Gravitational waves in multimetric gravity
An analysis of linearized multimetric gravity theories

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Motivation

- ΛCDM model: 95% of the universe are dark matter / dark energy.
- Constituents of dark universe are unknown.
- Idea: DM / DE effects from additional \textit{dark} standard model copies.
- Only interaction between standard model copies: repulsive gravity.
- Universe contains equal amounts of matter from all copies.
Motivation

- $\Lambda$CDM model: 95% of the universe are dark matter / dark energy.
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- Idea: DM / DE effects from additional dark standard model copies.
- Only interaction between standard model copies: repulsive gravity.
- Universe contains equal amounts of matter from all copies.
- Dark galaxies “push” visible matter & light towards visible galaxies.
  ⇒ Explanation of dark matter!
- Mutual repulsion between galaxies drives accelerating expansion.
  ⇒ Explanation of dark energy! [MH, M. Wohlfarth '10]
Construction principles

- \( N \geq 2 \) standard model copies \( \Psi^I \) governed by metrics \( g^I \).
- Each standard model copy \( \Psi^I \) couples only to its own metric \( g^I \):
  \[ \Rightarrow S_M[g^I, \Psi^I] = \int d^4x \sqrt{g^I} \mathcal{L}_M[g^I, \Psi^I]. \]
- Different sectors couple only gravitationally:
  \[ \Rightarrow S = S_G[g^1, \ldots, g^N] + \sum_{I=1}^{N} S_M[g^I, \Psi^I]. \]
- Field equations obtained from variation with respect to \( g^I \):
  \[ K^I_{ab} = 8\pi G_N T^I_{ab} \]
  Curvature tensor \( K^I_{ab} \) of second derivative order.
- Permutation symmetry of the sectors \((g^I, \Psi^I)\).
- Vacuum solution given by flat metrics \( g^I = \eta \).
Linearized multimetric gravity

- Perturbation ansatz: \( g^l = \eta + h^l \).
- Most general linearized geometry tensor:

\[
K_{ab} = P \cdot \partial^p \partial_{(a} h_{b)p} + Q \cdot \Box h_{ab} + R \cdot \partial_a \partial_b h + M \cdot \partial^p \partial^q h_{pq} \eta_{ab} + N \cdot \Box h \eta_{ab}
\]

- Parameter matrices \( P, Q, R, M, N \).
Perturbation ansatz: \( g^I = \eta + h^I \).

Most general linearized geometry tensor:

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K_{ab} = P \cdot \partial^p \partial_{(a} h_{b)p} + Q \cdot \Box h_{ab} + R \cdot \partial_a \partial_b h + M \cdot \partial^p \partial^q h_{pq} \eta_{ab} + N \cdot \Box h \eta_{ab}
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Parameter matrices \( P, Q, R, M, N \).

Permutation symmetry of the field equations:

\[
P^{IJ} = (P^+ - P^-) \delta^{IJ} + P^-, \ldots
\]

Simultaneously diagonalize parameter matrices.

\[\Rightarrow\] Field equations decouple:

\[
\mathcal{R}^1_{ab} = P_1 \partial^p \partial_{(a} h^1_{b)p} + Q_1 \Box h^1_{ab} + R_1 \partial_a \partial_b h^1 + M_1 \partial^p \partial^q h^1_{pq} \eta_{ab} + N_1 \Box h^1 \eta_{ab}
\]

\[
\mathcal{R}^i_{ab} = P_0 \partial^p \partial_{(a} h^i_{b)p} + Q_0 \Box h^i_{ab} + R_0 \partial_a \partial_b h^i + M_0 \partial^p \partial^q h^i_{pq} \eta_{ab} + N_0 \Box h^i \eta_{ab}
\]

\[\Rightarrow\] 10 parameters \( P_1, P_0, Q_1, Q_0, R_1, R_0, M_1, M_0, N_1, N_0 \).
Gauge invariance

- Gauge transformations $\triangleq$ diffeomorphisms.
Gauge invariance

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- Vacuum field equations:
  - Gauge invariance of the curvature tensors \( K^I \).
  - \( P_1 + 2Q_1 = P_1 + 2R_1 = M_1 + N_1 = 0 \)

