The universe as a whole in teleparallel gravity

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Center of Excellence “The Dark Side of the Universe”

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Outline

1 Overview

2 Teleparallel gravity and cosmology

3 Symmetric teleparallel gravity and cosmology

4 Conclusion
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2 Teleparallel gravity and cosmology

3 Symmetric teleparallel gravity and cosmology

4 Conclusion
Open questions in cosmology and gravity:

- Accelerating phases in the history of the Universe - dark energy, inflation?
- Relation between gravity, quantum theory and gauge theories?

Teleparallel gravity and symmetric teleparallel gravity:

- Based on a different (flat) connection - gravity is not mediated by curvature.
- Teleparallel gravity: interaction is mediated by the torsion.
- Symmetric teleparallel gravity: interaction is mediated by the non-metricity.

Modified gravity theories based on (symmetric) teleparallel gravity:

- Contains $f(T)$ gravity [Bengochea, Ferraro '09] and $f(Q)$ gravity [Beltran Jimenez, Heisenberg, Koivisto '17].
- Contains new GR [Hayashi, Shirafuji '79] and newer GR [Beltran Jimenez, Heisenberg, Koivisto '17].
- Contains teleparallel dark energy [Geng '11].
- Contains scalar-torsion gravity in covariant formulation [MH, Järv, Ualikhanova '18].
- Contains scalar-non-metricity gravity [Järv, Rünkla, Saal, Vilson '18].

Teleparallel cosmology - how to describe the Universe as a whole:

- Flat cosmology allows for de Sitter attractors [MH, Järv, Ualikhanova '17].
- Make use of cosmological symmetry in order to find further solutions?
- Modified Friedmann equations for non-flat models?
- How to distinguish and exclude models based on cosmological observables?
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  - How to distinguish and exclude models based on cosmological observables?
The trinity of geometric models of gravity

- Riemann-Cartan: $Q_{\rho \mu \nu} = 0$
- Riemann: $\left\{ \begin{array}{l} T^\lambda_{\mu \nu} = 0, \\
\mathrm{LC} \quad Q_{\rho \mu \nu} = 0 \end{array} \right.$
- Torsion free: $T^\lambda_{\mu \nu} = 0$
- Weitzenböck: $\begin{array}{l} w^w Q_{\rho \mu \nu} = 0, \\
\mathrm{w} \quad w^w R^\sigma_{\rho \mu \nu} = 0 \end{array}$
- Minkowski: symmetric teleparallel $\begin{array}{l} \mathrm{STP} \quad T^\lambda_{\mu \nu} = 0, \\
\mathrm{STP} \quad R^\sigma_{\rho \mu \nu} = 0 \end{array}$
- Teleparallel: $R^\sigma_{\rho \mu \nu} = 0$
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2. Teleparallel gravity and cosmology
3. Symmetric teleparallel gravity and cosmology
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Modified teleparallel gravity: $f(T)$ theory action and field equations

- **Gravitational action** [Bengochea, Ferraro '09]:

\[
S = \frac{1}{2\kappa^2} \int_M f(T) \theta d^4x + S_m[\theta^a, \chi^I].
\]
Gravitational action \cite{Bengochea:Ferraro'09}:

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Field equations:

- Symmetric part of the tetrad field equations:
  \[ \frac{1}{2} fg_{\mu\nu} + \overset{\circ}{\nabla}_\rho \left( f_T S_{(\mu\nu)}^{\rho}\right) - \frac{1}{2} f_T S_{(\mu}^{\rho\sigma} T_{\nu)\rho\sigma} = \kappa^2 \Theta_{\mu\nu}, \]

- Antisymmetric part of the tetrad field equations = connection equations:
  \[ \partial_{[\rho} f_T T^{\rho}_{\mu\nu]} = 0. \]
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- **Terms appearing in the action and field equations:**
  - Superpotential: \[ S_{\rho}^{\mu\nu} = \frac{1}{2} \left( T^{\nu\mu}_{\rho} + T_{\rho}^{\mu\nu} - T^{\mu\nu}_{\rho} \right) - \delta^\mu_{\rho} T_{\sigma}^{\sigma\nu} + \delta^\nu_{\rho} T_{\sigma}^{\sigma\mu}. \]
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- Torsion scalar: \[ T = \frac{1}{2} T_{\rho}^{\nu} \mu, S_{\rho}^{\mu\nu}. \]
Modified teleparallel gravity: $f(T)$ theory action and field equations

