

Algebra I 4. praktikumi vastused ja näpunäited:
kompleksarvud

1. a) $x = \frac{9}{5}, y = \frac{2}{5}$; b) $x = -\frac{2}{3}, y = \frac{5}{9}$; c) $x = 9, y = \frac{8}{3}$; d) $x = \frac{9}{17}, y = \frac{22}{17}$.
2. a) $\frac{1+2i}{5}$; b) $\frac{-4-3i}{5}$; c) $\frac{-3+4i}{5}$; d) $-1 - 2i$; e) $\frac{13-i}{2}$; f) $14 - 5i$; g) $-2i$.
3. a) $-8 - i$; b) $7 - \sqrt{6} + (3\sqrt{3} + 4\sqrt{2})i$; c) 4 ; d) $42 + 41i$.
5. Võib küll: $i^2 + i^2 = -2$.
8. Juhtudel, kus $a = \pm b$ ja $a \neq 0$.
9. Kompleksarvule z vastava komplekstasandi punkti a) kaugus nullpunktist; b) projektsiooni pikkus reaalteljel; c) projektsiooni pikkus imaginaarteljel.
12. Juhul, kui z_1 ja z_2 argumendid: a) on võrdsed; b) erinevad π radiaani võrra.
13. a) $-i$; c) 1 ; e) $-i$;
 b) $i^n = \begin{cases} 1, & \text{kui } n \equiv 0 \pmod{4} \\ i, & \text{kui } n \equiv 1 \pmod{4} \\ -1, & \text{kui } n \equiv 2 \pmod{4} \\ -i, & \text{kui } n \equiv 3 \pmod{4} \end{cases}$; d) $\sum_{k=1}^n i^k = \begin{cases} 1, & \text{kui } n \equiv 0 \pmod{4} \\ 0, & \text{kui } n \equiv 1 \pmod{4} \\ -i, & \text{kui } n \equiv 2 \pmod{4} \\ 1 - i, & \text{kui } n \equiv 3 \pmod{4} \end{cases}$.
14. a) 3 ; b) $2i$; c) $-1 + \sqrt{3}i$; d) $1 + i$; e) $2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$; f) 0 ; g) i ; h) $5 - 12i$.
15. a) $32(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})$; b) 2^{1006} ; c) $\frac{i}{2^{15}}$; d) $\frac{\sqrt{2}}{2^8}(\cos \frac{23\pi}{12} + i \sin \frac{23\pi}{12})$;
 e) $(\frac{2}{3})^n(\cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3})$.
16. a) $-6 + 6\sqrt{3}i$; b) -12 ; c) 35 ; d) i ; e) $\cos 190^\circ + i \sin 190^\circ$; f) $\frac{5}{6} + \frac{5}{6}\sqrt{3}i$.
17. a) $\pm\sqrt{\frac{3}{2}} \mp \frac{i}{\sqrt{2}}$; b) $\pm(1 + i)$; c) $\pm(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i), \pm(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i)$;
 d) $\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}, \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}, \cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12}$;
 e) $1 + i, \sqrt{2}(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}), \sqrt{2}(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12})$;
 f) $\frac{\sqrt{3}}{2} + \frac{i}{2}, -\frac{\sqrt{3}}{2} + \frac{i}{2}, -i$;
 g) $\sqrt[4]{2}(\cos \frac{5\pi}{24} + i \sin \frac{5\pi}{24}), \sqrt[4]{2}(\cos \frac{13\pi}{24} + i \sin \frac{13\pi}{24}), \sqrt[4]{2}(\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}),$
 $\sqrt[4]{2}(\cos \frac{29\pi}{24} + i \sin \frac{29\pi}{24}), \sqrt[4]{2}(\cos \frac{37\pi}{24} + i \sin \frac{37\pi}{24}), \sqrt[4]{2}(\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8})$;
 h) $\sqrt[6]{2}(\cos \frac{\pi}{18} + i \sin \frac{\pi}{18}), \sqrt[6]{2}(\cos \frac{7\pi}{18} + i \sin \frac{7\pi}{18}), \sqrt[6]{2}(\cos \frac{13\pi}{18} + i \sin \frac{13\pi}{18}),$
 $\sqrt[6]{2}(\cos \frac{19\pi}{18} + i \sin \frac{19\pi}{18}), \sqrt[6]{2}(\cos \frac{25\pi}{18} + i \sin \frac{25\pi}{18}), \sqrt[6]{2}(\cos \frac{31\pi}{18} + i \sin \frac{31\pi}{18})$;
 i) $\pm(\sqrt{3} + i), \pm(-1 + \sqrt{3}i)$;
 j) $\frac{1}{\sqrt[6]{2}}(\cos \frac{\pi}{36} + i \sin \frac{\pi}{36}), \frac{1}{\sqrt[6]{2}}(\cos \frac{25\pi}{36} + i \sin \frac{25\pi}{36}), \frac{1}{\sqrt[6]{2}}(\cos \frac{49\pi}{36} + i \sin \frac{49\pi}{36})$;
 k) $\frac{1}{\sqrt[8]{2}}(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}), \frac{1}{\sqrt[8]{2}}(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}), \frac{1}{\sqrt[8]{2}}(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12}),$
 $\frac{1}{\sqrt[8]{2}}(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12})$.

