Matemaatiline analüüs I
6. praktikum

Olgu X intervall (st. tõkestatud või tõkestamata lõik, poollõik või vahemik).
Olgu $a \in X^\circ$ (sisepunkt), olgu $f\colon X \to \mathbb{R}$.
$f'(a) \overset{\text{def}}{=} \lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a} = \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ (f-ni f *tuletis* punktis a).

f *diferentseeruv* punktis a $\overset{\text{def}}{\iff}$ $f'(a) \in \mathbb{R}$.
Olgu $f, g\colon X \to \mathbb{R}$, olgu $a \in X^\circ$.

Lause 5.2. f ja g dif-vad punktis a $\implies \begin{cases} f \pm g \text{ on dif-v punktis } a, \\ (f\pm g)'(a) = f'(a) \pm g'(a). \end{cases}$

Lause 5.3.
$$\left.\begin{array}{r} f \text{ dif-v punktis } a, \\ \lambda \in \mathbb{R} \end{array}\right\} \implies \begin{cases} \lambda \cdot f \text{ on dif-v punktis } a, \\ (\lambda\cdot f)'(a) = \lambda \cdot f'(a). \end{cases}$$

Lause 5.4. f ja g dif-vad punktis a $\implies \begin{cases} f \cdot g \text{ on dif-v punktis } a, \\ (f\cdot g)'(a) = f'(a)\cdot g(a) + f(a)\cdot g'(a). \end{cases}$

Lause 5.5.
$$\left.\begin{array}{r} f \text{ ja } g \text{ dif-vad punktis } a, \\ g(a) \neq 0 \end{array}\right\} \implies \begin{cases} \dfrac{f}{g} \text{ on dif-v punktis } a, \\ \left(\dfrac{f}{g}\right)'(a) = \dfrac{f'(a)\cdot g(a) - f(a)\cdot g'(a)}{g(a)^2}. \end{cases}$$

Olgu $f\colon X \to \mathbb{R}$, $h\colon E \to \mathbb{R}$, kusjuures $f(X) \subset E$, $a \in X^\circ$, $f(a) \in E^\circ$.
Lause 5.6 (liitfunktsiooni dif-vus)**.**
$$\left.\begin{array}{r} f \text{ dif-v punktis } a, \\ h \text{ dif-v punktis } f(a) \end{array}\right\} \implies \begin{cases} h\circ f \text{ on dif-v punktis } a, \\ \big(h\circ f\big)'(a) = h'(f(a))\cdot f'(a). \end{cases}$$

Olulisi tuletisi:

$$c' = 0 \quad (c \in \mathbb{R}) \qquad (\sin x)' = \cos x \qquad (\operatorname{sh} x)' = \operatorname{ch} x$$
$$x' = 1 \qquad (\cos x)' = -\sin x \qquad (\operatorname{ch} x)' = \operatorname{sh} x$$
$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2},\ x \neq 0 \qquad (\tan x)' = \frac{1}{\cos^2 x},\ x \neq (2k\pm 1)\pi \qquad (\operatorname{th} x)' = \frac{1}{\operatorname{ch}^2 x}$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}},\ x > 0 \qquad (\cot x)' = -\frac{1}{\sin^2 x},\ x \neq k\pi \qquad (\operatorname{cth} x)' = -\frac{1}{\operatorname{sh}^2 x}$$
$$(x^a)' = a x^{a-1} \qquad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}},\ |x| < 1 \qquad (\operatorname{arsh} x)' = \frac{1}{\sqrt{1+x^2}}$$
$$(e^x)' = e^x \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}},\ |x| < 1 \qquad (\operatorname{arch} x)' = \frac{1}{\sqrt{x^2-1}},\ x > 1$$
$$(a^x)' = a^x \ln a \qquad (\arctan x)' = \frac{1}{1+x^2} \qquad (\operatorname{arth} x)' = \frac{1}{1-x^2},\ |x| < 1$$
$$(\ln|x|)' = \frac{1}{x},\ x \neq 0 \qquad (\operatorname{arccot} x)' = -\frac{1}{1+x^2}, \qquad (\operatorname{arcth} x)' = \frac{1}{1-x^2},\ |x| > 1$$
$$(\log_a |x|)' = \frac{1}{x \ln a},\ x \neq 0$$