Dark and visible lenses in bimetric gravity

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Daniel Blixt, Manuel Hohmann

Laboratory of Theoretical Physics - Institute of Physics - University of Tartu
Center of Excellence “The Dark Side of the Universe”

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$$S_{\mathrm{GR}} = m_{\mathrm{g}}^2 \int \mathrm{d}^4x \sqrt{-g} R(g)$$

spin 0	scalar field ϕ	$\mathcal{L}_\phi = -\partial_\mu\phi\partial^\mu\phi - m^2\phi^2$
spin $1/2$	spinor field ψ^α	$\mathcal{L}_\psi = -\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$
spin 1	vector field A_μ	$\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{m^2}{2}A^\mu A_\mu$

Hassan Rosen Bimetric Gravity:

$$S_{\mathrm{HR}} = m_{\mathrm{g}}^2 \int \mathrm{d}^4x \sqrt{-g} R(g) + m_{\mathrm{f}}^2 \int \mathrm{d}^4x \sqrt{-f} R(f) - 2m^4 \int \mathrm{d}^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1} f} \right)$$

Equations of Motion:

$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) + \frac{m^4}{m_g^2}V^g_{\mu\nu}(g, f; \beta_n) = 0,$$

$$R_{\mu\nu}(f) - \frac{1}{2}f_{\mu\nu}R(f) + \frac{m^4}{m_f^2}V^f_{\mu\nu}(g, f; \beta_n) = 0$$

Couplings to matter

$$S_m = \int \mathrm{d}^4x \sqrt{-g}\mathcal{L}_m\left(g, \phi_g\right) + \int \mathrm{d}^4x \sqrt{-f}\tilde{\mathcal{L}}_m\left(f, \phi_f\right)$$

Stress-Energy Tensor:

$$T^g_{\mu\nu} \equiv -\frac{1}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathcal{L}_m\left(g, \phi_g\right)\right)}{\delta g^{\mu\nu}}$$

$$T^f_{\mu\nu} \equiv -\frac{1}{\sqrt{-f}} \frac{\delta\left(\sqrt{-f}\tilde{\mathcal{L}}_m\left(f, \phi_f\right)\right)}{\delta f^{\mu\nu}}$$

Dark matter phenomenology

- Observational evidence for dark matter:
  - Galaxy rotation curves: dark matter in galactic halos.
  - Peculiar motion in clusters: dark matter in clusters.
  - Galactic mergers: dark matter outside of galaxies.

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⇒ Matter density:

$$T^g_{00} = \rho^g = M^g\delta(\vec{x})\,, \quad T^f_{00} = \rho^f = M^f\frac{\delta(\vec{x})}{c^3}\,.$$

Post-Newtonian metric ansatz

- Proportional background metric ansatz:

$$g^{(0)}_{\mu\nu} = \eta_{\mu\nu}\,, \quad f^{(0)}_{\mu\nu} = c^2\eta_{\mu\nu}\,; \quad c > 0\,.$$

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- First order perturbation around background metric:

$$\begin{aligned} g_{00} &= -1 + 2G_v\frac{M^g}{r} + 2G_d\frac{cM^f}{r}\,, \\ g_{ij} &= \left(1 + 2G_v\gamma_v\frac{M^g}{r} + 2G_d\gamma_d\frac{cM^f}{r}\right)\delta_{ij}\,. \end{aligned}$$

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  - G_v: Newtonian gravity caused by visible matter.

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  - γ_v: Light deflection caused by visible matter.

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Values of PPN parameters

- Calculated values of PPN parameters:

$$G_v = \frac{3\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2 e^{-\mu r}}{24\pi\tilde{m}_g{}^2(\tilde{m}_f{}^2 + \tilde{m}_g{}^2)}\,, \qquad \gamma_v = \frac{3\tilde{m}_g{}^2 + 2\tilde{m}_f{}^2 e^{-\mu r}}{3\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2 e^{-\mu r}}\,,$$

$$G_d = \frac{3 - 4e^{-\mu r}}{24\pi(\tilde{m}_f{}^2 + \tilde{m}_g{}^2)}\,, \qquad \gamma_d = 1 + \frac{2(\tilde{m}_g{}^2 + 4\tilde{m}_f{}^2)}{3\tilde{m}_f{}^2(3e^{\mu r} - 4)}\,.$$

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- Constants appearing in PPN parameters:
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  - Massive spin 2 field mass:

$$\mu = m^2 \sqrt{\left(\tilde{\beta}_1 + 2\tilde{\beta}_2 + \tilde{\beta}_3\right)\left(\frac{1}{\tilde{m}_f^2} + \frac{1}{\tilde{m}_g^2}\right)}.$$

Solar system consistency

- Cassini tracking experiment (Shapiro delay by the sun):
  - Effective interaction distance: $r_0 \approx 1.6 R_\odot \approx 7.44 \cdot 10^{-3}$AU.
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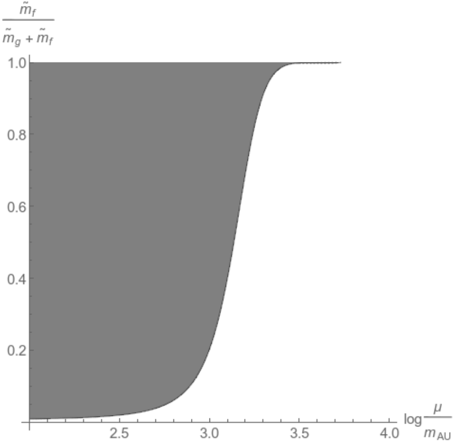
- Gray area excluded at 2σ (with $m_{\text{AU}} = 1\text{AU}^{-1} \approx 1.32 \cdot 10^{-18} \frac{\text{eV}}{c^2}$):



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- What matters?
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- Question:
  How can we measure the ratio of light deflection and Newtonian gravity for dark matter?

β_n are just coefficients to the elementary symmetric polynomials of the eigenvalues λ_n of the matrix $\sqrt{g^{-1}f}$:

$$e_0\left(\sqrt{g^{-1}f}\right) = 1, \tag{1}$$

$$e_1\left(\sqrt{g^{-1}f}\right) = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4, \tag{2}$$

$$e_2\left(\sqrt{g^{-1}f}\right) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4, \tag{3}$$

$$e_3\left(\sqrt{g^{-1}f}\right) = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4, \tag{4}$$

$$e_4\left(\sqrt{g^{-1}f}\right) = \lambda_1\lambda_2\lambda_3\lambda_4 = \det\sqrt{g^{-1}f} \tag{5}$$