18. a) $i, \pm \frac{\sqrt{3}}{2} - \frac{i}{2};$

b) $\sqrt[16]{2}(\cos \frac{7\pi}{32} + i \sin \frac{7\pi}{32}), \sqrt[16]{2}(\cos \frac{15\pi}{32} + i \sin \frac{15\pi}{32}), \sqrt[16]{2}(\cos \frac{23\pi}{32} + i \sin \frac{23\pi}{32}),$
 $\sqrt[16]{2}(\cos \frac{31\pi}{32} + i \sin \frac{31\pi}{32}), \sqrt[16]{2}(\cos \frac{39\pi}{32} + i \sin \frac{39\pi}{32}), \sqrt[16]{2}(\cos \frac{47\pi}{32} + i \sin \frac{47\pi}{32}),$
 $\sqrt[16]{2}(\cos \frac{55\pi}{32} + i \sin \frac{55\pi}{32}), \sqrt[16]{2}(\cos \frac{63\pi}{32} + i \sin \frac{63\pi}{32});$

c) $0, \pm 1, \pm i;$

d) $\sqrt[4]{13}(\cos \frac{\pi + \arctan \frac{-3}{2}}{2} + i \sin \frac{\pi + \arctan \frac{-3}{2}}{2}), \sqrt[4]{13}(\cos \frac{3\pi + \arctan \frac{-3}{2}}{2} + i \sin \frac{3\pi + \arctan \frac{-3}{2}}{2});$

e) $0, \cos \frac{2k\pi}{n+1} + i \sin \frac{2k\pi}{n+1}, k = 0, \dots, n,$ v.a. juht $n = 3,$ kus sobivad
 $r(\cos \frac{2k\pi}{n+1} + i \sin \frac{2k\pi}{n+1}), k = 0, \dots, n, r \geq 0.$

19. a) $x = \frac{169-117i}{50}, y = \frac{-304-78i}{50};$ b) $x = \frac{138+59i}{53}, y = \frac{-54+23i}{53}.$

24. Summa on 0, korrutis on $(-1)^{n-1}.$

25. Summa on 0, v.a. juhul, kui $z = 1,$ kui see on $\frac{n(n-1)}{2}.$

26. $\sin nx = \sum_{k=0}^n C_n^k \cos^k x \sin^{n-k} x \sin(\frac{1}{2}(n-k)\pi),$
 $\cos nx = \sum_{k=0}^n C_n^k \cos^k x \sin^{n-k} x \cos(\frac{1}{2}(n-k)\pi).$

27. $\sum_{k=1}^n \sin kx = \frac{\cos \frac{x}{2} - \cos(n+\frac{1}{2})x}{2 \sin \frac{x}{2}}.$

29. a) $2 \pm 2i;$ b) $1 \pm 2\sqrt{2}i;$ c) $3 \pm \sqrt{17} \cos(\pi + \arctan \frac{-8}{15}) + (2 \pm \sqrt{17} \sin(\pi + \arctan \frac{-8}{15}))i.$