Observer space geometry of Finsler spacetimes

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References

- Cartan geometry of observer space:
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- Finsler spacetimes:
  - C. Pfeifer and M. N. R. Wohlfarth,
    "Causal structure and electrodynamics on Finsler spacetimes,"
    Phys. Rev. D **84** (2011) 044039 [arXiv:1104.1079 [gr-qc]].
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    "Finsler geometric extension of Einstein gravity,"
    Phys. Rev. D **85** (2012) 064009 [arXiv:1112.5641 [gr-qc]].

Motivation

- A simple experiment:
  - Supernova occurs at $x_0 \in M$ and sends light along geodesic γ.
  - Observer at $x \in M$ with 4-velocity f_0 and frame f_α observes light.
  - Measured data $\Leftrightarrow$ tensor components wrt observer frame.

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- Possible generalization of the "spacetime picture":
  - Focus on the space of allowed observers $O \ni (x, f_0)$.
  - Describe physics in terms of observer frames $P \ni (x, f_0, f_\alpha)$.
  - $\Rightarrow$ Consider geometric theory on observer bundle $\pi : P \to O$.
  - O and P are not necessarily derived from some spacetime.

Cartan geometry on observer space

- Ingredients of a Cartan geometry:
  - Lie group G with a closed subgroup $H \subset G$.
  - Principal H-bundle $\pi : P \to M$.
  - 1-form $A \in \Omega^1(P, \mathfrak{g})$ on P with values in $\mathfrak{g} \Leftrightarrow$ Cartan connection.
  - A has an "inverse" $\underline{A} : \mathfrak{g} \to \Gamma(TP) \Leftrightarrow$ fundamental vector fields.

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- Example: Cartan geometry of spacetime:
  - Let (M, g) be a Lorentzian manifold.
  - Choose Lie groups: $G = \mathrm{ISO}_0(3,1)$, $H = \mathrm{SO}_0(3,1)$.
  - Orthonormal frame bundle $\tilde{\pi} : P \to M$ is principal H-bundle.
  - Let $A = e + \omega$: solder form e, Levi-Civita connection ω.

  $\Rightarrow$ Cartan geometry modeled on G/H.

  $\Rightarrow$ Lorentzian spacetime can be reconstructed from Cartan geometry.

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- Example: Cartan geometry of observer space: [S. Gielen, D. Wise '12]
  - Choose Lie group $K = \mathrm{SO}(3)$.
  - Let O be the unit timelike vectors on M.

  $\Rightarrow$ Principal K-bundle $\pi : P \to O$.

  $\Rightarrow$ Cartan geometry modeled on G/K.

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Finsler spacetimes

- Finsler geometry: generalized length functional:

$$\tau = \int_{t_1}^{t_2} F(x(t), \dot{x}(t)) dt \,.$$

- Finsler function $F : TM \to \mathbb{R}^+$.
- Finsler metric

$$g^F_{ab}(x, y) = \frac{1}{2} \frac{\partial}{\partial y^a} \frac{\partial}{\partial y^b} F^2(x, y) \,.$$

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- Unit vectors $y \in T_xM$ defined by

$$F^2(x, y) = g^F_{ab}(x, y)y^a y^b = 1\,.$$

$\Rightarrow$ Observer space O of unit timelike vectors.

$\Rightarrow$ Principal K-bundle $\pi : P \to O$ of orthonormal frames.

Cartan connection and fundamental vector fields

- Translational part $e \in \Omega^1(P, \mathfrak{z})$ given by solder form:

$$e^i = f^{-1}{}^i{}_a dx^a \,.$$

- Boost / rotational part $\omega \in \Omega^1(P, \mathfrak{h})$ given by Cartan linear connection:

$$\omega^i{}_j = f^{-1}{}^i{}_a \left[df_j^a + f_j^b \left(dx^c F^a{}_{bc} + (dx^d N^c{}_d + df_0^c) C^a{}_{bc} \right) \right] \,.$$

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$\Rightarrow$ $A = \omega + e$ is a Cartan connection on $\pi : P \to O$.

