Post-Newtonian limit of massive bimetric and scalar-tensor gravity

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Outline

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Motivation

- Not yet understood cosmological observations:
  - Homogeneity of the cosmic microwave background.
  - Accelerating expansion of the universe at present time.
  - Motion of galaxies in clusters and galaxy rotation curves.
  - Galactic mergers (“trainwreck cluster” Abell 520).

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  - Bigravity:
    - Contains additional, massive graviton and allows two matter sectors.
    - Both massive gravitons and additional matter sector are dark matter candidates.
    - Additional matter sector may contain interacting dark matter.

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    - Non-zero vacuum value of scalar field potential acts as cosmological constant.

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- Solar system consistency of theories with massive extra degrees of freedom?

Post-Newtonian approximation

- Perfect fluid energy-momentum tensor:

$$T^{\mu\nu} = (\rho + \rho\Pi + p)u^{\mu}u^{\nu} + pg^{\mu\nu}\,.$$

  - Four-velocity u^{μ}.
  - Matter density ρ.
  - Specific internal energy Π.
  - Pressure p.

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- Slow-moving source matter:

$$v^i = \frac{u^i}{u^0} \ll 1 \,.$$

- Assign velocity orders $|v^i|^n \sim \mathcal{O}(n)$ based on solar system.

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- Perturbative expansion of the metric:

$$g_{00} = -1 + h^{(2)}_{00} + h^{(4)}_{00} + \mathcal{O}(6)\,, \quad g_{0j} = h^{(3)}_{0j} + \mathcal{O}(5)\,, \quad g_{ij} = \delta_{ij} + h^{(2)}_{ij} + \mathcal{O}(4)\,.$$

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- Superscripts correspond to velocity orders: $h^{(n)}_{\mu\nu} \sim \mathcal{O}(n)$.
- Quasi-static solution: time dependence enters only through motion of source matter.

⇒ Assign additional velocity order $\partial_0 \sim \mathcal{O}(1)$ to every time derivative.

Standard parametrized post-Newtonian (PPN) formalism

- PPN metric perturbations in standard gauge:

$$
\begin{aligned}
h^{(2)}_{00} &= 2U\,,\\
h^{(2)}_{ij} &= 2\gamma U\delta_{ij}\,,\\
h^{(3)}_{0i} &= -\frac{1}{2}(3 + 4\gamma + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i\,,\\
h^{(4)}_{00} &= -2\beta U^2 - 2\xi\Phi_W + (2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(1 + 3\gamma - 2\beta + \zeta_2 + \xi)\Phi_2\\
&\quad + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A}\,.
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h_{ij}^{(2)} &= 2\gamma U\delta_{ij}\,, \\
h_{0i}^{(3)} &= -\frac{1}{2}(3+4\gamma+\alpha_1-\alpha_2+\zeta_1-2\xi)V_i-\frac{1}{2}(1+\alpha_2-\zeta_1+2\xi)W_i\,, \\
h_{00}^{(4)} &= -2\beta U^2-2\xi\Phi_W+(2+2\gamma+\alpha_3+\zeta_1-2\xi)\Phi_1+2(1+3\gamma-2\beta+\zeta_2+\xi)\Phi_2 \\
&\quad +2(1+\zeta_3)\Phi_3+2(3\gamma+3\zeta_4-2\xi)\Phi_4-(\zeta_1-2\xi)\mathcal{A}\,.
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- PPN potentials depend on source only:
  - U: Newtonian potential
  - V_i, W_i: moving source matter
  - $\Phi_1, \mathcal{A}$: kinetic energy
  - Φ_2: gravitational self-energy
  - Φ_3: internal energy
  - Φ_4: pressure
  - Φ_W: anisotropic interaction

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- PPN parameters depend on theory:
  - γ: spatial curvature per unit mass
  - β: non-linearity of Newton's law
  - ξ: preferred location effects
  - $\alpha_1, \alpha_2, \alpha_3$: preferred frame effects
  - $\alpha_3, \zeta_1, \zeta_2, \zeta_3$: violation of conservation laws

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- Field content: metrics g, f; matter fields Ψ^g, Ψ^f.

