Observable effects in Finsler cosmology

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Outline

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Motivation

- Modify spacetime geometry to address open problems:
  - Origin of dark matter and dark energy.
  - Homogeneity of the cosmic microwave background and inflation.
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  - Divide tangent spaces into space-, time-, lightlike vectors.
  - Provide notions of future and past.
  - Distinguish curves corresponding to physical trajectories.
  - Define proper time along physical trajectories.
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  - Finsler geodesics determine notion of free fall.
  - Gravity theory on Finsler spacetimes exists.

From metric to Finsler geometry

The clock postulate

Proper time along a curve in Lorentzian spacetime:

$$ds^2 = -g_{\mu\nu}dx^\mu dx^\nu \quad \Rightarrow \quad s[\gamma] = \int_{t_1}^{t_2} \sqrt{-g_{\mu\nu}(\gamma(t))\dot{\gamma}^\mu(t)\dot{\gamma}^\nu(t)}dt\,.$$

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Generalized clock postulate: Finsler length measure

- Finsler geometry: use a more general length functional:

$$s[\gamma] = \int_{t_1}^{t_2} F(\gamma(t), \dot{\gamma}(t))\,dt\,.$$

- Finsler function $F : TM \to \mathbb{R}^+$.
- Parametrization invariance requires homogeneity:

$$F(x, \lambda y) = \lambda F(x, y) \quad \forall \lambda > 0\,.$$

Geodesic motion

Cartan non-linear connection

- Extremal curve of length functional satisfies geodesic equation:

$$\ddot{\gamma}^{\mu}(t) + N^{\mu}{}_{\nu}(\gamma(t), \dot{\gamma}(t)) = 0\,.$$

- $N^{\mu}{}_{\nu}$: coefficients of Cartan non-linear connection.
- Horizontal-vertical split of $TTM = HTM \oplus VTM$:

$$\delta_{\mu} = \frac{\partial}{\partial x^{\mu}} - N^{\nu}{}_{\mu}\frac{\partial}{\partial y^{\nu}}\,, \quad \bar{\partial}_{\mu} = \frac{\partial}{\partial y^{\mu}}\,.$$

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Geodesic spray

- Canonical lift $\Gamma^{a} = (\gamma^{\mu}, \dot{\gamma}^{\mu})$ of geodesic to TM satisfies

$$\dot{\Gamma}^{a}(t) - \mathbf{S}^{a}(\Gamma(t)) = 0$$

- $\mathbf{S}(x, y) = y^{\mu}\delta_{\mu} \in \mathrm{Vect}(TM)$: geodesic spray.

Outline

Cosmological symmetry

Cosmological coordinates on TM [MH '15]

- Spherical coordinates t, r, ϑ, φ on M.
- Coordinates y, u, v, w on each T_xM:

$$y\partial_t + w\left[\cos u\sqrt{1-kr^2}\partial_r + \frac{\sin u}{r}\left(\cos v\partial_\vartheta + \frac{\sin v}{\sin\vartheta}\partial_\varphi\right)\right] \in T_xM\,.$$

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Cosmologically symmetric Finsler spacetime

- Symmetry under rotations and translations (six vector fields).
- Most general Finsler function: $F(t, y, w)$.
- Homogeneity condition: $F(t, \lambda y, \lambda w) = \lambda F(t, y, w)$.
- Express Finsler function as $F(t, y, w) = y\tilde{F}(t, w/y)$.

Geodesics on cosmological background

Geodesic equation

$$\dot{t} = y\,, \quad \dot{r} = w\sqrt{1-kr^2}\cos u\,, \quad \dot{u} = -\frac{w\sqrt{1-kr^2}\sin u}{r}$$
$$\dot{\theta} = \frac{w\sin u\cos v}{r}\,, \quad \dot{\varphi} = \frac{w\sin u\sin v}{r\sin\theta}\,, \quad \dot{v} = -\frac{w\sin u\sin v}{r\tan\theta}\,,$$
$$\dot{y} = -y^2\frac{\tilde{F}_{ww}\tilde{F}_t - \tilde{F}_w\tilde{F}_{tw}}{\tilde{F}\tilde{F}_{ww}}\,, \quad \dot{w} = -y\frac{w\tilde{F}_t\tilde{F}_{ww} + y\tilde{F}\tilde{F}_{tw} - w\tilde{F}_w\tilde{F}_{tw}}{\tilde{F}\tilde{F}_{ww}}\,.$$

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Radial geodesics

- Purely radial motion: $\vartheta = \pi/2, \varphi = 0, u = 0, v = 0$.
- Co-moving velocity:

$$\frac{dr}{dt} = \frac{\dot{r}}{\dot{t}} = \frac{w}{y}\sqrt{1-kr^2}\,.$$

Conservation of the Finsler function

- Geodesic spray leaves Finsler function invariant: $\mathcal{L}_{\mathbf{S}} F = 0$.

$\Rightarrow$ Finsler function is constant along geodesics.

- Finsler function satisfies $F \equiv 0$ along null geodesics.

Light propagation

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Solution of null direction condition

- Solve $0 = F = y\tilde{F}(t, w/y)$ with $\dot{t} = y > 0$ for all $t \in \mathbb{R}$.