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  - ⇒ Restriction on parameter matrices:
    \[ P_1 + 2Q_1 = P_1 + 2R_1 = M_1 + N_1 = 0 \]
- Gravitational action:
  - Bianchi identity for $\mathfrak{h}^1$.
  - Geometric identity satisfied by all perturbations $\mathfrak{h}^1$.
  - ⇒ Restriction on parameter matrices:
    \[ P_1 + 2Q_1 = P_1 + 2M_1 = R_1 + N_1 = 0 \]
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- Gravitational action:
  - Bianchi identity for \( h^1 \).
  - Geometric identity satisfied by all perturbations \( h^1 \).
  - \( \Rightarrow \) Restriction on parameter matrices:
    \[
    P_1 + 2Q_1 = P_1 + 2M_1 = R_1 + N_1 = 0
    \]

- Matter action:
  - Energy-momentum conservation: \( \nabla^l_a T^{l,ab} = 0 \).
  - Field equations imply Bianchi identities for \( h^i \).
  - Satisfied by all solutions \( h^i \) of the field equations.
  - \( \Rightarrow \) Not a geometric identity, no restriction on parameter matrices.
Consider only physical degrees of freedom.

Algebraic $3 + 1$-split and differential decomposition of $h_{ab}^I$:

- $4N$ scalars $I_1^I, I_2^I, I_3^I, I_4^I$
- $2N$ divergence-free vectors $I_{\alpha}^I, I'_{\alpha}^I$
- $N$ trace-free, divergence-free tensors $I_{\alpha\beta}^I$

Gauge invariance:

- $I_3^I, I_4^I, I_{\alpha}^I$: pure gauge quantities
- Remaining potentials $I$: gauge invariants
- Curvature tensor $\mathcal{R}_{ab}^I$ depends on gauge invariants only!

Consider wave-like solution:

$(x) = \hat{I} e^{ik_a x_\alpha} \Rightarrow$ Gravitational vacuum field equations solved only for $k_a k_a = 0$. 

$\Rightarrow$ Gravitational waves propagate at the speed of light!
Consider only physical degrees of freedom.

Algebraic 3 + 1-split and differential decomposition of $\mathcal{h}_{ab}^l$:

- $4N$ scalars $\mathcal{I}_1^l, \mathcal{I}_2^l, \mathcal{I}_3^l, \mathcal{I}_4^l$
- $2N$ divergence-free vectors $\mathcal{J}_{\alpha}^l, \mathcal{J}'_{\alpha}^l$
- $N$ trace-free, divergence-free tensors $\mathcal{I}_{\alpha\beta}^l$

Gauge invariance:

- $\mathcal{I}_3^1, \mathcal{I}_4^1, \mathcal{J}_{\alpha}^1$: pure gauge quantities
- Remaining potentials $\mathcal{J}$: gauge invariants
- Curvature tensor $\mathcal{R}_{ab}^l$ depends on gauge invariants only!

Consider wave-like solution:

$$\mathcal{J}(x) = \hat{\mathcal{J}} e^{ik_a x^a}$$

$\Rightarrow$ Gravitational vacuum field equations solved only for $k_a k^a = 0$.

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Polarizations and E(2) class

- Express Riemann tensor & $h^i$ in Newman-Penrose frame.
- Polarizations classified by reps. of E(2). [Eardley, Lee, Lightman et al. '73]
- E(2) class depends on $P_1, R_1, M_1$ for $h_1$ or $P_0, R_0, M_0$ for $h^i$.

\[ P + 2R \neq 0 \rightarrow \begin{cases} P = 0 \rightarrow N_2 = 0 \quad 2 \text{ tensors} \\ M = 0 \rightarrow N_3 \neq 0 \quad +1 \text{ scalar} \end{cases} \]

\[ P = 0 \rightarrow \begin{cases} M = 0 \rightarrow III_5 \neq 0 \quad +2 \text{ vectors} \\ \end{cases} \]
A multimetric example

Gravitational action with parameters $x, y, u, v, w, r, s$ [MH ~ PRD 03/12]:

$$S_G = \frac{1}{16\pi} \int d^4x \sqrt{g_0} \left[ x \sum_{I,J=1}^{N} g^{l\,ij} R^{j}_{\,ij} + \sum_{I=1}^{N} g^{l\,ij} \left( y R^{l}_{\,ij} + u \tilde{S}^{l}_{\,i} \tilde{S}^{l}_{\,j} + v \tilde{S}^{l}_{\,k} \tilde{S}^{l}_{\,ij} + w \tilde{S}^{l}_{\,k} \tilde{S}^{l}_{\,im} \tilde{S}^{l}_{\,jk} + \tilde{S}^{l}_{\,k} \tilde{S}^{l}_{\,m} \right) \right].$$

Connection difference tensors:

$$S^{l\,I\,J\,i}_{jk} = \Gamma^{l\,I\,i}_{jk} - \Gamma^{J\,I\,i}_{jk}, \quad S^{l\,I\,J\,j}_{i} = S^{l\,J\,k\,i}_{jk},$$

$$\tilde{S}^{J\,i}_{jk} = \frac{1}{N} \sum_{l=1}^{N} S^{l\,I\,J\,i}_{jk}, \quad \tilde{S}^{J\,i}_{jk} = \tilde{S}^{J\,k\,i}_{jk}.$$

Volume form:

$$g_0 = \prod_{l=1}^{N} \left( g^l \right)^{\frac{1}{N}}.$$
Calculate linearized field equations.

