Gravitational waves in teleparallel theories of gravity

Manuel Hohmann
(with Martin Krššák, Christian Pfeifer, Jackson Levi Said, Ulbossyn Ualikhanova)

Laboratory of Theoretical Physics - Institute of Physics - University of Tartu
Center of Excellence “The Dark Side of the Universe”

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The third Zeldovich meeting - 25. April 2018

Wise expectations

International Center for Relativistic Astrophysics Network

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ICRANet

THE THIRD ZELDOVICH MEETING SCIENTIFIC OBJECTIVES

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International Center for Relativistic Astrophysics Network (ICRANet) together with the National Academy of Sciences of Belarus organize an international conference to be held in Minsk, Belarus in April 23-27, 2018. Participation from neighboring countries such as Estonia, Latvia, Lithuania, Poland, Russia and Ukraine as well as from Balkan countries, Eastern and Western Europe and the Americas is expected.
Exceptionally wide research interests of Ya. B. Zeldovich ranging from chemical physics, elementary particle and nuclear physics to astrophysics and cosmology provide the topics to be covered at the conference:
Early cosmology, large scale structure, cosmic microwave background;
Neutron stars, black holes, gamma-ray bursts, supernovae, hypernovae;
Ultra high energy particles;
Gravitational waves.
Many lectors at the conference will be the members of the world-famous scientific school in astrophysics and cosmology, founded by Ya. B. Zeldovich, who now became leading scientists in these fields in many countries worldwide including Germany, Italy, USA and Russia.
This conference will follow a very successful international conferences in honor of Ya. B. Zeldovich, held in Minsk in 2009 and in 2014.

For suggestions&comments write to the Webmaster

Zeldovich in Estonia (end of 1970s)

Summer school on cosmology, Tõravere 1962

Outline

1. Introduction
2. Waves in torsion gravity
3. Waves in non-metricity gravity
4. Conclusion

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  - Accelerating phases in the history of the Universe?
  - Relation between gravity and gauge theories?
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  - Based on tetrad and flat spin connection.
  - Describes gravity as gauge theory of the translation group.
  - First order action, second order field equations.
  - Spin connection as Lorentz gauge degree of freedom.

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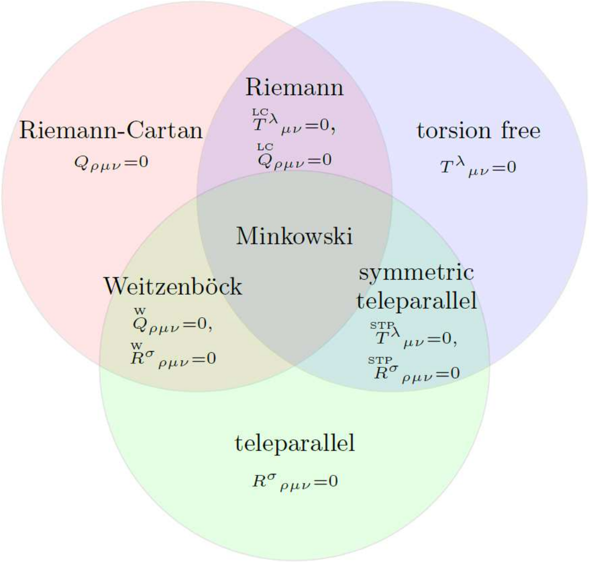
- Gravity formulated as gauge theories.

Overview of geometries



Newman-Penrose formalism

- Complex double null basis of the tangent bundle:

$$l = \partial_t + \partial_z\,, \quad n = \frac{\partial_t - \partial_z}{2}\,, \quad m = \frac{\partial_x + i\partial_y}{\sqrt{2}}\,, \quad \bar{m} = \frac{\partial_x - i\partial_y}{\sqrt{2}}\,.$$

