Fluid dynamics
on generalized geometric backgrounds

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Outline

1. Motivation

2. Finsler geometry and observer space

3. Fluids on observer space

4. Conclusion
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1 Motivation

2 Finsler geometry and observer space

3 Fluids on observer space

4 Conclusion
Fluids are everywhere

- **Perfect fluid:**
  - No shear stress, no friction.
  - Characterized by density $\rho$ and pressure $p$.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
    - Dark energy: $p < -\frac{1}{3}\rho$.
  - Used in cosmology, parameterized post-Newtonian formalism...
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- **Collisionless fluid:**
  - Model for dark matter.
  - Used in structure formation...
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  - Collisions described by Boltzmann equation.
  - Used in structure formation, atmosphere dynamics...
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- **Charged, multi-component gas:**
  - Plasma, interacting gas including recombination / ionization.
  - Used in stellar dynamics, pre-CMB era models...
Fluid dynamics naturally lift to tangent bundle:

- Fluids conveniently modeled by particle dynamics (SPH...).
- Physical fluids constituted by particles.
- Particle trajectories lift to tangent bundle: \( \gamma \mapsto (\gamma, \dot{\gamma}) \).

\[ \Rightarrow \] Dynamics on the tangent bundle described by first order ODE.
From spacetime to observer space

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- Velocity dependence of physical measurements:
  - Physical observables are tensor components.
  - Measured tensor components depend on observer velocity.
  - Physical observer velocities are future unit timelike vectors.
  $\Rightarrow$ Observer space is space of physical velocities.
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- **Modified gravity theories** may have more general observer spaces.
  - Physical observables become functions on observer space!
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  - Physical observables become functions on observer space!
- Space of observers corresponds to particle tangent vectors.
  - Consider fluid dynamics on observer space!
Finsler spacetimes

- Finsler geometry of space widely used in physics:
  - Approaches to quantum gravity
  - Electrodynamics in anisotropic media
  - Modeling of astronomical data

Finsler geometry generalizes Riemannian geometry: Geometry described by Finsler function on the tangent bundle. Finsler function measures length of tangent vectors. Well-defined notions of connections, curvature, parallel transport. . .

Finsler spacetimes are suitable backgrounds for:
- Gravity
- Electrodynamics
- Other matter field theories

Possible explanations of yet unexplained phenomena:
- Fly-by anomaly
- Galaxy rotation curves
- Accelerating expansion of the universe
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The clock postulate

Proper time along a curve in Lorentzian spacetime:

\[ \tau = \int_{t_1}^{t_2} \sqrt{-g_{ab}(x(t))\dot{x}^a(t)\dot{x}^b(t)}\,dt. \]
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  \[
  \tau = \int_{t_1}^{t_2} \sqrt{-g_{ab}(x(t))\dot{x}^a(t)\dot{x}^b(t)} \, dt.
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- Finsler geometry: use a more general length functional:
  \[
  \tau = \int_{t_1}^{t_2} F(x(t), \dot{x}(t)) \, dt.
  \]

- Finsler function \( F : TM \rightarrow \mathbb{R}^+ \).

- Parametrization invariance requires homogeneity:
  \[
  F(x, \lambda y) = \lambda F(x, y) \quad \forall \lambda > 0.
  \]
Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]
- Finsler metric with Lorentz signature:

  \[ g^F_{ab}(x, y) = \frac{1}{2} \frac{\partial}{\partial y^a} \frac{\partial}{\partial y^b} F^2(x, y). \]

- Notion of timelike, lightlike, spacelike tangent vectors.
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- Unit vectors \( y \in T_x M \) defined by

\[ F^2(x, y) = g^F_{ab}(x, y)y^a y^b = 1. \]

⇒ Set \( \Omega_x \subset T_x M \) of unit timelike vectors at \( x \in M \).
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⇒ Set \( \Omega_x \subset T_x M \) of unit timelike vectors at \( x \in M. \)

- \( \Omega_x \) contains a closed connected component \( S_x \subset \Omega_x. \)

\( \sim \) Causality: \( S_x \) corresponds to physical observers.
Geometry on the tangent bundle

- Cartan non-linear connection:

\[ N^a_b = \frac{1}{4} \bar{\partial}_b \left[ g^{F ac} (y^d \partial_d \bar{\partial}_c F^2 - \partial_c F^2) \right] \]
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\[ \Rightarrow \] Split of the tangent and cotangent bundles:

- Tangent bundle: \( TTM = HTM \oplus VTM \)

\[ \delta_a = \partial_a - N^b_a \bar{\partial}_b, \quad \bar{\partial}_a \]

- Cotangent bundle: \( T^* TM = H^* TM \oplus V^* TM \)

\[ dx^a, \quad \delta y^a = dy^a + N^a_b dx^b \]
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- Sasaki metric:
  \[ G = -g_{ab}^F dx^a \otimes dx^b - \frac{g_{ab}^F}{F^2} \delta y^a \otimes \delta y^b \]
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- Geodesic spray:
  \[ S = y^a \delta_a \]
Recall from the definition of Finsler spacetimes:

- Set $\Omega_x \subset T_x M$ of unit timelike vectors at $x \in M$.
- Physical observers correspond to $S_x \subseteq \Omega_x$.

Definition of observer space:

$$O = \bigcup_{x \in M} S_x \subset TM.$$
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Sasaki metric $\tilde{G}$ on $O$ given by pullback of $G$ to $O$.

Volume form $\Sigma$ of Sasaki metric $\tilde{G}$.