Eigenvalues of the parameter matrices:

\[
\begin{align*}
P_1 &= -2Q_1 = -2R_1 = -2M_1 = 2N_1 = Nx + y, \\
P_0 &= -Nx + y - w + r - 2s, \\
Q_0 &= \frac{Nx - y + w - 3r}{2}, \\
R_0 &= M_0 = \frac{Nx - v + 2s}{2}, \\
N_0 &= \frac{-Nx - y - u + v - s}{2}.
\end{align*}
\]

⇒ Field equations are gauge invariant. ✓

⇒ Gravitational action is diffeomorphism invariant. ✓
Obtain “fingerprint” of metric gravity theories. [Thorne, Will ’71; Will ’93]
  - Characterize gravity theories by 10 parameters.
  - PPN parameters can be measured by solar system experiments.

Extension to multimetric gravity theories. [MH, M. Wohlfarth ’10]
  - Additional 14 unobserved parameters.
  - 8 parameters can be obtained from linearized field equations.
Parametrized post-Newtonian (PPN) formalism

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- PPN formalism applied to example theory.

- Restriction of input parameters:
  \[
  y = \frac{1}{2 - N} - Nx, \quad v = \frac{6 - N}{4 - 2N} - Nx + 2u, \\
  w = -\frac{6 - N}{4 - 2N} + Nx - 3u, \quad r = -\frac{1}{2 - N} + Nx - u.
  \]

⇒ Three free parameters \( x, u, s \) remain.
⇒ Consistent with solar system experiments up to linear PPN order.
PPN consistent multimetric example

Parameter matrices:

\[ P_1 = -2Q_1 = -2R_1 = -2M_1 = 2N_1 = \frac{1}{2 - N}, \]
\[ P_0 = \frac{6 - N}{4 - 2N} - 2Nx + 2u - 2s, \quad N_0 = \frac{4 - N}{8 - 4N} + \frac{-Nx + u - s}{2}, \]
\[ Q_0 = -\frac{1}{4}, \quad R_0 = M_0 = -\frac{6 - N}{8 - 4N} + Nx - u + s. \]
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\[
Q_0 = -\frac{1}{4}, \quad R_0 = M_0 = -\frac{6 - N}{8 - 4N} + Nx - u + s.
\]

- Dependent only on single parameter \( p := Nx - u + s. \)

Generic case:

- E(2) class for \( h^1 \): \( N_2 \)
- E(2) class for \( h^i \): \( N_2 \)

\( \Rightarrow \) Effective E(2) class: \( N_2 \)

Special case \( p = \frac{6 - N}{8 - 4N} \):

- E(2) class for \( h^1 \): \( N_2 \)
- E(2) class for \( h^i \): \( \Pi_6 \)

\( \Rightarrow \) Effective E(2) class: \( \Pi_6 \)
Multimetric gravity:
- Designed to explain dark matter / dark energy effects
- $N \geq 2$ metrics and standard model copies
- Different standard model copies couple only gravitationally

Linearized multimetric gravity:
- Completely defined by parameter matrices
- Parameters restricted by gauge invariance

Gravitational waves:
- Propagation at the speed of light
- Polarizations given by representations of $E(2)$

$E(2)$ classes:
- $N_2, N_3, \Pi_5, \Pi_6$

Example theory:
- Defined by gravitational action
- Consistent up to linear PPN level
- 3 free parameters
- $E(2)$ class either $N_2$ or $\Pi_6$
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Production of gravitational waves:

- Slow-moving sources
- Binary systems
  - Expansion in multipole coefficients
  - Make use of tensor solid harmonics \(\Rightarrow\) algebraic equations
  \(\leadsto\) Watch the arXiv!

- Numerical simulations
Outlook

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- **Deflection of gravitational waves:**
  - Consider waves in a galactic background metric.
  - Calculate Shapiro delay for light and gravitational waves.
  - Crucial for arrival time measurements!
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$\Rightarrow$ Experimental data from gravitational wave experiments!