Aspects of multimetric gravity

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4. september 2012
Einstein gravity

- Gravity is described by metric tensor $g_{ab}$.
- Einstein-Hilbert action:
  \[ S_G = \frac{1}{2} \int \omega R. \]
  - Volume form $\omega$.
  - Scalar curvature $R$.
- Minimally coupled matter action:
  \[ S_M = \int \omega \mathcal{L}_M. \]
- Einstein equations:
  \[ R_{ab} - \frac{1}{2} R g_{ab} = T_{ab}. \]
4.6% visible matter.

[Komatsu et al. '09]
Application to the universe

- 4.6% visible matter.
  [Komatsu et al. '09]
- 22.8% dark matter.
  - Galaxy rotation curves.
    [de Blok, Bosma '02]
  - Anomalous light deflection.
    [Wambsganss '98]
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  - Accelerating expansion.
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⇒ Problem: What are dark matter and dark energy?
Explanations for the dark universe

Particle physics:

- **Dark matter:** [Bertone, Hooper, Silk '05]
  - Weakly interacting massive particles (WIMPs). [Ellis et al. '84]
  - Axions. [Preskill, Wise, Wilczek '83]
  - Massive compact halo objects (MACHOs). [Paczynski '86]

- **Dark energy:** [Copeland, Sami, Tsujikawa '06]
  - Quintessence. [Peebles, Ratra '88]
  - K-essense. [Chiba, Okabe, Yamaguchi '00; Armendariz-Picon, Mukhanov, Steinhardt '01]
  - Chaplygin gas. [Kamenshchik, Moschella, Pasquier '01]

Gravity:

- Modified Newtonian dynamics (MOND). [Milgrom '83]
- Tensor-vector-scalar theories. [Bekenstein '04]
- Curvature corrections. [Schuller, Wohlfarth '05; Sotiriou, Faraoni '05]
- Dvali-Gabadadze-Porrati (DGP) model. [Dvali, Gabadadze, Porrati '00, Lue '06]
- Non-symmetric gravity. [Moffat '95]
- Area metric gravity. [Punzi, Schuller, Wohlfarth '07]
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  - **New idea:** repulsive gravity ⇔ negative mass!
Three types of mass! [Bondi '57]

- Active gravitational mass $m_a$ - source of gravity: $\phi = -G_N \frac{m_a}{r}$.
- Passive gravitational mass $m_p$ - reaction on gravity: $\vec{F} = -m_p \vec{\nabla} \phi$.
- Inertial mass $m_i$ - relates force to acceleration: $\vec{F} = m_i \ddot{a}$.
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Theory relates the different types of mass:
- Momentum conservation: $m_a \sim m_p$.
- Weak equivalence principle: $m_p \sim m_i$. 

Gravity is always attractive.

Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.

Observations exist for visible mass only.
Mass in Newtonian gravity

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Dark universe from negative mass

- Idea for dark universe: standard model with $m_a = m_p = -m_i < 0$.
- Both copies couple only through gravity $\Rightarrow$ “dark”.
- Preserves momentum conservation.
- Breaks weak equivalence principle only for cross-interaction.
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$\Rightarrow$ Advantage: Dark copy $\Psi^-$ of well-known standard model $\Psi^+$:
- No new parameters.
- No unknown masses.
- No unknown couplings.
Positive and negative test masses follow different trajectories.
Two types of test mass trajectories $\Rightarrow$ two types of observers.
Observer trajectories are autoparallels of two connections $\nabla^\pm$.
Observers attach parallely transported frames to their curves.
Frames are orthonormalized using two metric tensors $g_{ab}^\pm$. 

No-go theorem forbids bimetric repulsive gravity.

[MH, M. Wohlfarth '09]

Solution:
$N \geq 3$ metrics $g_{Iab}$ and standard model copies $\Psi_I$. 

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Solution: $N \geq 3$ metrics $g_{ab}^I$ and standard model copies $\Psi^I$. 
Action and equations of motion

- $N$ metric tensors and $N$ standard model copies.
- Matter action: sum of standard model actions.
- Gravitational action:

$$S_G[g^1, \ldots, g^N] = \frac{1}{2} \int d^4x \sqrt{g_0} \left[ \sum_{I,J=1}^{N} (x + y \delta^{IJ}) g^{lij} R_{ij}^J + F(S^{IJ}) \right].$$

- Symmetric volume form $g_0 = (g^1 g^2 \ldots g^N)^{1/N}$.
- $F(S^{IJ})$ quadratic in connection difference tensors $S^{IJ} = \Gamma^I - \Gamma^J$. 

\[\Rightarrow\] Equations of motion:

$$T^I_{ab} = \sqrt{g_0} g^{I} \left[ - \frac{1}{2} N g^{I} \sum_{J,K=1}^{N} (x + y \delta^{JK}) g_{Jij} R_{Kj}^I + \sum_{J=1}^{N} (x + y \delta^{IJ}) g_{Iab} \right].$$

\[\Rightarrow\] Repulsive Newtonian limit for $N \geq 3$. 

