

Parameterized post-Newtonian limit of Horndeski's gravity theory

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- So far unexplained cosmological observations:
 - Accelerating expansion of the universe
 - Homogeneity of cosmic microwave background

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 - Modification of the laws of gravity?
 - Scalar field in addition to metric mediating gravity?
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 - **Scalar field in addition to metric mediating gravity?**
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- **Horndeski gravity** [G. W. Horndeski '74]:
 - **Scalar-tensor theory of gravity.**
 - **Most general STG with second order field equations.**
 - **Healthy, ghost-free theory.**
 - **Contains many interesting cases (Galileons, Higgs inflation. . .).**

Gravitational action

- Action functional [T. Kobayashi, M. Yamaguchi, J. 'i. Yokoyama '11]:

$$S = \sum_{i=2}^5 \int d^4x \sqrt{-g} \mathcal{L}_i[g_{\mu\nu}, \phi] + S_m[g_{\mu\nu}, \chi_m].$$

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- Gravitational Lagrangian with $X = -\nabla_\mu \phi \nabla^\mu \phi$:

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X) \square \phi,$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) \left[(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right],$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi$$

$$- \frac{1}{6} G_{5X}(\phi, X) \left[(\square \phi)^3 - 3(\square \phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right].$$

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- Free functions K, G_3, G_4, G_5 .

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- Background solution:
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- Taylor expansion of free functions:

$$K(\phi, X) = \sum_{m,n=0}^{\infty} K_{(m,n)} \psi^m X^n.$$

- Expansion coefficients:

$$K_{(m,n)} = \frac{1}{m!n!} \left. \frac{\partial^{m+n}}{\partial \phi^m \partial X^n} K(\phi, X) \right|_{\phi=\Phi, X=0}.$$

- Similar expansion for G_3, G_4, G_5 .

Post-Newtonian approximation

- Perfect fluid energy-momentum tensor:

$$T^{\mu\nu} = (\rho + \rho\Pi + p)u^\mu u^\nu + pg^{\mu\nu} .$$

- Four-velocity u^μ .
- Matter density ρ .
- Specific internal energy Π .
- Pressure p .

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- Perfect fluid energy-momentum tensor:

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- Four-velocity u^μ .
- Matter density $\rho \sim \mathcal{O}(2)$.
- Specific internal energy $\Pi \sim \mathcal{O}(2)$.
- Pressure $p \sim \mathcal{O}(4)$.
- Slow-moving source matter:

$$v^i = \frac{u^i}{u^0} \ll 1 .$$

- Assign velocity orders $|v^i|^n \sim \mathcal{O}(n)$ based on solar system.

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- Relevant terms for dynamical fields:

$$h_{00}^{(2)} , \quad h_{ij}^{(2)} , \quad h_{0j}^{(3)} , \quad h_{00}^{(4)} , \quad \psi^{(2)} , \quad \psi^{(4)} .$$

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- Time dependence only through motion of source matter.
- ⇒ Assign time derivative $\partial_0 \sim \mathcal{O}(1)$.

Spherically symmetric solution

- Static, point-like mass source:

$$\rho = M\delta(\vec{x}), \quad \Pi = 0, \quad p = 0, \quad v_j = 0.$$

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- Spherically symmetric metric:

$$g_{00} = -1 + 2G_{\text{eff}}(r)U(r) - 2G_{\text{eff}}^2(r)\beta(r)U^2(r) + \Phi^{(4)}(r) + \mathcal{O}(6),$$

$$g_{0j} = \mathcal{O}(5),$$

$$g_{ij} = [1 + 2G_{\text{eff}}(r)\gamma(r)U(r)]\delta_{ij} + \mathcal{O}(4).$$

- Newtonian potential: $U(r) = M/r$.
- Gravitational self energy $\Phi^{(4)}(r)$.
- Effective gravitational constant $G_{\text{eff}}(r)$.
- PPN parameters $\gamma(r)$ and $\beta(r)$.

