

Algebra I 8. praktikumi vastused ja näpunäited:
polünoomid ja jäägiga jagamine

1. a) $q(x) = 5x^2 - 15x + 34, r(x) = -72x - 62$;
 b) $q(x) = 0, r(x) = f(x)$;
 c) $q(x) = 2x^2 + 3x + 11, r(x) = 25x - 5$.
2. Jagatis on $q(x) = x^2 + 2$, ringis $\mathbb{Z}_3[X]$ on $r(x) = 0$, ringis $\mathbb{Z}_5[X]$ on $r(x) = 3x + 2$, ringis $\mathbb{Q}[X]$ on $r(x) = 3x - 3$.
3. a) $a = \sqrt{2}, q = 1$;
 b) $a = 3, q = -8$;
 c) $(a, p, q) = (\pm\sqrt{c}, \pm\sqrt{c}(c-2), c-1), c \in [0, \infty)$;
 d) kui $A := -\frac{1}{3} \left(\sqrt[3]{\frac{189+3\sqrt{3873}}{2}} + \sqrt[3]{\frac{189-3\sqrt{3873}}{2}} \right)$, siis $(a, q) = (A, A^2 - 1)$.
4. a) Jah; b) ei, c) ei.
5. Taandumatud on $x, \bar{2}x, x + \bar{1}, x + \bar{2}, \bar{2}x + \bar{1}, \bar{2}x + \bar{2}, x^2 + \bar{1}, x^2 + x + \bar{2}, x^2 + \bar{2}x + \bar{2}, \bar{2}x^2 + \bar{2}, \bar{2}x^2 + x + \bar{1}, \bar{2}x^2 + \bar{2}x + \bar{1}$.
8. a) $\text{SÜT}(f, g) = x + 1, \text{VÜK}(f, g) = x^6 + x^5 - 4x^4 - 5x^3 + 2x^2 + 4x + 1$;
 b) $\text{SÜT}(f, g) = x^2 + 1, \text{VÜK}(f, g) = 3x^8 - 28x^6 + 24x^5 + 49x^4 - 77x^3 + 21x^2 + 49x - 49$;
 c) $\text{SÜT}(f, g) = 1, \text{VÜK}(f, g) = x^7 + 2x^6 + 3x^5 + 10x^4 + 6x^3 + 6x^2 + 10x + 4$.
9. a) $\text{SÜT}(f, g) = x^2 - 2 = -(x + 1)f + (x + 2)g$;
 b) $\text{SÜT}(f, g) = 1 = xf + (-3x^2 - x + 1)g$;
 c) $\text{SÜT}(f, g) = 1 = -(\frac{1}{2}x^2 - \frac{3}{2})f + (\frac{1}{2}x^4 - x^2 - 1)g$.
10. a) $1 = (4 - 3x)f + (3x^2 + 2x + 1)g$;
 b) $1 = (\frac{1}{16}x^2 - \frac{3}{8}x + \frac{9}{16})f + (-\frac{1}{16}x^3 + \frac{3}{16}x^2 + \frac{1}{4})g$;
 c) $1 = (-\frac{16}{3}x^2 + \frac{37}{3}x + \frac{26}{3})f + (\frac{16}{3}x^3 - \frac{53}{3}x^2 - \frac{37}{3}x - \frac{23}{3})g$.
11. Nõutud polünoom on a) $x^3 - 3x + 3$; b) $4x^4 - 27x^3 + 6x^2 - 65x + 24$; c) x^2 .
12. a) Üle \mathbb{Z}_3 on $\text{SÜT}(f, g) = x + \bar{2}$ ja $\text{VÜK}(f, g) = x^4 + x^2 + \bar{1}$, üle \mathbb{Z}_5 on $\text{SÜT}(f, g) = \bar{1}$ ja $\text{VÜK}(f, g) = x^5 + \bar{2}x^4 + \bar{4}x^3 + \bar{4}x + \bar{3}$ ning üle \mathbb{Q} on $\text{SÜT}(f, g) = 1$ ja $\text{VÜK}(f, g) = x^5 + 2x^4 + 4x^3 + 5x^2 + 4x + 3$;
 b) üle \mathbb{Z}_3 on $\text{SÜT}(f, g) = x + \bar{1}$ ja $\text{VÜK}(f, g) = x^6 + \bar{2}x^2 + \bar{2}x + 2$, üle \mathbb{Z}_5 on $\text{SÜT}(f, g) = x^2 + \bar{3}x + \bar{2}$ ja $\text{VÜK}(f, g) = x^5 + \bar{3}x^4 + x^3 + \bar{3}x^2 + x + \bar{2}$ ning üle \mathbb{Q} on $\text{SÜT}(f, g) = x + 1$ ja $\text{VÜK}(f, g) = x^6 - 3x^4 + 2x^2 - x - 1$;
 c) üle \mathbb{Z}_3 on $\text{SÜT}(f, g) = \bar{1}$ ja $\text{VÜK}(f, g) = x^7 + \bar{2}x^6 + x^5 + \bar{2}x^4 + \bar{2}x^3 + \bar{2}x + \bar{1}$, üle \mathbb{Z}_5 on $\text{SÜT}(f, g) = x + \bar{3}$ ja $\text{VÜK}(f, g) = x^6 + x^5 + x^2 + x + \bar{1}$ ning üle \mathbb{Q} on $\text{SÜT}(f, g) = 1$ ja $\text{VÜK}(f, g) = x^7 - x^6 - 2x^5 + 5x^4 - 4x^3 + 2x - 2$.

13. a) $\text{SÜT}(f, g) = x^2 + x + \bar{1} = (x + \bar{1})f + x^2g$;
b) $\text{SÜT}(f, g) = x + \bar{1} = xf + (x^2 + \bar{1})g$;
c) $\text{SÜT}(f, g) = \bar{1} = (x^3 + x)f + (x^4 + x + \bar{1})g$.
16. a) $\text{SÜT}(975, 645) = 15$, $\text{VÜK}(975, 645) = 41925$;
b) $\text{SÜT}(975, 645) = \text{VÜK}(975, 645) = 1$.
17. a) $\text{SÜT}(3, 1 + \sqrt{-5}) = 1$, $\text{VÜK}(3, 1 + \sqrt{-5})$ ei ole olemas;
b) ei $\text{SÜT}(6, 3 + 3\sqrt{-5})$ ega $\text{VÜK}(6, 3 + 3\sqrt{-5})$ ei ole olemas;
c) $\text{SÜT}(5, 1 + \sqrt{-5}) = 1$, $\text{VÜK}(5, 1 + \sqrt{-5}) = 5 + 5\sqrt{-5}$.