Experimental Tests of Multimetric Gravity
Gravitational Waves and the Cosmos

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DESY Theory Workshop
29 September 2011
ΛCDM model: only 5% visible matter. [Komatsu et al. '09]

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- Dark energy needed to explain accelerating expansion.
- Constituents of dark universe are unknown!
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  \item Only interaction between both copies: repulsive gravity.
  \item Universe contains equal amounts of both types of mass.
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- Dark galaxies “push” visible matter & light towards visible galaxies.
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- Mutual repulsion between galaxies drives accelerating expansion.
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Idea

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- Simple idea: two metrics $g^{\pm}_{ab}$, two standard model copies $\Psi^{\pm}$.
- No-go theorem forbids bimetric repulsive gravity! [MH, M. Wohlfarth ’09]
- Solution: $N \geq 3$ metrics $g^{I}_{ab}$ and standard model copies $\Psi^{I}$. 
Action and equations of motion

- Matter action: sum of standard model actions.
- Gravitational action:

\[
S_G[g^1, \ldots, g^N] = \frac{1}{2} \int d^4x \sqrt{g_0} \left[ \sum_{I,J=1}^{N} (x + y \delta^{IJ}) g^{lij} R^J_{ij} + F(S^{IJ}) \right].
\]

- Symmetric volume form \( g_0 = (g^1 g^2 \ldots g^N)^{1/N} \).
- \( F(S^{IJ}) \) quadratic in connection difference tensors \( S^{IJ} = \Gamma^I - \Gamma^J \).
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⇒ Equations of motion:

\[
T^I_{ab} = \sqrt{\frac{g_0}{g^I}} \left[ -\frac{1}{2N} g^I_{ab} \sum_{J,K=1}^{N} (x + y \delta^{JK}) g^{Jij} R^K_{ij} + \sum_{J=1}^{N} (x + y \delta^{IJ}) R^J_{ab} \right]
+ \text{ terms linear in } \nabla^I S^{JK} + \text{ terms quadratic in } S^{IJ}.
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\( \Rightarrow \) Equations of motion:

\[
T^l_{ab} = \sqrt{\frac{g_0}{g^l}} \left[ -\frac{1}{2N} g^l_{ab} \sum_{J,K=1}^{N} (x + y \delta^{JK}) g^{IJ} R^{K}_{IJ} + \sum_{J=1}^{N} (x + y \delta^{IJ}) R^{J}_{ab} \right]
\]

\[+ \text{ terms linear in } \nabla^l S^{JK} + \text{ terms quadratic in } S^{IJ} .\]

\( \Rightarrow \) Repulsive Newtonian limit for \( N \geq 3 \). [MH, M. Wohlfarth '10]

\( \Rightarrow \) Post-Newtonian limit consistent with solar system. [MH, M. Wohlfarth '10]
Simple cosmological model

- **Standard cosmology:**
  - Homogeneous and isotropic universe (FLRW metric).
  - Matter content: perfect fluid.
  - Early universe: radiation; late universe: dust.
  - Copernican principle: common evolution for all matter sectors.

\[
\rho = \frac{3}{2\dot{a}^2 a^2 + k a^2}.
\]

⇒ Positive matter density \(\rho > 0\) requires 
\(k = -1\) and \(\dot{a}^2 < 1\).

⇒ No solutions for 
\(k = 0\) or 
\(k = 1\).

- **Acceleration equation:**
  \[\ddot{a} a = \frac{N - 2}{6 (\rho + 3p)}\]

⇒ Acceleration must be positive.
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Explicit solution

- Equation of state: $\rho = \omega \rho$; dust: $\omega = 0$, radiation: $\omega = 1/3$.
- General solution using conformal time $\eta$ defined by $dt = a \, d\eta$:

$$a = a_{\text{min}} \left( \cosh \left( \frac{3\omega + 1}{2} \eta \right) \right)^{\frac{2}{3\omega + 1}}$$

$$\rho = \frac{3}{(N - 2) a_{\text{min}}^2} \left( \cosh \left( \frac{3\omega + 1}{2} \eta \right) \right)^{-\frac{6\omega + 6}{3\omega + 1}}.$$

$\Rightarrow$ Big bounce at $t = 0$. [MH, M. Wohlfarth '10]
Cosmological parameters

- Friedmann equation: \((2 - N)\Omega_M + \Omega_K = 1\).
- Matter density:
  \[
  \Omega_M = \frac{\rho_0}{3H_0^2} \sim \sinh^{-2} \left( \frac{3\omega + 1}{2} \eta_0 \right).
  \]
- Curvature parameter:
  \[
  \Omega_K = -\frac{k}{a_0^2 H_0^2} = \frac{1}{\dot{a}^2(t_0)} \rightarrow 1.
  \]
- Fitting of supernova data: [Amanullah et al. '10]
Structure formation

- Structure formation in $\Lambda$CDM not fully understood:
  - Missing dwarf problem. [Moore et al. '99]
  - Core-cusp-problem. [Dubinski, Carlberg '91; Navarro et al. '96]
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Structure formation in multimetric gravity:
- Perturbation of cosmological background solution.
- Model dust matter by point particles.
- Interaction between point particles given by Newtonian limit.

Implementation:
Large particle number requires high computing power.
⇒ Use GPU computing!

Results:
- Galactic clusters and filament-like structures.
- Seemingly empty voids contain "invisible" matter.
  ⇒ Repulsive gravity effects from galactic voids.
  ⇒ Negative gravitational lenses in galactic voids?
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Gravitational waves

- Gravitational waves are null. [MH '11]
- Polarizations classified by reps. of E(2). [Eardley, Lee, Lightman et al. '73]
- E(2) class of multimetric gravity depends on 3 parameters: [MH '11]

\[ P + 2R = 0 \]

\[ P = 0 \]

\[ M = 0 \]

2 tensors +1 scalar +2 vectors +1 scalar
Idea: Repulsive gravity might explain dark matter & dark energy.

- Linearly PPN consistent multimetric repulsive gravity for $N \geq 3$.
- Cosmology features late-time acceleration and big bounce.
- Structure formation features clusters and voids.
- Voids contain repulsive mass concentrations.
- Gravitational waves are null.
- $E(2)$ class can be one of $N_2$, $N_3$, $III_5$, $II_6$. 
Outlook

- **Solar system physics:**
  - Develop full non-linear multimetric PPN formalism.
  - Examine further exact solutions (single point mass...).

- **Cosmology:**
  - Calculate further cosmological matching parameters.
  - Perform cosmological perturbation analysis to understand CMB.

- **Structure formation:**
  - Advanced simulation of structure formation including thermodynamics using GADGET-2 (Millennium Simulation).
  - Search for repulsive gravity sources in galactic voids through gravitational lensing.

- **Gravitational waves:**
  - Emission of gravitational waves from various sources.
  - Shapiro delay of gravitational waves in weak gravitational fields.