Geodesic spray $\mathbf{S}$ restricts to Reeb vector field $\mathbf{r}$ on $O$. 
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- Volume form $\Sigma$ of Sasaki metric $\tilde{G}$.
- Geodesic spray $\mathbf{S}$ restricts to Reeb vector field $\mathbf{r}$ on $O$.
- Geodesic hypersurface measure $\omega = \iota_\mathbf{r} \Sigma$.
- Note that $\mathcal{L}_r \Sigma = 0$ and $d\omega = 0$. 
From metric to Finsler geometry

Tangent bundle geometry:

- Finsler function:
  \[ F(x, y) = \sqrt{|g_{ab}(x) y^a y^b|} \]

- Finsler metric:
  \[ g^{F}_{ab}(x, y) = \begin{cases} 
  -g_{ab}(x) & y \text{ timelike} \\
  g_{ab}(x) & y \text{ spacelike} 
  \end{cases} \]

- Cartan non-linear connection:
  \[ N^a_{\ b}(x, y) = \Gamma^a_{\ bc}(x) y^c \]
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- **Cartan non-linear connection:**
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- **Observer space:**
  - Space \( \Omega_x \) of unit timelike vectors at \( x \in M \).
  - Space \( S_x \) of future unit timelike vectors at \( x \in M \).
  - Observer space \( O \): union of shells \( S_x \).
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Definition of fluids

- Single-component fluid:
  - Constitute by classical, relativistic particles.
  - Particles have equal properties (mass, charge, ...).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.
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- Multi-component fluid: multiple types of particles.
Geodesics on observer space

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime $M$:

$$\ddot{x}^a + N^a_{\ b}(x, \dot{x})\dot{x}^b = 0.$$
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- Canonical lift of curve to tangent bundle $TM$:
  \[
  x, \quad y = \dot{x}.
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- Lift of geodesic equation:
  \[
  \dot{x}^a = y^a, \quad \dot{y}^a = -N^a_b(x, y)y^b.
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⇒ Solutions are integral curves of vector field:
  $$y^a \partial_a - y^b N^a_b \bar{\partial}_a = y^a \delta_a = \mathbf{S}.$$
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  \]
- Tangent vectors are future unit timelike: $(x, y) \in O$.
  \[\Rightarrow\] Particle trajectories are piecewise integral curves of $r$ on $O$. 
One-particle distribution function

- Recall: \( \omega = \iota_r \Sigma \in \Omega^6(O) \) unique 6-form such that:
  - \( \omega \) non-degenerate on every hypersurface not tangent to \( r \).
  - \( d\omega = 0 \).
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- Recall: $\omega = \iota_r \Sigma \in \Omega^6(O)$ unique 6-form such that:
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- Define one-particle distribution function $\phi : O \rightarrow \mathbb{R}^+$ such that:

For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_\sigma \phi \omega$$

# of particle trajectories through $\sigma$.

- Counting of particle trajectories respects hypersurface orientation.
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  - Counting of particle trajectories respects hypersurface orientation.
  - For multi-component fluids: $\phi_i$ for each component $i$. 
Collision in spacetime $\leftrightarrow$ interruption in observer space.

For any open set $V \in O$,

$$\int_{\partial V} \phi \omega = \int_V d(\phi \omega) = \int_V L_r \phi \Sigma$$

# of outbound trajectories - # of inbound trajectories.

$\Rightarrow$ Collision density measured by $L_r \phi$.

Collisionless fluid: trajectories have no endpoints, $L_r \phi = 0$.

$\Rightarrow$ Simple, first order equation of motion for collisionless fluid.

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Collisions & the Liouville equation

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Examples of fluids

Geodesic dust fluid:
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Interacting fluid: 
\( \mathcal{L}_r \phi \neq 0 \).
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Interacting fluid:
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Averaged quantities

- Volume form $\Pi_x$ on unit timelike shells $S_x$ induced by $\tilde{\mathcal{G}}$. 
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- Volume form $\Pi_x$ on unit timelike shells $S_x$ induced by $\tilde{G}$.
- Averaged rest mass current density:
  \[ J^a(x) = m \int_{S_x} \phi y^a \Pi_x \]
- Averaged particle energy momentum tensor:
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⇒ Connection to well-known spacetime observables.
⇒ Connection to measurements.
Symmetric solutions

- Infinitesimal diffeomorphism described by vector field $\xi$ on $M$.
- Canonical lift of $\xi$ to vector field on $TM$:

$$\hat{\xi} = \xi^a \partial_a + y^a \partial_a \xi^b \bar{\partial}_b$$

- Killing vector field $\xi$:

$$L_{\hat{\xi}} F = 0 \Rightarrow \hat{\xi} \text{ is tangent to observer space } O \subset TM.$$

- Symmetric fluid solution:

$$L_{\hat{\xi}} \phi = 0$$

Symmetry provides simplification of 7-dimensional $O$:

- Spherical symmetry: 4 dimensions remain.
- Static spherical symmetry: 3 dimensions remain.
- Cosmological symmetry: 2 dimensions remain.
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- Model fluids by particle trajectories.
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- Describe geometry of observer space using Finsler geometry.
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Summary

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Classical fluid variables obtained via averaging:
- Particle flux density.
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Symmetries defined “as usual” by Killing vector fields.
Coupling of fluids to non-metric gravity theories.
Cosmological solutions with non-metric geometry.
Extension of parameterized post-Newtonian formalism.
...
Kinetic theory on the tangent bundle:


Finsler spacetimes:


Kiitos!