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Resulting curve on spacetime M

$$\frac{dr}{dt} = \mathring{W}(t)\sqrt{1 - kr^2} \Rightarrow \int_{r_e}^{r_o} \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_e}^{t_o} \mathring{W}(t)\, dt\,.$$

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Redshift of a light signal

Propagation of two wave packets

- Source and observer and fixed co-moving coordinates r_e and r_o.
- Wave packets emitted at times $t_{e,1}$ and $t_{e,2}$ from r_e.
- Wave packets observed at times $t_{o,1}$ and $t_{o,2}$ at r_o.

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Both packets travel identical coordinate distances

$$0 = \int_{t_{e,2}}^{t_{o,2}} \mathring{W}(t)dt - \int_{t_{e,1}}^{t_{o,1}} \mathring{W}(t)dt = \int_{t_{o,1}}^{t_{o,2}} \mathring{W}(t)dt - \int_{t_{e,1}}^{t_{e,2}} \mathring{W}(t)dt \approx \mathring{W}(t_o)\Delta t_o - \mathring{W}(t_e)\Delta t_e\,.$$

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Cosmological redshift

Compare period of emitted and observed signals:

$$1 + z = \frac{\Delta t_o}{\Delta t_e} = \frac{\mathring{W}(t_e)}{\mathring{W}(t_o)} \,.$$

Magnitude of a distant source

Ratio P/L of received vs. emitted power

- Rate of photons decreased by factor $1 + z$.
- Energy of each photon decreased by factor $1 + z$.

$\Rightarrow$ Ratio $P/L = (1 + z)^{-2}$.

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Area of illuminated sphere

- Finsler metric for co-moving receiver (with $\tilde{F}\tilde{F}_{ww} < 0$ for Lorentzian signature):

$$g^F_{ab}\,dx^a \otimes dx^b = \tilde{F}^2\,dt \otimes dt + \tilde{F}\tilde{F}_{ww}\left[\frac{dr \otimes dr}{1 - kr^2} + r^2\left(d\theta \otimes d\theta + \sin^2\theta\,d\phi \otimes d\phi\right)\right].$$

- Surface area of co-moving illuminated sphere: $A = 4\pi r^2|\tilde{F}\tilde{F}_{ww}|$.

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Observed magnitude

$$m = -\frac{5}{2}\log_{10}\frac{P}{A} + \text{const.} = 5\log_{10}[r(1+z)] + \frac{5}{2}\log_{10}|\tilde{F}\tilde{F}_{ww}| - \frac{5}{2}\log_{10}L + \text{const.}$$

Relating magnitude and redshift

Procedure

- Consider fixed observation time t_o.
- Express emission time t_e and distance by redshift z.
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Taylor expansion around observation time

$$\mathring{W}(t) = \mathring{W}(t_o) + \left.\frac{d\mathring{W}}{dt}\right|_{t_o}(t - t_o) + \frac{1}{2}\left.\frac{d^2\mathring{W}}{dt^2}\right|_{t_o}(t - t_o)^2 + \frac{1}{6}\left.\frac{d^3\mathring{W}}{dt^3}\right|_{t_o}(t - t_o)^3 + \ldots$$

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Taylor expansion around observation time

$$\mathring{W}(t) = \mathring{W}_0 + \mathring{W}_1(t - t_o) + \frac{1}{2}\mathring{W}_2(t - t_o)^2 + \frac{1}{6}\mathring{W}_3(t - t_o)^3 + \dots$$

Magnitude-redshift relation

$$m(z) = 5\log_{10} z + \frac{5}{2\ln 10}\left(3 - \frac{\mathring{W}_0\mathring{W}_2}{\mathring{W}_1^2}\right) z - \frac{5}{2}\log_{10} L + \text{const.} + \mathcal{O}(z^2)\,.$$

$\Rightarrow$ Relates observational data to the zeros of the geometry function.

Comparison with metric FLRW spacetime

Geometry

- Derive metric Finsler function from $g_{\mu\nu}$.
- Cosmology: FLRW metric $g = -dt \otimes dt + a^2(t)\gamma_{ij}[\kappa]dx^i \otimes dx^j$.
- Finsler function:

$$F = \sqrt{|g_{\mu\nu}y^\mu y^\nu|} = \sqrt{|y^2 - a^2(t)w^2|}$$

- Null curve solution: $\mathring{W}(t) = 1/a(t)$.

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Hubble and deceleration parameters

- Series expansion of $a(t)$ around observation time t_o

$$a(t) = a_0\left[1 + H_0(t - t_o) - \frac{1}{2}q_0 H_0^2(t - t_o)^2\right] + \mathcal{O}((t - t_o)^3)\,.$$

$\Rightarrow$ Coefficient in magnitude-redshift relation:

$$3 - \frac{\mathring{W}_0\mathring{W}_2}{\mathring{W}_1^2} = 1 - q_0\,.$$

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Summary

- Finsler spacetimes:
  - Based on Finsler length function.
  - Make use of tensors on the tangent bundle.
  - Generalize standard notions of causality, observers and gravity.
- Cosmologically symmetric Finsler spacetimes:
  - Geometry defined by function $\tilde{F}(t, w/y)$.
  - Simple form of geodesic equation (first order ODE).
  - Light propagation from $\tilde{F}(t, \mathring{W}(t)) = 0$ for all t.
- Magnitude-redshift relation:
  - Expressed through Taylor coefficients of $\mathring{W}(t)$.
  - Allows probing of spacetime geometry via light propagation.
  - FLRW metric geometry: standard result for deceleration parameter.

Outlook and references

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- References:
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