- **Gravitational action** [Bengochea, Ferraro '09]:
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  S = \frac{1}{2\kappa^2} \int_M f(T) \theta \, d^4x + S_m[\theta^\alpha, \chi^\prime].
  \]

- **Field equations**:
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  - Superpotential: $S_\rho^{\mu\nu} = \frac{1}{2} \left( T^{\nu\mu} \rho + T^{\mu\nu} \rho - T^{\mu\nu} \rho \right) - \delta^{\mu}_{\rho} T^{\sigma\nu} + \delta^{\nu}_{\rho} T^{\sigma\mu}$.
  - Torsion scalar: $T = \frac{1}{2} T^{\rho}_{\mu\nu} S_\rho^{\mu\nu}$.
  - Energy-momentum tensor $\Theta_{\mu\nu}$ derived from the matter part $S_m$ of the action.
Spatially flat ($k = 0$) $f(T)$ cosmology as a dynamical system

- Ansatz for spatially flat ($k = 0$) cosmology:

$$
\theta^a_{\mu} = \text{diag} \left( 1, a(t), a(t), a(t) \right), \quad \omega^a_{\mu} = 0 \quad \Rightarrow \quad g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \delta_{ij} dx^i dx^j.
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- Ansatz for perfect fluid matter:
  \[ \Theta^{\mu\nu} = (\rho + p) u^\mu u^\nu + pg^{\mu\nu}, \quad u^\mu = (1, 0, 0, 0). \]
Spatially flat \((k = 0)\) \(f(T)\) cosmology as a dynamical system

- Ansatz for spatially flat \((k = 0)\) cosmology:
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  \theta^a_{\mu} = \text{diag}(1, a(t), a(t), a(t)) \quad \text{and} \quad \omega^a_{b\mu} = 0 \quad \Rightarrow \quad g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\delta_{ij}dx^i dx^j.
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- Cosmological field equations using \(T = -6H^2\):
  \[
  12H^2 f_T + f = 2\kappa^2 \rho, \\
  48H^2 \dot{H} f_{TT} - (12H^2 + 4\dot{H})f_T - f = 2\kappa^2 p,
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- Consider mixture of dust and radiation matter:
  \[
  \rho = \rho_m + \rho_r, \quad p = \rho_m + p_r, \quad \rho_m = 0, \quad p_r = \frac{1}{3}\rho_r.
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  \[ \rho = \rho_m + \rho_r, \quad p = p_m + \rho_r, \quad \rho_m = 0, \quad \rho_r = \frac{1}{3} \rho_r. \]

- Cosmological dynamics as a dynamical system [MH, Järv, Ualikhanova '17]:
  
  \[ W(H) = 12H^2 f_T + f, \quad X = \frac{\rho_r}{\rho_r + \rho_m} \Rightarrow \dot{X} = HX(X - 1), \quad \dot{H} = -\frac{(X + 3) H}{(\ln W)_H}. \]
Consider simple power law model:

\[ f(T) = T + \alpha (-T)^n. \]
Example: \( f(T) = T + \alpha(-T)^n \) cosmology and evolution

- Consider simple power law model:
  \[
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- Function in dynamical system:
  \[
  W = 6H^2 + (1 - 2n)\alpha(6H^2)^n.
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  \[ f(T) = T + \alpha(-T)^n. \]

- Function in dynamical system:
  \[ W = 6H^2 + (1 - 2n)\alpha(6H^2)^n. \]

- For $\alpha > 0$, $\frac{1}{2} < n < 1$ or $\alpha < 0$, $n < \frac{1}{2}$:
  - Big bang at $H = \infty$, $X = 1$.
  - Transition from $\ddot{a} < 0$ to $\ddot{a} > 0$.
  - De Sitter attractor at $H = H^*$, $X = 0$.
  - Phantom or non-phantom, no crossing.
The non-flat case: $k = 1$ cosmology

- Ansatz for $k = 1$ tetrad:

$$
\theta^a_\mu = \text{diag} \left( 1, \frac{a(t)}{\sqrt{1 - r^2}}, a(t) r, a(t) r \sin \vartheta \right).
$$