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- Action:

$$S = \int_M d^4x \left[\frac{m_g^2}{2}\sqrt{-\det g}R^g + \frac{m_f^2}{2}\sqrt{-\det f}R^f \right.$$
$$\left. - m^4\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right) + \sqrt{-\det g}\mathcal{L}_m^g(g,\Psi^g) + \sqrt{-\det f}\mathcal{L}_m^f(f,\Psi^f) \right].$$

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- Field equations:

$$m_g^2\left(R^g_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^g\right) + m^4 V^g_{\mu\nu} = T^g_{\mu\nu},$$
$$m_f^2\left(R^f_{\mu\nu} - \frac{1}{2}f_{\mu\nu}R^f\right) + m^4 V^f_{\mu\nu} = T^f_{\mu\nu}.$$

Flat, proportional background solution

- Assume existence of vacuum solution with $c > 0$:

$$g^{(0)}_{\mu\nu} = \eta_{\mu\nu}\,, \quad f^{(0)}_{\mu\nu} = c^2\eta_{\mu\nu}\,.$$

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⇒ Consider only models which satisfy

$$\tilde{\beta}_0 = -3\tilde{\beta}_1 - 3\tilde{\beta}_2 - \tilde{\beta}_3\,, \quad \tilde{\beta}_4 = -\tilde{\beta}_1 - 3\tilde{\beta}_2 - 3\tilde{\beta}_3\,.$$

⇒ New free parameter $c > 0$ instead of β_0, β_4 in the action.

Static point mass source

- Energy-momentum tensors for perfect bi-fluid:

$$T^{g\,\mu\nu} = (\rho^g + \rho^g\Pi^g + p^g)u^{g\,\mu}u^{g\,\nu} + p^g g^{\mu\nu}\,,$$
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- Static point mass source including both dark and visible matter:

$$\rho^g = M^g\delta(\vec{x})\,, \qquad \Pi^g = 0\,, \qquad p^g = 0\,, \qquad u^g \sim \partial_t\,,$$
$$\rho^f = M^f\frac{\delta(\vec{x})}{c^3}\,, \qquad \Pi^f = 0\,, \qquad p^f = 0\,, \qquad u^f \sim \partial_t\,.$$

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- Total visible mass M^g, total dark mass M^f.
- Rescaling of mass parameters:

$$\tilde{m}_g = m_g\,, \quad \tilde{m}_f = cm_f \quad \tilde{M}^g = M^g\,, \quad \tilde{M}^f = cM^f\,.$$

Post-Newtonian metric ansatz

- PPN metric ansatz:

$$\begin{aligned}
g_{00} &= -1 + 2\frac{\tilde{\alpha}^{gg}\tilde{M}^g + \tilde{\alpha}^{gf}\tilde{M}^f}{r}\,,\\
g_{ij} &= \delta_{ij} + 2\frac{\tilde{\gamma}^{gg}\tilde{M}^g + \tilde{\gamma}^{gf}\tilde{M}^f}{r}\delta_{ij} + 2\frac{\tilde{\theta}^{gg}\tilde{M}^g + \tilde{\theta}^{gf}\tilde{M}^f}{r^3}x_ix_j\,,\\
c^{-2}f_{00} &= -1 + 2\frac{\tilde{\alpha}^{fg}\tilde{M}^g + \tilde{\alpha}^{ff}\tilde{M}^f}{r}\,,\\
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- PPN parameters (in general depend on interaction distance r):
  - $\tilde{\alpha}^{gg}, \tilde{\alpha}^{gf}, \tilde{\alpha}^{fg}, \tilde{\alpha}^{ff}$: Newtonian interaction.

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  - $\tilde{\theta}^{gg}, \tilde{\theta}^{gf}, \tilde{\theta}^{fg}, \tilde{\theta}^{ff}$: Off-diagonal contribution.
- Gauge choice: $\tilde{\theta}^{gg} = \tilde{\theta}^{ff} = 0$.

PPN parameters

- Solution for PPN parameters:

$$\tilde{\alpha}^{gg} = \frac{3\tilde{m}_g^2 + 4\tilde{m}_f^2 e^{-\mu r}}{24\pi\tilde{m}_g^2(\tilde{m}_f^2 + \tilde{m}_g^2)}, \quad \tilde{\alpha}^{ff} = \frac{3\tilde{m}_f^2 + 4\tilde{m}_g^2 e^{-\mu r}}{24\pi\tilde{m}_f^2(\tilde{m}_f^2 + \tilde{m}_g^2)},$$

$$\tilde{\alpha}^{gf} = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f^2 + \tilde{m}_g^2)}, \quad \tilde{\alpha}^{fg} = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f^2 + \tilde{m}_g^2)},$$

