	Wire j	Encrypted wire k
$w_i^{(0)}$	$w_j^{(0)}$	$E_{w_i^{(0)},w_j^{(0)}}(w_k^{(0)})$
$w_i^{(0)}$	$w_j^{(1)}$	$E_{w_i^{(0)},w_j^{(1)}}(w_k^{(0)})$
$w_i^{(1)}$	$w_j^{(0)}$	$E_{w_i^{(1)},w_j^{(0)}}(w_k^{(0)})$
$w_i^{(1)}$	$w_j^{(1)}$	$E_{w_i^{(1)},w_j^{(1)}}(w_k^{(1)})$

Garbling

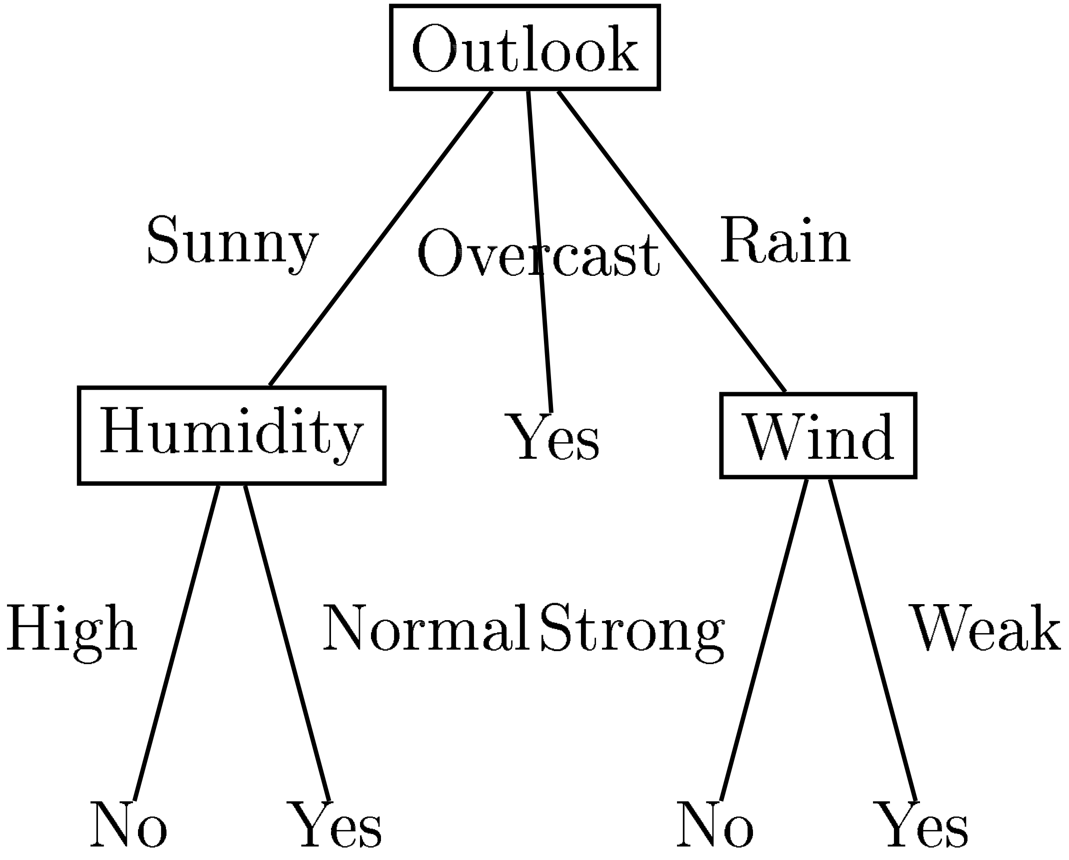
- Bob sends the gate tables and the garbled versions of his input wires to Alice.
- Alice uses 1-out-of-2 OT to obtain the garbled versions of her input wires.
- Alice computes the circuit.
- Bob translates the computed garbled values to the real ones.

Data-mining

- Every data mining algorithm can be descried as a function of the data $f(D)$.
- However, the circuit representation of such a function and the size of its inputs is usually enormous, thus we can't use Yao's algorithm directly and have to make up special optimizations for each data-mining algorithm separately.
- A simple example: calculating the mean of a large set of numbers.
- A more interesting example: ID3.