Resulting FLRW metric:

$$
g_{\mu \nu} \, dx^\mu \, dx^\nu = dt^2 - \frac{a^2(t)}{4} \left( dr^2 + \frac{1-r^2}{r^2} \left( d\vartheta^2 + \sin^2 \vartheta \, d\phi^2 \right) \right).
$$

Solve antisymmetric part of the field equations using non-vanishing spin connection:

\begin{align*}
\bar{\omega}^1_2 \vartheta &= -\sqrt{1-r^2}, \\
\bar{\omega}^1_3 \vartheta &= r, \\
\bar{\omega}^2_3 r &= -\frac{1}{\sqrt{1-r^2}} \\
\bar{\omega}^2_3 \phi &= -\cos \vartheta.
\end{align*}
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$$g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - r^2} + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \right].$$
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- Solve antisymmetric part of the field equations using non-vanishing spin connection:
  \[
  \begin{align*}
  \dot{\omega}^1_{2\vartheta} &= -\omega^2_{1\vartheta} = -\sqrt{1 - r^2}, & \dot{\omega}^1_{2\phi} &= -\omega^2_{1\phi} = -r \sin \vartheta, & \dot{\omega}^1_{3\vartheta} &= -\omega^3_{1\vartheta} = r, \\
  \dot{\omega}^1_{3\phi} &= -\omega^3_{1\phi} = -\sqrt{1 - r^2} \sin \vartheta, & \dot{\omega}^2_{3r} &= -\omega^3_{2r} = -\frac{1}{\sqrt{1 - r^2}}, & \dot{\omega}^2_{3\phi} &= -\omega^3_{2\phi} = -\cos \vartheta.
  \end{align*}
  \]
The non-flat case: $k = 1$ Friedmann equations

- Friedmann equations:

\[
\begin{align*}
\dot{f} + 12f_T H^2 &= 2\kappa^2 \rho, \\
\dot{f} - 48f_{TT} \left(\dot{H} + \frac{1}{a^2}\right) H^2 + 12f_T H^2 + 4f_T \left(\dot{H} - \frac{1}{a^2}\right) &= -2\kappa^2 p.
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- Constraint equation is essentially unchanged compared to $k = 0$. 
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\end{align*}
\]

- Constraint equation is essentially unchanged compared to $k = 0$.
- Dynamical equation now also depends on the scale factor.

$\Rightarrow$ Additional dimension in dynamical systems analysis.
The non-flat case 2: $k = -1$ cosmology

- Ansatz for $k = -1$ tetrad:

$$\theta^a_\mu = \text{diag} \left( 1, \frac{a(t)}{\sqrt{1 + r^2}}, a(t)r, a(t)r \sin \vartheta \right).$$
The non-flat case 2: $k = -1$ cosmology

- Ansatz for $k = -1$ tetrad:

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- Resulting FLRW metric:

\[ g_{\mu\nu} dx^{\mu} dx^{\nu} = dt^2 - a^2(t) \left[ \frac{dr^2}{1 + r^2} + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \right]. \]
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- Solve antisymmetric part of the field equations using non-vanishing spin connection [MH, Järv, Ualikhanova '18]:

$$\begin{align*}
\dot{\omega}^0_{1r} &= \dot{\omega}^1_{0r} = \frac{1}{\sqrt{1 + r^2}}, & \dot{\omega}^0_{2\vartheta} &= \dot{\omega}^2_{0\vartheta} = r, & \dot{\omega}^0_{3\varphi} &= \dot{\omega}^3_{0\varphi} = r \sin \vartheta, \\
\dot{\omega}^1_{2\vartheta} &= -\dot{\omega}^2_{1\vartheta} = -\sqrt{1 + r^2}, & \dot{\omega}^1_{3\varphi} &= -\dot{\omega}^3_{1\varphi} = -\sqrt{1 + r^2} \sin \vartheta, & \dot{\omega}^2_{3\varphi} &= -\dot{\omega}^3_{2\varphi} = -\cos \vartheta.
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- Alternative complex choice of the spin connection [Capozziello, Luongo, Richard Pincak, Ravanpak '18]:

\[
\begin{align*}
\omega^1_{2\vartheta} &= -\omega^2_{1\vartheta} = -\sqrt{1 - r^2}, & \omega^1_{2\varphi} &= -\omega^2_{1\varphi} = -ir \sin \vartheta, & \omega^1_{3\vartheta} &= -\omega^3_{1\vartheta} = ir, \\
\omega^1_{3\varphi} &= -\omega^3_{1\varphi} = -\sqrt{1 - r^2 \sin \vartheta}, & \omega^2_{3r} &= -\omega^3_{2r} = -\frac{i}{\sqrt{1 - r^2}}, & \omega^2_{3\varphi} &= -\omega^3_{2\varphi} = -\cos \vartheta.
\end{align*}
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Trouble ahead? Non-unique evolution in $k = -1$ cosmology