$$\tilde{\gamma}^{gg} = \frac{3\tilde{m}_g^2 + 2\tilde{m}_f^2 e^{-\mu r}}{24\pi\tilde{m}_g^2(\tilde{m}_f^2 + \tilde{m}_g^2)}, \quad \tilde{\gamma}^{ff} = \frac{3\tilde{m}_f^2 + 4\tilde{m}_g^2 e^{-\mu r}}{24\pi\tilde{m}_f^2(\tilde{m}_f^2 + \tilde{m}_g^2)},$$

$$\tilde{\gamma}^{gf} = \frac{9\tilde{m}_f^2 + 2(\tilde{m}_g^2 - 2\tilde{m}_f^2)e^{-\mu r}}{72\pi\tilde{m}_f^2(\tilde{m}_f^2 + \tilde{m}_g^2)} - \frac{\mu r(\mu r + 3) + 3}{36\pi\tilde{m}_f^2\mu^2 r^2}e^{-\mu r},$$

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$$\tilde{\theta}^{gf} = \frac{\mu r(\mu r + 3) + 3}{12\pi\tilde{m}_f^2\mu^2 r^2}e^{-\mu r}, \quad \tilde{\theta}^{fg} = \frac{\mu r(\mu r + 3) + 3}{12\pi\tilde{m}_g^2\mu^2 r^2}e^{-\mu r}.$$

Physical interpretation

- Physical meaning of PPN parameters:

	Newtonian gravity	light deflection
by visible matter	$\tilde{\alpha}^{gg} = \frac{3\tilde{m}_g{}^2+4\tilde{m}_f{}^2e^{-\mu r}}{24\pi\tilde{m}_g{}^2(\tilde{m}_f{}^2+\tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gg}}{\tilde{\alpha}^{gg}} = \frac{3\tilde{m}_g{}^2+2\tilde{m}_f{}^2e^{-\mu r}}{3\tilde{m}_g{}^2+4\tilde{m}_f{}^2e^{-\mu r}}$
by dark matter	$\tilde{\alpha}^{gf} = \frac{3-4e^{-\mu r}}{24\pi(\tilde{m}_f{}^2+\tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gf}+\tilde{\theta}^{gf}/3}{\tilde{\alpha}^{gf}} = 1 + \frac{2(\tilde{m}_g{}^2+4\tilde{m}_f{}^2)}{3\tilde{m}_f{}^2(3e^{\mu r}-4)}$

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by dark matter	$\tilde{\alpha}^{gf} = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f{}^2 + \tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gf} + \tilde{\theta}^{gf}/3}{\tilde{\alpha}^{gf}} = 1 + \frac{2(\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2)}{3\tilde{m}_f{}^2(3e^{\mu r} - 4)}$

- Constants appearing in PPN parameters:
  - Graviton mass:

$$\mu = m^2\sqrt{\left(\tilde{\beta}_1 + 2\tilde{\beta}_2 + \tilde{\beta}_3\right)\left(\frac{1}{\tilde{m}_f{}^2} + \frac{1}{\tilde{m}_g{}^2}\right)}\,.$$

Physical interpretation

- Physical meaning of PPN parameters:

	Newtonian gravity	light deflection
by visible matter	$\tilde{\alpha}^{gg} = \frac{3\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2 e^{-\mu r}}{24\pi \tilde{m}_g{}^2(\tilde{m}_f{}^2 + \tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gg}}{\tilde{\alpha}^{gg}} = \frac{3\tilde{m}_g{}^2 + 2\tilde{m}_f{}^2 e^{-\mu r}}{3\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2 e^{-\mu r}}$
by dark matter	$\tilde{\alpha}^{gf} = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f{}^2 + \tilde{m}_g{}^2)}$	$\frac{\tilde{\gamma}^{gf} + \tilde{\theta}^{gf}/3}{\tilde{\alpha}^{gf}} = 1 + \frac{2(\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2)}{3\tilde{m}_f{}^2(3e^{\mu r} - 4)}$

- Constants appearing in PPN parameters:
  - Graviton mass:

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  - Effective Planck masses $\tilde{m}_g$, $\tilde{m}_f$.