- For $a = z^i \mathcal{Z}_i + h^i{}_j \mathcal{H}_i{}^j \in \mathfrak{g}$ we have the fundamental vector field

$$\underline{A}(a) = z^i f^a_i \left(\partial_a - f^b_j F^c{}_{ab} \bar{\partial}^j_c \right) + \left(h^i{}_j f^a_i - h^i{}_0 f^b_i f^c_j C^a{}_{bc} \right) \bar{\partial}^j_a \,.$$

$\Rightarrow$ For all $p \in P$, $A|_p \circ \underline{A}|_p = \mathrm{id}_{\mathfrak{g}}$ and $\underline{A}|_p \circ A|_p = \mathrm{id}_{T_pP}$.

Time translation

- Consider the fundamental vector field

$$\mathbf{t} = \underline{A}(\mathcal{Z}_0) = f_0^a \partial_a - f_j^a N^b{}_a \bar{\partial}_b^j \qquad \Leftrightarrow \qquad \omega^i{}_j(\mathbf{t}) = 0\,, \quad e^i(\mathbf{t}) = \delta_0^i\,.$$

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- From $\omega^\alpha{}_\beta(\mathbf{t}) = 0$ follows:

$$0 = \dot{f}_\alpha^a + f_\alpha^b \left(\dot{x}^c F^a{}_{bc} + (\dot{x}^d N^c{}_d + \dot{f}_0^c) C^a{}_{bc}\right) = \nabla_{(\dot{x}, \dot{f}_0)} f_\alpha^a\,.$$

$\Rightarrow$ Frame f is parallely transported.

Split of the tangent bundle TP

- Consider adjoint representation $\text{Ad} : K \subset G \to \text{Aut}(\mathfrak{g})$ of K on $\mathfrak{g}$.
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- $\mathfrak{g}$ splits into irreducible subrepresentations of Ad.
- Induced decompositions of A and TP:

$$\begin{array}{ccccccccc} \mathfrak{g} & = & \mathfrak{k} & \oplus & \mathfrak{y} & \oplus & \vec{\mathfrak{z}} & \oplus & \mathfrak{z}_0 \\ \uparrow A & = & \uparrow \Omega & + & \uparrow b & + & \uparrow \vec{e} & + & \uparrow e^0 \\ T_pP & = & R_pP & \oplus & B_pP & \oplus & \vec{H}_pP & \oplus & H^0_pP \\ & & \Updownarrow & & \Updownarrow & & \Updownarrow & & \Updownarrow \\ & & \text{rotations} & & \text{boosts} & & \text{spatial translations} & & \text{temporal translations} \end{array}$$

Reconstruction of spacetime from Cartan geometry

- Condition: boost distribution $VP = RP \oplus BP$ is integrable.

$\Rightarrow$ *VP* can be integrated to a foliation $\mathcal{F}$ with leaf space *M*.

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- Tangent spaces (with $o = \pi(p)$ and $x = \pi'(o) = \tilde{\pi}(p)$):

$$\begin{array}{ccccccc}
R_pP & \oplus & B_pP & \oplus & H_pP & = & T_pP \\
\downarrow \pi_* & & \downarrow \pi_* & & \downarrow \pi_* & & \\
0 & & B_oO & \oplus & H_oO & = & T_oO \\
 & & \downarrow \pi'_* & & \downarrow \pi'_* & & \\
 & & 0 & & T_xM & = & T_xM
\end{array}$$

Reconstruction of the Finsler metric

- For $p \in P$ and $w, w' \in H_pP$ define the horizontal metric on P:

$$\begin{array}{rccc} g_P(p) & : & H_pP \otimes H_pP & \to & \mathbb{R} \\ & & (w, w') & \mapsto & -\eta_{ij}e^i(w)e^j(w') \end{array} .$$

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- Use equivariance of A and isomorphism $\pi_* : H_pP \rightarrow H_oO$.