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- Gravitational self energy $\Phi^{(4)}(r)$.
- Effective gravitational constant $G_{\text{eff}}(r)$.
- PPN parameters $\gamma(r)$ and $\beta(r)$.
- Consistency condition:

$$K_{(0,0)} = K_{(1,0)} = 0.$$

- Scalar field equation at $\mathcal{O}(2)$ is screened Poisson equation:

$$\psi_{,ii}^{(2)} - m_\psi^2 \psi^{(2)} = -c_\psi \rho .$$

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- Constants:

$$m_\psi = \sqrt{\frac{-2K_{(2,0)}}{K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G_{4(1,0)}^2}{G_{4(0,0)}}}},$$

$$c_\psi = \frac{G_{4(1,0)}}{2G_{4(0,0)}} \left(K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G_{4(1,0)}^2}{G_{4(0,0)}} \right)^{-1}.$$

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- Constants:

$$c_1 = -2 \frac{G_{4(1,0)} K_{(2,0)}}{G_{4(0,0)}} \left(K_{(0,1)} - 2G_{3(1,0)} + 3 \frac{G_{4(1,0)}^2}{G_{4(0,0)}} \right)^{-1},$$

$$c_2 = \frac{1}{G_{4(0,0)}} \left[\frac{1}{2} + \frac{G_{4(1,0)}^2}{2G_{4(0,0)}} \left(K_{(0,1)} - 2G_{3(1,0)} + 3 \frac{G_{4(1,0)}^2}{G_{4(0,0)}} \right)^{-1} \right].$$

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- Solve and read off PPN parameter γ :

$$\gamma(r) = \frac{c_4 + \frac{c_3 c_\psi}{m_\psi^2} (e^{-m_\psi r} - 1)}{c_2 + \frac{c_1 c_\psi}{m_\psi^2} (e^{-m_\psi r} - 1)}.$$

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- Constants:

$$c_3 = 2 \frac{G_{4(1,0)} K_{(2,0)}}{G_{4(0,0)}} \left(K_{(0,1)} - 2G_{3(1,0)} + 3 \frac{G_{4(1,0)}^2}{G_{4(0,0)}} \right)^{-1},$$

$$c_4 = \frac{1}{G_{4(0,0)}} \left[\frac{1}{2} - \frac{G_{4(1,0)}^2}{2G_{4(0,0)}} \left(K_{(0,1)} - 2G_{3(1,0)} + 3 \frac{G_{4(1,0)}^2}{G_{4(0,0)}} \right)^{-1} \right].$$

- Metric field equation:

$$h_{ij,kk}^{(2)} = (c_3 \psi^{(2)} - c_4 \rho) \delta_{ij}.$$

- Solve and read off PPN parameter γ :

$$\gamma(r) = \frac{2\omega + 3 - e^{-m_\psi r}}{2\omega + 3 + e^{-m_\psi r}}.$$

- Constants:

$$\omega = \frac{G_{4(0,0)}}{2G_{4(1,0)}^2} (K_{(0,1)} - 2G_{3(1,0)}),$$

$$m_\psi = \sqrt{\frac{-2K_{(2,0)}}{K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G_{4(1,0)}^2}{G_{4(0,0)}}}}.$$

- Calculate β from fourth order solution:

$$\beta(r) = 1 + \frac{1}{(2\omega + 3 + e^{-m_\psi r})^2} \left\{ \frac{\omega + \tau - 4\omega\sigma}{2\omega + 3} e^{-2m_\psi r} \right. \\ + (2\omega + 3)m_\psi r \left[e^{-m_\psi r} \ln(m_\psi r) - \frac{1}{2} e^{-2m_\psi r} \right. \\ \left. \left. - (m_\psi r + e^{m_\psi r}) \text{Ei}(-2m_\psi r) \right] \right. \\ \left. + \frac{6\mu r + 3(3\omega + \tau + 6\sigma + 3)m_\psi^2 r}{2(2\omega + 3)m_\psi} \left[e^{m_\psi r} \text{Ei}(-3m_\psi r) \right. \right. \\ \left. \left. - e^{-m_\psi r} \text{Ei}(-m_\psi r) \right] \right\},$$

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- Constants $m_\psi, \omega, \tau, \sigma, \mu$.