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- Consider plane null wave with $k_\mu = -\omega l_\mu$ and $u = t - z$:

$$h_{\mu\nu} = H_{\mu\nu} e^{ik_\mu x^\mu} = H_{\mu\nu} e^{i\omega u}\,.$$

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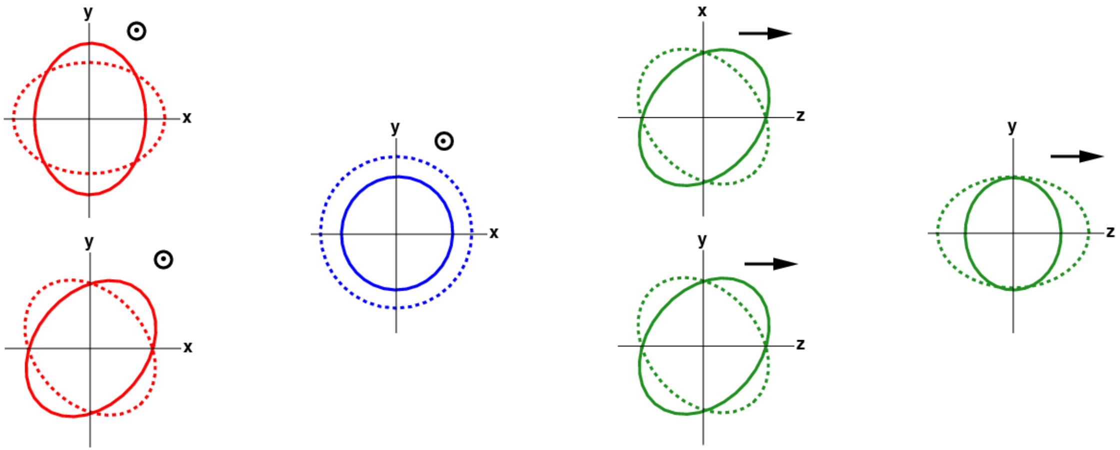
- Riemann tensor determined by “electric” components:

$$\Psi_2 = -\frac{1}{6}R_{nlnl} = \frac{1}{12}\ddot{h}_{ll}\,, \qquad \Psi_3 = -\frac{1}{2}R_{nln\bar{m}} = \frac{1}{4}\ddot{h}_{l\bar{m}}\,,$$

$$\Psi_4 = -R_{n\bar{m}n\bar{m}} = \frac{1}{2}\ddot{h}_{\bar{m}\bar{m}}\,, \qquad \Phi_{22} = -R_{nmn\bar{m}} = \frac{1}{2}\ddot{h}_{m\bar{m}}\,.$$

Polarisations of gravitational waves



$\Psi_4, \bar{\Psi}_4$ Φ_{22} $\Psi_3, \bar{\Psi}_3$ Ψ_2

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Field content and geometry

- Fundamental fields in the gravity sector:
  - Coframe field $\theta^a = \theta^a{}_\mu \mathrm{d}x^\mu$.
  - Flat spin connection $\overset{\bullet}{\omega}{}^a{}_b = \overset{\bullet}{\omega}{}^a{}_{b\mu} \mathrm{d}x^\mu$.

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- Derived quantities:
  - Frame field $e_a = e_a{}^\mu \partial_\mu$ with $\iota_{e_a}\theta^b = \delta^b_a$.
  - Metric $g_{\mu\nu} = \eta_{ab}\theta^a{}_\mu \theta^b{}_\nu$.
  - Volume form $\theta \mathrm{d}^4x = \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3$.
  - Levi-Civita connection

$$\overset{\circ}{\omega}_{ab} = -\frac{1}{2}(\iota_{e_b}\iota_{e_c}\mathrm{d}\theta_a + \iota_{e_c}\iota_{e_a}\mathrm{d}\theta_b - \iota_{e_a}\iota_{e_b}\mathrm{d}\theta_c)\theta^c\,.$$

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  - Torsion $T^a = \mathrm{d}\theta^a + \overset{\bullet}{\omega}{}^a{}_b \wedge \theta^b$.
- Gauge fixing
  - Perform local Lorentz transformation:

$$\theta'^a = \Lambda^a{}_b\theta^b\,, \quad \overset{\bullet}{\omega}{}'^a{}_b = \Lambda^a{}_c\overset{\bullet}{\omega}{}^c{}_d\Lambda_b{}^d + \Lambda^a{}_c\mathrm{d}\Lambda_b{}^c\,.$$

$\Rightarrow$ Weitzenböck gauge: set $\overset{\bullet}{\omega}{}^a{}_b \equiv 0$.