$\Rightarrow$ Horizontal metric on O independent of $p \in \pi^{-1}(o)$:

$$\begin{array}{rcccc} g_O(o) & : & H_oO \otimes H_oO & \rightarrow & \mathbb{R} \\ & & (\pi_*(w), \pi_*(w')) & \mapsto & g_P(p)(w, w') \end{array} .$$

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- Use isomorphism $\pi'_* : H_oO \to T_xM$ for $v, v' \in H_oO$.

$\Rightarrow$ Observer dependent metric at "observer location" x:

$$g(o) \quad : \quad \begin{array}{ccc} T_xM \otimes T_xM & \to & \mathbb{R} \\ (\pi'_*(v), \pi'_*(v')) & \mapsto & g_O(o)(v, v') \end{array} .$$

$\Rightarrow$ Metric has Lorentz signature.

Embedding of observers into TM

- Fundamental vector field $\mathbf{t} = \underline{A}(\mathcal{Z}_0) \in \Gamma(TP)$ of time translation.

$\Rightarrow$ Vector field $\mathbf{r} \in \Gamma(TO)$ independent of $p \in \pi^{-1}(o)$:

$$\mathbf{r}(o) = \pi_*(\mathbf{t}(p))\,.$$

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$\Rightarrow$ $\sigma(o)$ is unit timelike:

$$g(o)(\sigma(o), \sigma(o)) = 1 .$$

$\Rightarrow$ Image $\sigma(\pi'^{-1}(x)) \subset T_xM$ is connected and fixes time orientation.

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- σ is in general not an embedding.
- Impose this as another condition.

$\Rightarrow$ Finsler spacetime reconstructed from Cartan geometry.

Gravity from Cartan to Finsler

- MacDowell-Mansouri gravity on observer space: [S. Gielen, D. Wise '12]

$$S_G = \int_O \epsilon_{\alpha\beta\gamma}\,\mathrm{tr}_{\mathfrak{h}}(F_{\mathfrak{h}} \wedge \star F_{\mathfrak{h}}) \wedge b^\alpha \wedge b^\beta \wedge b^\gamma$$

  - Hodge operator $\star$ on $\mathfrak{h}$.
  - Non-degenerate H-invariant inner product $\mathrm{tr}_{\mathfrak{h}}$ on $\mathfrak{h}$.

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  - Hodge operator $\star$ on $\mathfrak{h}$.
  - Non-degenerate H-invariant inner product $\operatorname{tr}_{\mathfrak{h}}$ on $\mathfrak{h}$.
- Translate terms into Finsler language (with $R = d\omega + \frac{1}{2}[\omega, \omega]$):
  - Curvature scalar:

$$[e, e] \wedge \star R \rightsquigarrow g^{F\,ab} R^c{}_{acb}\, dV\,.$$

  - Cosmological constant:

$$[e, e] \wedge \star [e, e] \rightsquigarrow dV\,.$$

  - Gauss-Bonnet term:

$$R \wedge \star R \rightsquigarrow \epsilon^{abcd} \epsilon^{efgh} R_{abef} R_{cdgh}\, dV\,.$$

$\Rightarrow$ Gravity theory on Finsler spacetime.

Gravity from Finsler to Cartan

- Finsler gravity action: [C. Pfeifer, M. Wohlfarth '11]

$$S_G = \int_O d^4x\, d^3y\, \sqrt{-\tilde{G}} R^a{}_{ab} y^b \,.$$

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$$d^4x\, d^3y\, \sqrt{-\tilde{G}} = \epsilon_{ijkl}\epsilon_{\alpha\beta\gamma}\, e^i \wedge e^j \wedge e^k \wedge e^l \wedge b^\alpha \wedge b^\beta \wedge b^\gamma \,,$$

$$R^a{}_{ab} y^b = b^\alpha[\underline{A}(\mathcal{Z}_\alpha), \underline{A}(\mathcal{Z}_0)] \,.$$

$\Rightarrow$ Gravity theory on observer space.

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- Future projects:
  - Consistent matter coupling.
  - Study of exact solutions.
  - Effects of deviations from metric geometry?
  - …