Fall 2016

Combinatorics.

Problem set 3: Graphs I.

1. (MN 4.1.4) Show that a graph G with n vertices is asymmetric (its only automorphism is the identity mapping) if and only if $n!$ distinct graphs on the set $V(G)$ are isomorphic to G.

2. (MN 4.1.6) How many graphs on the vertex set $\{1, 2, ..., 2n\}$ are isomorphic to the graph consisting of n vertex-disjoint edges (i.e. with edge set $\{\{1, 2\}, \{3, 4\}, ..., \{2n - 1, 2n\}\}$?

3. (MN 4.2.2) What is the maximum possible number of edges of a graph with n vertices and k components?

4. (MN 4.2.4) Prove that a graph is bipartite if and only if it contains no cycle of odd length.

5. (MN 4.3.2) Construct an example of a sequence of length $n \geq 5$ in which each term is some of the numbers $1, 2, ..., n-1$ and which has an even number of odd terms, and yet the sequence is not a graph score.

6. (MN 4.3.5) Draw all nonisomorphic graphs with score $(6, 3, 3, 3, 3, 3, 3)$. Prove that none was left out!

7. (MN 4.4.2) Characterize graphs that have a tour, not necessarily a closed one, covering all edges.

8. (MN 4.4.7) Construct two connected graphs with the same score, one with and one without a Hamiltonian cycle.

9*. (MN 4.2.6) Describe all graphs containing no path of length 4.