Fall 2016

Combinatorics.

Problem set 2: Counting II.

1. (MN 3.3.9) How many k-element subsets of $\{1, 2, ..., n\}$ exist containing no two consecutive numbers?

2. (MN 3.5.2) Using Fact 3.5.4, prove that for all $n \geq 1$

$$\left(1+\frac{1}{n}\right)^{n} \leq e \leq \left(1+\frac{1}{n}\right)^{n+1}.$$

3. (MN 3.6.1). Prove the estimate $\binom{n}{k} \leq (en/k)^k$ directly from Theorem 3.5.5.

4. (MN 3.7.3) (Sieve of Eratosthenes) How many numbers are left in the set 1,2, . . . ,1000 after all multiples of 2, 3, 5, and 7 are crossed out?

5. (MN 3.7.6) How many ways are there to arrange 4 Americans, 3 Russians, and 5 Chinese into a queue, in such a way that no nationality forms a single consecutive block?

6. (MN 3.8.6) How many permutations of the numbers 1, 2, . . . , 10 exist that map no even number to itself?

7. (MN 3.8.12) For a given natural number N, determine the probability that two numbers $m, n \in \{1, 2, ..., N\}$ chosen independently at random are relatively prime.

8. (MN 3.8.14) How many ways are there to seat n married couples at a round table with $2n$ chairs in such a way that the couples never sit next to each other?

9*. ($\sim$MN 3.7.5) How many orderings of the letters A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P are there such that we cannot obtain any of the words BAD, DEAF, APE by crossing out some letters? What if we also forbid BEAMING?