Dark and visible lenses in bimetric gravity

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\[ S_{GR} = m_g^2 \int d^4 x \sqrt{-g} R(g) \]
| Spin 0 | Scalar field $\phi$ | $\mathcal{L}_\phi = -\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2$ |
| Spin 1/2 | Spinor field $\psi^\alpha$ | $\mathcal{L}_\psi = -\bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$ |
| Spin 1 | Vector field $A_\mu$ | $\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} A_\mu A^\mu$ |
Hassan Rosen Bimetric Gravity:

\[ S_{HR} = m_g^2 \int d^4 x \sqrt{-g} R(g) + m_f^2 \int d^4 x \sqrt{-f} R(f) \]

\[ -2m^4 \int d^4 x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n \left( \sqrt{g^{-1} f} \right) \]
Equations of Motion:

\[ R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) + \frac{m^4}{m_g^2} V^g_{\mu\nu}(g, f; \beta_n) = 0, \]

\[ R_{\mu\nu}(f) - \frac{1}{2} f_{\mu\nu} R(f) + \frac{m^4}{m_f^2} V^f_{\mu\nu}(g, f; \beta_n) = 0 \]
Couplings to matter

\[ S_m = \int d^4x \sqrt{-g} \mathcal{L}_m (g, \phi_g) + \int d^4x \sqrt{-f} \tilde{\mathcal{L}}_m (f, \phi_f) \]
Stress-Energy Tensor:

\[ T^g_{\mu\nu} \equiv -\frac{1}{\sqrt{-g}} \delta \left( \sqrt{-g} \mathcal{L}_m (g, \phi_g) \right) \frac{\delta g^{\mu\nu}}{\delta g_{\mu\nu}} \]

\[ T^f_{\mu\nu} \equiv -\frac{1}{\sqrt{-f}} \delta \left( \sqrt{-f} \check{\mathcal{L}}_m (f, \phi_f) \right) \frac{\delta f^{\mu\nu}}{\delta f_{\mu\nu}} \]
Observational evidence for dark matter:
- Galaxy rotation curves: dark matter in galactic halos.
- Peculiar motion in clusters: dark matter in clusters.
- Galactic mergers: dark matter outside of galaxies.
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Dark matter phenomenology

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- Simple ansatz for matter source:
  - Point-like matter distribution (consider galaxy as point mass).
  - Contains both visible and dark matter.
  - Dark matter is constituted by second matter sector $T^f_{\mu\nu}$.
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⇒ Matter density:

$$T^g_{00} = \rho^g = M^g \delta(\vec{x}) , \quad T^f_{00} = \rho^f = M^f \frac{\delta(\vec{x})}{c^3} .$$
Post-Newtonian metric ansatz

- Proportional background metric ansatz:

\[ g_{\mu\nu}^{(0)} = \eta_{\mu\nu}, \quad f_{\mu\nu}^{(0)} = c^2 \eta_{\mu\nu}; \quad c > 0. \]
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- First order perturbation around background metric:

  \[
  g_{00} = -1 + 2G_v \frac{M^g}{r} + 2G_d \frac{cM^f}{r},
  \]

  \[
  g_{ij} = \left(1 + 2G_v \gamma_v \frac{M^g}{r} + 2G_d \gamma_d \frac{cM^f}{r}\right) \delta_{ij}.
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- Observable PPN parameters:
  - \( G_v \): Newtonian gravity caused by visible matter.
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- Observable PPN parameters:
  - \( G_v \): Newtonian gravity caused by visible matter.
  - \( G_d \): Newtonian gravity caused by dark matter.
  - \( \gamma_v \): Light deflection caused by visible matter.
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Calculated values of PPN parameters:

\[ G_v = \frac{3\tilde{m}_g^2 + 4\tilde{m}_f^2 e^{-\mu r}}{24\pi \tilde{m}_g^2 (\tilde{m}_f^2 + \tilde{m}_g^2)}, \]

\[ G_d = \frac{3 - 4 e^{-\mu r}}{24\pi (\tilde{m}_f^2 + \tilde{m}_g^2)}, \]

\[ \gamma_v = \frac{3\tilde{m}_g^2 + 2\tilde{m}_f^2 e^{-\mu r}}{3\tilde{m}_g^2 + 4\tilde{m}_f^2 e^{-\mu r}}, \]

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Constants appearing in PPN parameters:

- Effective Planck masses \( \tilde{m}_g = m_g, \tilde{m}_f = cm_f. \)
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Constants appearing in PPN parameters:

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- Massive spin 2 field mass:

\[
\mu = m^2 \sqrt{\left(\tilde{\beta}_1 + 2\tilde{\beta}_2 + \tilde{\beta}_3\right)\left(\frac{1}{\tilde{m}_f^2} + \frac{1}{\tilde{m}_g^2}\right)}.
\]
Solar system consistency

- Cassini tracking experiment (Shapiro delay by the sun):
  - Effective interaction distance: \( r_0 \approx 1.6R_\odot \approx 7.44 \cdot 10^{-3}\text{AU} \).
  - Measured PPN parameter: \( \gamma_v - 1 = (2.1 \pm 2.3) \cdot 10^{-5} \).
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  - Gray area excluded at $2\sigma$ (with $m_{\text{AU}} = 1\text{AU}^{-1} \approx 1.32 \cdot 10^{-18}\text{eV/c}^2$):
Summary:

- **Dark matter:**
  - Non-interacting component passes through (Bullet cluster).
  - Interacting component undergoes shock in merger (Abell 520).

- **Bimetric gravity:**
  - Non-interacting dark matter could be dark spin 2 field.
  - Interacting dark matter could be sector coupled to second metric.

- **Test hypothesis using light deflection:**
  - Visible matter observations yield bounds on theory parameters.
  - Study gravitational effects of dark matter on masses / light.

- **What matters?**
  - Dark matter influences both light and visible matter by its gravity.
  - This gravitational influence may differ from that of visible matter.

- **Question:**
  - How can we measure the ratio of light deflection and Newtonian gravity for dark matter?
Conclusion

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$\beta_n$ are just coefficients to the elementary symmetric polynomials of the eigenvalues $\lambda_n$ of the matrix $\sqrt{g^{-1}f}$:

$$e_0 \left( \sqrt{g^{-1}f} \right) = 1,$$  \hspace{1cm} (1)

$$e_1 \left( \sqrt{g^{-1}f} \right) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4,$$  \hspace{1cm} (2)

$$e_2 \left( \sqrt{g^{-1}f} \right) = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4,$$  \hspace{1cm} (3)

$$e_3 \left( \sqrt{g^{-1}f} \right) = \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4,$$  \hspace{1cm} (4)

$$e_4 \left( \sqrt{g^{-1}f} \right) = \lambda_1 \lambda_2 \lambda_3 \lambda_4 = \det \sqrt{g^{-1}f} \hspace{1cm} (5)$$