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1. Introduction
2. Multimetric cosmology
3. Simulation of structure formation
4. Post-Newtonian consistency
5. Gravitational waves
6. Conclusion
Cosmological symmetry

- **Standard cosmology:** Robertson–Walker metrics
  \[ g_I = -n_I^2(t) dt \otimes dt + a_I^2(t) \gamma_{\alpha\beta} dx^\alpha \otimes dx^\beta. \]
  - Lapse functions \( n_I \).
  - Scale factors \( a_I \).
  - Spatial metric \( \gamma_{\alpha\beta} \) of constant curvature \( k \in \{-1, 0, 1\} \) and Riemann tensor \( R(\gamma)_{\alpha\beta\gamma\delta} = 2k \gamma_{\alpha[\gamma\gamma\delta]\beta}. \)

- **Perfect fluid matter:**
  \[ T^{l\,ab} = (\rho_l + p_l) u^{la} u^{lb} + p_l g^{lab}. \]
- **Normalization:** \( g^{l}_{ab} u^{la} u^{lb} = -1. \)
Simple cosmological model

- Early universe: radiation; late universe: dust.
- Copernican principle: common evolution for all matter sectors.
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- Copernican principle: common evolution for all matter sectors.
  \[ \Rightarrow \] Single effective energy-momentum tensor \( T^I_{ab} = T_{ab} \).
  \[ \Rightarrow \] Single effective metric \( g^I_{ab} = g_{ab} \).
  \[ \Rightarrow \] Common scale factors \( a^I = a \) and lapse functions \( n^I = n \).
  \[ \Rightarrow \] Rescale cosmological time to set \( n = 1 \).
  \[ \Rightarrow \] Ricci tensors \( R^I_{ab} = R_{ab} \) become equal.
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  \[ \Rightarrow \text{Connection differences } S^{IJ}i_{jk} = 0 \text{ vanish.} \]
  \[ \Rightarrow \text{Equations of motion simplify:} \]
  \[ (2 - N)T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}. \]
  \[ \Rightarrow \text{Negative effective gravitational constant for early / late universe.} \]
Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

\[
\rho = \frac{3}{2 - N} \left( \frac{\ddot{a}^2}{a^2} + \frac{k}{a^2} \right),
\]

\[
p = -\frac{1}{2 - N} \left( 2\frac{\dot{a}}{a} + \frac{\ddot{a}^2}{a^2} + \frac{k}{a^2} \right).
\]

⇒ Positive matter density \( \rho > 0 \) requires \( k = -1 \) and \( \ddot{a}^2 < 1 \).

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- Acceleration equation:

\[
\frac{\ddot{a}}{a} = \frac{N - 2}{6} (\rho + 3p).
\]

⇒ Acceleration must be positive for standard model matter.
Explicit solution

- Equation of state: \( p = \omega \rho \); dust: \( \omega = 0 \), radiation: \( \omega = 1/3 \).
- General solution using conformal time \( \eta \) defined by \( dt = a \, d\eta \):

\[
a = a_0 \left( \cosh \left( \frac{3\omega + 1}{2} (\eta - \eta_0) \right) \right)^{\frac{2}{3\omega + 1}},
\]

\[
\rho = \frac{3}{(N - 2)a_0^2} \left( \cosh \left( \frac{3\omega + 1}{2} (\eta - \eta_0) \right) \right)^{-\frac{6\omega + 6}{3\omega + 1}}.
\]

\( \Rightarrow \) Positive minimal radius \( a_0 \). [MH, M. Wohlfarth '10]
Cosmological evolution
Formation of galactic structures not fully understood:
- Missing dwarf problem. [Moore et al. '99]
- Core-cusp-problem. [Dubinski, Carlberg '91; Navarro et al. '96]

Structure formation in multimetric gravity:
- Perturbation of cosmological background solution.
- Model dust matter by point particles.
- Interaction between point particles given by Newtonian limit.

Implementation:
- Large particle number requires high computing power. ⇒ Use GPU computing!

Results:
- Galactic clusters and filament-like structures.
- Seemingly empty voids contain “invisible” matter. ⇒ Repulsive gravity effects from galactic voids.
- Negative gravitational lenses in galactic voids?
Structure formation - all matter types
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Structure formation - only visible matter
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Outline

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Obtain “fingerprint” of single-metric gravity theories. [Thorne, Will ’71; Will ’93]  
⇒ 10 parameters, constrained by solar system experiments.
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Extension to multimetric gravity theories. [MH, M. Wohlfarth ’10]

- Additional 14 unobserved parameters.