Limiting cases

- $m_\psi \rightarrow 0$, all other constants fixed and finite:

$$\gamma = \frac{\omega + 1}{\omega + 2}, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{(2\omega + 3)(2\omega + 4)^2}.$$

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- $\omega \rightarrow \infty$, all other constants fixed and finite:

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- $m_\psi r \rightarrow \infty$, large distance from the matter source:

$$\gamma = \beta = 1.$$

Example: scalar-tensor gravity with potential

- Gravitational action:

$$S_G = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(\phi R - \frac{\omega(\phi)}{\phi} \partial_\rho \phi \partial^\rho \phi - 2\kappa^2 V(\phi) \right).$$

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- PPN parameters [MH, L. Järvi, P. Kuusk, E. Randla '13]:

$$\gamma(r) = \frac{2\omega_0 + 3 - e^{-m_\psi r}}{2\omega_0 + 3 + e^{-m_\psi r}},$$

$$\beta(r) = 1 + \frac{1}{(2\omega_0 + 3 + e^{-m_\psi r})^2} \left\{ \frac{\Phi\omega_1}{2\omega_0 + 3} e^{-2m_\psi r} + (2\omega_0 + 3)m_\psi r \right. \\ \times \left[e^{-m_\psi r} \ln(m_\psi r) - (m_\psi r + e^{m_\psi r}) \text{Ei}(-2m_\psi r) - \frac{1}{2} e^{-2m_\psi r} \right] \\ \left. + \frac{3m_\psi r}{2} \left(1 - \frac{\Phi V_3}{V_2} + \frac{\Phi\omega_1}{2\omega_0 + 3} \right) [e^{m_\psi r} \text{Ei}(-3m_\psi r) - e^{-m_\psi r} \text{Ei}(-m_\psi r)] \right\}.$$

Example: Higgs inflation

- Gravitational action [F. L. Bezrukov, M. Shaposhnikov '08]:

$$S_G = \int d^4x \sqrt{-g} \left(\frac{M_{\text{Pl}}^2 - \xi \phi^2}{2} R + X - V(\phi) \right).$$

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- PPN parameters:

$$\begin{aligned} \gamma &= 1 - 4\xi^2 e^{-m_\psi r} \frac{\Phi^2}{M_{\text{Pl}}^2} + \mathcal{O}\left(\frac{\Phi^3}{M_{\text{Pl}}^3}\right), \\ \beta &= 1 + \left\{ 2\xi^3 e^{-2m_\psi r} - \xi^2 m_\psi r \left[e^{-2m_\psi r} - 2e^{-m_\psi r} \ln(m_\psi r) \right. \right. \\ &\quad \left. \left. + 2(m_\psi r + e^{m_\psi r}) \text{Ei}(-2m_\psi r) \right] \right\} \frac{\Phi^2}{M_{\text{Pl}}^2} + \mathcal{O}\left(\frac{\Phi^3}{M_{\text{Pl}}^3}\right). \end{aligned}$$

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 - Models of Higgs inflation.
 - Galileons.

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- Example theories:
 - Classical scalar-tensor gravity with arbitrary potential.
 - Models of Higgs inflation.
 - Galileons.
- PPN parameters:
 - Most general theory: obtained PPN parameters $\gamma(r)$ and $\beta(r)$.
 - Massless scalar field: only γ and β potentially deviate.
 - Reproduces and generalizes well-known results.

- Extend analysis to more general theories:
 - Allow time-dependent scalar background field $\dot{\phi} \neq 0$.
 - Theories beyond Horndeski / G^3 -inflation.
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 - Allow time-dependent scalar background field $\dot{\phi} \neq 0$.
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- Take screening mechanisms into account:
 - Vainshtein mechanism.
 - Chameleon mechanism.
 - Symmetron mechanism.

Acknowledgments

- Estonian Research Council
 - ERMOS115 “Geometric extensions of general relativity - foundations and phenomenology”
 - PUT790 “Geometric foundations of gravity and their comparison to observations”
- Archimedes Foundation
 - TK133 “The Dark Side of the Universe”

Job openings supported by these grants:

- PhD student position (deadline: 20. March 2016)
- PostDoc position (deadline: 10. April 2016)

<http://www.fi.ut.ee/en/postdoc-and-phd-in-gravity-theory>