ID3

- ID3 is an algorithm by Quinlan that constructs *decision trees* from data.



ID3: The algorithm

Input: a database D. Output: a decision tree.

1. If D is empty, return an empty tree.
2. If all the transactions in D have the same class value, return a leaf node indicating this class value.
3. Otherwise, determine the attribute A that *best* classifies the transactions in D.
4. Create a new tree node N, that splits the data on the values of this attribute.
5. For each value a_i of the attribute A attach to N a subtree returned by $ID3(D[A = a_i])$.

Entropy and Mutual Information

- How to find the attribute A that best predicts the class C at node N?
- Use the attribute A that has maximal *information gain*, which is its *mutual information* with C:

$$\mathrm{Gain}(A) = I_D(A;C) = H_D(C) - H_D(C|A)$$

$$H(C) = -\sum_{c \in C} P(c) \log P(c)$$

$$H(C|A=a) = -\sum_{c \in C} P(c|A=a) \log P(A=a)$$

$$H(C|A) = \sum_{a \in A} P(a) H(C|A=a)$$

Secure ID3

- Note that in order to secure ID3 we must secure only the step where the attribute A is computed by maximizing gain.
- Maximizing gain is same as minimizing $H_D(C|A)$.

$$H_D(C|A) = \sum_i \frac{|D([A = a_i])|}{|D|} \times$$

$$\times \sum_k -\frac{|D([A = a_i, C = c_k])|}{|D([A = a_i])|} \log \frac{|D([A = a_i, C = c_k])|}{|D([A = a_i])|}$$

Secure ID3

$$H_D(C|A) = \frac{1}{|D|}\left(-\sum_i \sum_k |D(a_i, c_k)| \log |D(a_i, c_k)| + \right.$$

$$\left. + \sum_k |D(a_i)| \log |D(a_i)| \right)$$

The constant $\frac{1}{|D|}$ can be dropped and we are left with the sum of the form

$$\sum x \log x$$

Secure ID3

- We are left with the sum of the form $\sum x \log x$ where each x can be computed as a sum of two values, one of which is known to Alice, and another—to Bob:

$$\sum (a_i + b_i) \log(a_i + b_i)$$

- We can apply the Yao's algorithm now to compute this sum securely.
- Actually one additional optimization is used in order to make this computation practical. It is a bit too involved so we skip it and say that we are done.

Randomization

- Example situation: the clients submit their age to the server because the server needs to find out their average age.
- The clients could as well add a zero-mean random variable to their age before submitting. If the number of clients is large, the server will get more-or-less the same average.
- More generally: the clients have values x_i from a distribution F_X. They add a shift y_i from distribution F_Y and submit $z_i = x_i + y_i$ to the server.
- The server now wants to restore F_X knowing F_Z and F_Y.

Randomization

Algorithm for restoring F_X proposed by Agrawal:

- Denote by f_X the density of F_X, by f_Y - the density of F_Y.
- Start with initial guess $f_X^0 = U$ and iterate the following step until convergence:

$$f_X^{j+1}(a) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_Y(z_i - a) f_X^j(a)}{\int_{-\infty}^{\infty} f_Y(z_i - z) f_X^j(z) dz}$$

Privacy breaches

- The major difference of the randomization approach is that it always leaks some data.
- This brings the additional requirements to measure privacy and the notion of *privacy breach*.
- The most simple way to measure privacy is using the a-posteriory confidence intervals on x_i-s.
- But:
  - Suppose we add a uniform random number from $[-50, 50]$ to the submitted age. The 100% confidence interval is 100 units wide.
  - However, if the client submits the value 125 we may state with 90% probability that his age is near 75.

Privacy breaches

- Another way is to use information-theoretical terms like mutual information.
- Yet another possibility is to specify explicitly the probabilities of possible privacy breaches.
- The different choices of measures of privacy and the privacy breaches make the randomization approach a bit too ad-hoc.

Summary

- PPDM is a way for performing data-mining securely.
- It is a viable alternative to methods based on magic or armed assault. The trusted server solution still has certain unbeaten advantages however.
- PPDM comes in 2 flavors: secure multi-party computation and randomization. The flavors are often associated with their cryptographic and statistical solutions correspondingly.
- For both flavors there exist effective algorithms for decision-tree induction, association rule mining, clustering and more.

Questions?