Contents

Strategic game

Players	1	2	$\cdots$	i	$\cdots$	N
Actions	A_1	A_2	$\cdots$	A_i	$\cdots$	A_N
Profiles	$(a_1,$	$a_2,$	$\ldots,$	$a_i,$	$\ldots,$	$a_N)$
Preferences	$\precsim_1$	$\precsim_2$	$\cdots$	$\precsim_i$	$\cdots$	$\precsim_N$

Actions are taken only once and simultaneously. Nash equilibrium: A profile $a* = (a_1*, \ldots, a_N*)$ where each a_i* is among the most preferable actions of the player i.

Examples

A play with two equilibria

	Chicken	Dare
Chicken	3,3	1,4
Dare	4,1	0,0

A play with no equilibria

	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

Strictly competitive games

- Utility function u_i for player i:

$$u_i(a) \leq u_i(b) \text{ if and only if } a \precsim_i b$$

- In strictly competitive games there are two players with

$$u_2 = -u_1$$

  Also called *zerosum* games.

- Nash equilibrium (if exists) determines the (highest?) payoff of the players.

Bayesian games

- Imperfect information about the ”state of the nature”
  - Each player has an *initial belief*, how probable the differents states are, and also receives a *signal* about the state.
  - The preference relation $\precsim_i$ now also consideres (the probability of) the state of the nature.

Example:

Player 1, Player 2

Possible states of nature: $\omega_1, \omega_2, \omega_3$

Prior beliefs: Both players consider all the states equally probable, that is, 1/3.

Two signals for each player:

- Player 1:

$$\begin{aligned} \omega_1, \omega_2 &\rightarrow \text{signal } 1 \\ \omega_3 &\rightarrow \text{signal } 2 \end{aligned}$$

- Player 2:

$$\begin{aligned} \omega_1 &\rightarrow \text{signal } 1 \\ \omega_2, \omega_3 &\rightarrow \text{signal } 2 \end{aligned}$$

Mixed strategy

- Introduction:
  - Players choices are nondeterministic.
  - The set of actions is replaced by probabilistic distributions on the actions.
  - Players compare the profiles of probabilistic distributions on actions instead of the profiles of actions.
- A mixed strategy Nash equilibrium of a strategic game is a Nash equilibrium of its mixed extension

Interpretations

- Mixed strategy is a choice of a player (so what?)
- Mixed strategy Nash equilibrium is a steady state, reached by observing actions of other players long enough.
- Mixed strategy models unconcious reasons of the players choises. Example: Bach or Stravinsky in different moods.
- Perturbed game: small random variations effect the payoffs the players. Reaches a mixed strategy Nash equilibrium over time.

Mixed strategy Nash equilibrium

- Every finite strategic game has a mixed strategy Nash equilibrium.
  In such an equilibrium $\alpha*$, any possible action of player i
  in α_i*, is a best response.
  All the actions yield the same payof.
- The set of Nash equilibria of a strategic game is a subset of its set of mixed strategy Nash equilibria.
- Example

Correlated equilibrium

- Compare to Bayesian games
- In Corretalet games, the signals are not private and independent
- Information partition
- Example: Bach or Stravinsky
- Correlated equilibria contains the set of mixed strategy Nash equilibria

Evolutionary equilibrium

- Players actions are deterministic by the forces of evolution
- Each player gets its action from the same set B by inheritance or mutation
- The utility function corresponds to the ability to survive.
- Equilibrium: when all the players take the same action, no mutant can invade the population.