Most general action and corresponding field equations

- Most general action:

$$S = \frac{1}{2\kappa^2} \int d^4x\, e\, (c_1 T^{\mu\nu\rho} T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho} T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho} T_{\nu}{}^{\nu\rho}) \,.$$

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$$\partial_\sigma \left(F^{\mu\rho\sigma} + B^{\mu\rho\sigma}\right) = 0\,.$$

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$$\partial_\sigma \left(F^{\mu\rho\sigma} + B^{\mu\rho\sigma} \right) = 0 .$$

- Field tensors:
  - Symmetric perturbation part:

$$F^{\mu\rho\sigma} = (2c_1 + c_2)\left(\partial^\sigma \phi^{\mu\rho} - \partial^\rho \phi^{\mu\sigma}\right) + c_3 \left[(\partial^\sigma \phi^\alpha{}_\alpha - \partial_\alpha \phi^{\alpha\sigma})\eta^{\mu\rho} - (\partial^\rho \phi^\alpha{}_\alpha - \partial_\alpha \phi^{\alpha\rho})\eta^{\mu\sigma} \right] .$$

  - Antisymmetric perturbation part:

$$B^{\mu\rho\sigma} = (2c_1 - c_2)\left(\partial^\sigma a^{\mu\rho} - \partial^\rho a^{\mu\sigma}\right) + (2c_2 + c_3)\partial^\mu a^{\sigma\rho} .$$

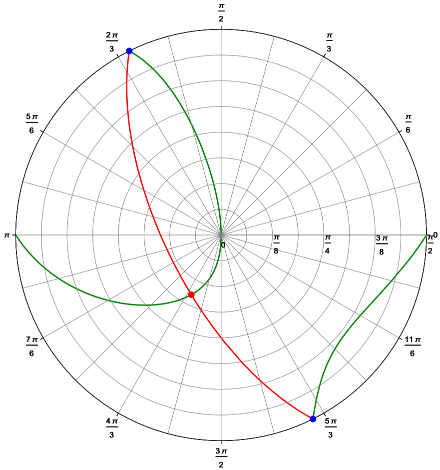
Newman-Penrose decomposition

Field equations expressed in Newman-Penrose basis

$$
\begin{aligned}
0 &= E_{nn} = (2c_1 + c_2 + c_3)\partial_n^2\phi_{nl} + 2c_3\phi_{m\bar{m}} + (2c_1 + c_2 + c_3)\partial_n^2 a_{nl}\,,\\
0 &= E_{mn} = (2c_1 + c_2)\partial_n^2\phi_{ml} + (2c_1 - c_2)\partial_n^2 a_{ml}\,,\\
0 &= E_{\bar{m}n} = (2c_1 + c_2)\partial_n^2\phi_{\bar{m}l} + (2c_1 - c_2)\partial_n^2 a_{\bar{m}l}\,,\\
0 &= E_{nm} = -c_3\partial_n^2\phi_{lm} - (2c_2 + c_3)\partial_n^2 a_{lm}\,,\\
0 &= E_{n\bar{m}} = -c_3\partial_n^2\phi_{l\bar{m}} - (2c_2 + c_3)\partial_n^2 a_{l\bar{m}}\\
0 &= E_{m\bar{m}} = -c_3\partial_n^2\phi_{ll}\,,\\
0 &= E_{ln} = (2c_1 + c_2)\partial_n^2\phi_{ll}\,,\,.
\end{aligned}
$$

Gravitational wave polarisations

$$c_1 = \sin\theta\cos\phi$$
$$c_2 = \sin\theta\sin\phi$$
$$c_3 = \cos\theta$$



- N_2
- N_3
- III_5
- II_6

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  - Flat, symmetric affine connection $\overset{\times}{\Gamma}{}^{\mu}{}_{\nu\rho}$.

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  - Levi-Civita connection

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(\partial_{\mu}g_{\sigma\nu} + \partial_{\nu}g_{\mu\sigma} - \partial_{\sigma}g_{\mu\nu}\right).$$

  - Non-metricity $Q_{\rho\mu\nu} = \overset{\times}{\nabla}_{\rho}g_{\mu\nu}$.