Solar system consistency

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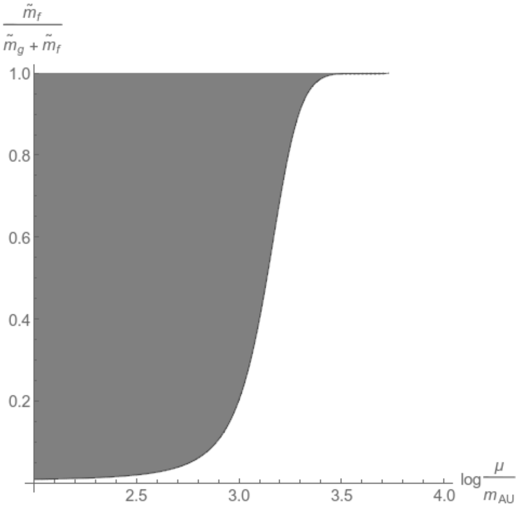
- Gray area excluded at 2σ (with $m_{\text{AU}} = 1\text{AU}^{-1} \approx 1.32 \cdot 10^{-18}\frac{\text{eV}}{c^2}$):



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- Field equations (with $\mathcal{F} \equiv \frac{2\mathcal{A}\mathcal{B}+3\mathcal{A}'^2}{4\mathcal{A}^2}$):

$$R_{\mu\nu} - \frac{\mathcal{A}'}{\mathcal{A}}\left(\nabla_\mu\nabla_\nu\Phi + \frac{1}{2}g_{\mu\nu}\square\Phi\right) - \left(\frac{\mathcal{A}''}{\mathcal{A}} + 2\mathcal{F} - \frac{3\mathcal{A}'^2}{2\mathcal{A}^2}\right)\partial_\mu\Phi\partial_\nu\Phi$$

$$- \frac{1}{2}g_{\mu\nu}\frac{\mathcal{A}''}{\mathcal{A}}g^{\rho\sigma}\partial_\rho\Phi\partial_\sigma\Phi - \frac{1}{\mathcal{A}}g_{\mu\nu}\kappa^2\mathcal{U} = \frac{\kappa^2}{\mathcal{A}}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)\,,$$

$$\mathcal{F}\square\Phi + \frac{1}{2}\left(\mathcal{F}' + 2\mathcal{F}\frac{\mathcal{A}'}{\mathcal{A}}\right)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{\mathcal{A}'}{\mathcal{A}^2}\kappa^2\mathcal{U} - \frac{1}{2\mathcal{A}}\kappa^2\mathcal{U}' = \kappa^2\frac{\mathcal{A}' - 2\mathcal{A}\alpha'}{4\mathcal{A}^2}T\,.$$

Invariant formalism

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- Define covariant / invariant quantities [Järv, Kuusk, Saal, Vilson '14]:
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$$\mathcal{I}_1 = \frac{e^{2\alpha}}{\mathcal{A}} = \bar{\mathcal{I}}_1\,, \quad \mathcal{I}_2 = \frac{\mathcal{U}}{\mathcal{A}^2} = \bar{\mathcal{I}}_1\,, \quad \mathcal{F} \equiv \frac{2\mathcal{A}\mathcal{B} + 3\mathcal{A}'^2}{4\mathcal{A}^2} = \left(\frac{\partial\bar{\Phi}}{\partial\Phi}\right)^2 \bar{\mathcal{F}}\,.$$

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$$g^{\mathfrak{E}}_{\mu\nu} = \mathcal{A}g_{\mu\nu}\,, \quad g^{\mathfrak{J}}_{\mu\nu} = e^{2\alpha}g_{\mu\nu}\,, \quad T^{\mathfrak{E}}_{\mu\nu} = \frac{T_{\mu\nu}}{\mathcal{A}}\,, \quad T^{\mathfrak{J}}_{\mu\nu} = \frac{T_{\mu\nu}}{e^{2\alpha}}\,.$$

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- Express field equations in terms of invariants [Järv, Kuusk, Saal, Vilson '14]:

$$R^{\mathfrak{E}}_{\mu\nu} - 2\mathcal{F}\partial_\mu\Phi\partial_\nu\Phi - \kappa^2 g^{\mathfrak{E}}_{\mu\nu}\mathcal{I}_2 = \kappa^2\bar{T}^{\mathfrak{E}}_{\mu\nu}\,,$$

$$\mathcal{F}g^{\mathfrak{E}\,\mu\nu}\nabla^{\mathfrak{E}}_\mu\partial_\nu\Phi + \frac{\mathcal{F}'}{2}g^{\mathfrak{E}\,\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{\kappa^2}{2}\mathcal{I}_2' = -\frac{1}{4}\kappa^2(\ln\mathcal{I}_1)'T^{\mathfrak{E}}\,.$$

Post-Newtonian metric ansatz for homogeneous spherical source

- Energy-momentum tensor for perfect fluid:

$$T^{\mathfrak{I}\,\mu\nu} = (\rho + \rho\Pi + p)u^{\mu}u^{\nu} + pg^{\mathfrak{I}\,\mu\nu}\,.$$