- 8 parameters can be obtained from linearized field equations.
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Example: multimetric action can be chosen such that

- $\alpha^+ = 1, \theta^+ = 0$: standard PPN gauge choice.
- $\gamma^+ = 1, \sigma^+_+ = -2$: experimental consistency.
- $\alpha^- = -1$: repulsive Newtonian limit.
- $\gamma^- = -1, \theta^- = 0, \sigma^-_- = 2$: additional “dark” PPN parameters.
Gravitational action with parameters $x, y, u, v, w, r, s$ [MH '12]:

$$S_G = \frac{1}{16\pi} \int d^4x \sqrt{g_0} \left[ x \sum_{I,J=1}^{N} g^{ij} R_{ij}^J + \sum_{l=1}^{N} g^{ij} \left( y R_{ij}^l + u \tilde{S}_{ij}^l \tilde{S}_{ij}^l ight) + v \tilde{S}_{ik}^l \tilde{S}_{ij}^l + w \tilde{S}_{ik}^m \tilde{S}_{mj}^l \tilde{S}_{jk}^m + g^{kl} g_{mn} \left( r \tilde{S}_{ik}^m \tilde{S}_{nj}^l + s \tilde{S}_{im}^l \tilde{S}_{jn}^k \tilde{S}_{kl}^m \right) \right].$$

Restriction of input parameters by PPN consistency:

$$y = \frac{1}{2 - N} - Nx, \quad v = \frac{6 - N}{4 - 2N} - Nx + 2u,$$

$$w = -\frac{6 - N}{4 - 2N} + Nx - 3u, \quad r = -\frac{1}{2 - N} + Nx - u.$$

$\Rightarrow$ PPN consistent theory with parameters $x, u, s$. 

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Perturbation ansatz: $g^l = \eta + h^l$.

Most general linearized vacuum field equations:

$$P \cdot \partial^p \partial_{(a h^b)p} + Q \cdot \Box h_{ab} + R \cdot \partial_a \partial_b h + M \cdot \partial^p \partial^q h_{pq} \eta_{ab} + N \cdot \Box h \eta_{ab} = 0$$


Diagonalize parameter matrices.

$\Rightarrow$ 10 parameters $P_1, P_0, Q_1, Q_0, R_1, R_0, M_1, M_0, N_1, N_0$. 
Perturbation ansatz: $g^I = \eta + h^I$.

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Calculate wave-like solutions of vacuum field equations.

⇒ Gravitational waves propagate at the speed of light.
Polarizations and E(2) class

- Polarizations classified by reps. of E(2). [Eardley, Lee, Lightman et al. '73]
- E(2) class depends on parameters $P_i, R_i, M_i$. 

$\begin{align*}
N_2 & \quad P = 0 \\
N_3 & \quad P + 2R \neq 0 \\
\text{III}_5 & \quad M = 0
\end{align*}$

2 tensors +1 scalar +2 vectors +1 scalar
PPN consistent multimetric example

Parameter values \cite{MH12}:

\[
\begin{align*}
    P_1 &= -2Q_1 = -2R_1 = -2M_1 = 2N_1 = \frac{1}{2 - N}, \\
    P_0 &= \frac{6 - N}{4 - 2N} - 2Nx + 2u - 2s, \\
    N_0 &= \frac{4 - N}{8 - 4N} + \frac{-Nx + u - s}{2}, \\
    Q_0 &= -\frac{1}{4}, \\
    R_0 &= M_0 = -\frac{6 - N}{8 - 4N} + Nx - u + s.
\end{align*}
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Parameter values [MH’12]:

\[ P_1 = -2Q_1 = -2R_1 = -2M_1 = 2N_1 = \frac{1}{2 - N}, \]

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\[ Q_0 = -\frac{1}{4}, \quad R_0 = M_0 = -\frac{6 - N}{8 - 4N} + Nx - u + s. \]

- Dependent only on single parameter \( p := Nx - u + s \).

Generic case:

- E(2) class for \( h^1 \): \( N_2 \)
- E(2) class for \( h^i \): \( N_2 \)

\[ \Rightarrow \text{Effective E}(2) \text{ class: } N_2 \]

Special case \( p = \frac{6-N}{8-4N} \):

- E(2) class for \( h^1 \): \( N_2 \)
- E(2) class for \( h^i \): \( \Pi_6 \)

\[ \Rightarrow \text{Effective E}(2) \text{ class: } \Pi_6 \]
Idea: Repulsive gravity might explain dark matter & dark energy.

⇒ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.
⇒ Cosmology features late-time acceleration and big bounce.
⇒ Structure formation features clusters and voids.
⇒ Repulsive gravity is consistent with solar system experiments.
⇒ Gravitational waves are null.
⇒ $E(2)$ class can be one of $N_2, N_3, III_5, II_6$. 
Outlook

- Work in progress:
  - Emission of gravitational waves from binary systems.
  - Post-Newtonian approximation of axially symmetric solutions.

Future work:
- Remaining PPN parameters from full multimetric PPN formalism.
- Restrict multimetric gravity theories by additional PPN bounds.
- Further construction principles, e.g., higher symmetries.
- Construct further exact solutions.
- Stability of cosmological solutions.
- Obtain restrictions from cosmological perturbation theory.
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