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  - Non-metricity $Q_{\rho\mu\nu} = \overset{\times}{\nabla}_\rho g_{\mu\nu}$.
- Gauge fixing
  - Perform local coordinate transformation:

$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}g_{\alpha\beta}\,, \quad \overset{\times}{\Gamma}'^{\rho}{}_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial x'^\rho}{\partial x^\gamma}\overset{\times}{\Gamma}^{\gamma}{}_{\alpha\beta} + \frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\frac{\partial x'^\rho}{\partial x^\alpha}\,.$$

$\Rightarrow$ Coincident gauge: set $\overset{\times}{\Gamma}^{\rho}{}_{\mu\nu} \equiv 0 \Rightarrow Q_{\rho\mu\nu} = \partial_\rho g_{\mu\nu}$.

Most general action and corresponding field equations

- Most general action:

$$S = -\int d^4x \frac{\sqrt{-g}}{2}\left[c_1 Q^\alpha{}_{\mu\nu} + c_2 Q_{(\mu}{}^\alpha{}_{\nu)} + c_3 Q^\alpha g_{\mu\nu} + c_4 \delta^\alpha_{(\mu}\tilde{Q}_{\nu)} + \frac{c_5}{2}\left(\tilde{Q}^\alpha g_{\mu\nu} + \delta^\alpha_{(\mu}Q_{\nu)}\right)\right]$$

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- Linearized field equations:

$$\begin{aligned} 0 = {} & 2c_1 \eta^{\alpha\sigma}\partial_\alpha\partial_\sigma h_{\mu\nu} + c_2 \eta^{\alpha\sigma}\left(\partial_\alpha\partial_\mu h_{\sigma\nu} + \partial_\alpha\partial_\nu h_{\sigma\mu}\right) + 2c_3 \eta_{\mu\nu}\eta^{\tau\omega}\eta^{\alpha\sigma}\partial_\alpha\partial_\sigma h_{\tau\omega} \\ & + c_4 \eta^{\omega\sigma}(\partial_\mu\partial_\omega h_{\nu\sigma} + \partial_\nu\partial_\omega h_{\mu\sigma}) + c_5 \eta_{\mu\nu}\eta^{\omega\gamma}\eta^{\alpha\sigma}\partial_\alpha\partial_\omega h_{\sigma\gamma} + c_5 \eta^{\omega\sigma}\partial_\mu\partial_\nu h_{\omega\sigma}\,. \end{aligned}$$

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- Terms involving c_1 and c_3 do not contribute for a null wave $\Box h_{\mu\nu} = 0$.

Field equations expressed in Newman-Penrose basis

$$\begin{aligned}
0 &= E_{nn} = -2(c_2\ddot{h}_{ln} + c_4\ddot{h}_{nl} + c_5\ddot{h}_{nl} - c_5\ddot{h}_{m\bar{m}})\,,\\
0 &= E_{mn} = E_{nm} = -(c_2 + c_4)\ddot{h}_{lm}\,,\\
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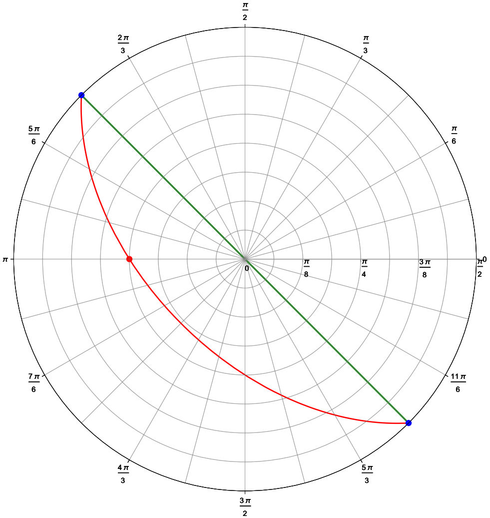

- N_2 (red)
- N_3 (white)
- III_5 (green)
- II_6 (blue)

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  - Fields are metric and flat, symmetric affine connection.
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  - Most general theory needs 5 parameters at linearized level.
- Results:
  - Gravitational waves propagate at the speed of light (not shown in this talk).
  - Polarisation classes N_2, N_3, III_5, II_6: tensor modes always exist, maybe more.

Acknowledgments

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  Eesti Teadusagentuur
  Estonian Research Council

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Teleparallel gravity workshop

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http://hexagon.fi.tartu.ee/~telegrav2018/