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- PPN metric ansatz:

$$\begin{aligned} g^{\mathfrak{J}}_{00} &= -1 + 2G_{\text{eff}}U - 2\beta G_{\text{eff}}^2 U^2 + 2G_{\text{eff}}^2(1 + 3\gamma - 2\beta)\Phi_2 + G_{\text{eff}}(2\Phi_3 + 6\gamma\Phi_4)\,, \\ g^{\mathfrak{J}}_{0i} &= 0\,, \\ g^{\mathfrak{J}}_{ij} &= (1 + 2\gamma G_{\text{eff}}U)\delta_{ij}\,. \end{aligned}$$

PPN parameters outside the source

- Effective gravitational constant:

$$G_{\text{eff}} = \frac{\kappa^2 l_1}{8\pi}\left[1 + 3\frac{mR\cosh(mR) - \sinh(mR)}{(2\omega + 3)m^3R^3}e^{-mr}\right], \quad \omega = 2F\frac{l_1^2}{l_1'^2} - \frac{3}{2}.$$

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- PPN parameter γ:

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- PPN parameter β (only limit $r \to \infty$ shown here due to length of full result):

$$\lim_{r\to\infty}\beta = 1 + 5\frac{\left[39 + m^2R^2(20mR - 33)\right] - 3(1+mR)[13 + mR(13+2mR)]e^{-2mR}}{16(2\omega+3)m^5R^5}.$$

- Result deviates from $\beta \to 1$ due to modification of gravitational self-energy [MH, Schärer '17].

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  - Effective interaction distance: $r_0 \approx 1.6 R_\odot \approx 7.44 \cdot 10^{-3}\text{AU}$.
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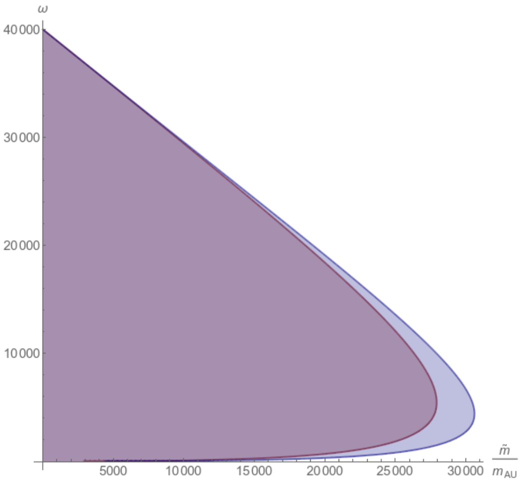

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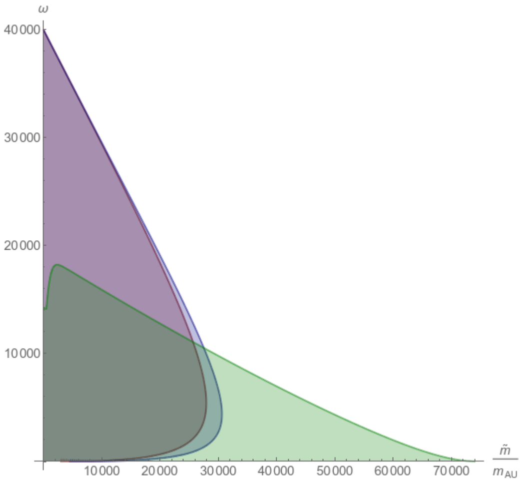

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Summary

- Bimetric gravity:
  - Theoretical background:
    - Unique ghost-free theory of two interacting metrics.
    - Natural candidates for dark energy and dark matter.
  - Post-Newtonian limit:
    - Matter source given by static point mass with dark and visible matter.
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  - Theoretical background:
    - Scalar field to mediate gravity besides metric tensor.
    - Theories often arise as effective theories or from phenomenology.
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    - Matter source given by static homogeneous sphere..
    - Effective gravitational constant and PPN parameters γ and β.
  - Experimental constraints:
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Outlook

- Bimetric gravity:
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- References:

  - MH, "Post-Newtonian parameter γ and the deflection of light in ghost-free massive bimetric gravity", **Phys. Rev. D 95 (2017) 124049 [arXiv:1701:07700 [gr-qc]]**.
  - MH and Andreas Schärer, "Post-Newtonian parameters γ and β of scalar-tensor gravity for a homogeneous gravitating sphere", **arXiv:1708:07851 [gr-qc]**.