- Friedmann equations derived using real spin connection [MH, Järv, Ualikhanova '18]:

$$f + 12f_T H^2 = 2\kappa^2 \rho,$$

$$f - 48f_{TT} \left( \dot{H} + \frac{H}{a} \right) \left( H - \frac{1}{a} \right)^2 + 12f_T H \left( H - \frac{1}{a} \right) + 4f_T \left( \dot{H} + \frac{1}{a^2} \right) = -2\kappa^2 \rho.$$
Trouble ahead? Non-unique evolution in $k = -1$ cosmology

- **Friedmann equations derived using real spin connection** [MH, Järv, Ualikhanova '18]:

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\]
\[
f - 48f_{TT} \left( \dot{H} + \frac{H}{a} \right) \left( H - \frac{1}{a} \right)^2 + 12f_T H \left( H - \frac{1}{a} \right) + 4f_T \left( \dot{H} + \frac{1}{a^2} \right) = -2\kappa^2 p.
\]

- **Friedmann equations derived using complex spin connection** [Capozziello, Luongo, Richard Pincak, Ravanpak '18]:

\[
f + 12f_T H^2 = 2\kappa^2 \rho,
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\[
f - 48f_{TT} \left( \dot{H} - \frac{1}{a^2} \right) H^2 + 12f_T H^2 + 4f_T \left( \dot{H} + \frac{1}{a^2} \right) = -2\kappa^2 p.
\]
Trouble ahead? Non-unique evolution in $k = -1$ cosmology

• Friedmann equations derived using **real** spin connection \([MH, \text{Järv, Ualikhanova ‘18}]:\)

\[
\begin{align*}
f + 12 f \dot{H}^2 &= 2\kappa^2 \rho, \\
f - 48 f_{TT} \left(\dot{H} + \frac{H}{a}\right) \left(H - \frac{1}{a}\right)^2 &+ 12 f \dot{H} \left(H - \frac{1}{a}\right) + 4 f \left(\dot{H} + \frac{1}{a^2}\right) = -2\kappa^2 p.
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\]

• Different field equations depending on choice of the (unobservable) spin connection.
• Evolution of the Universe depends on a gauge variable?
Add scalar fields $\phi = (\phi^A)$ to the set of dynamical variables.
The next step: scalar-torsion gravity action and field equations

- Add scalar fields $\phi = (\phi^A)$ to the set of dynamical variables.
- Example for gravitational action without derivative couplings [MH, L. Järv, U. Ualikhanova ’18]:

$$S = \frac{1}{2\kappa^2} \int_M \left[ f(T, \phi) + Z_{AB}(\phi) g^{\mu\nu} \phi^A_{,\mu} \phi^B_{,\nu} \right] \theta d^4x + S_m[\theta^a, \chi^I].$$
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Field equations:
- Symmetric part of the tetrad field equations:

\[
\frac{1}{2} fg_{\mu\nu} + \nabla_{\rho} \left( f_T S_{(\mu\nu)^\rho} \right) - \frac{1}{2} f_T S_{(\mu}^{\rho\sigma} T_{\nu)\rho\sigma} - Z_{AB} \phi_A^\rho \phi_B^\nu - \frac{1}{2} Z_{AB} \phi_A^\rho \phi_B^\sigma g^{\rho\sigma} g_{\mu\nu} = \kappa^2 \Theta_{\mu\nu},
\]

- Antisymmetric part of the tetrad field equations = connection equations:

\[
\partial[\rho f_T T^{\rho \mu\nu}] = 0.
\]

- Scalar field equation:

\[
f_{\phi^A} \left( 2Z_{AB,\phi^C} - Z_{BC,\phi^A} \right) g^{\mu\nu} \phi_B^\mu \phi_C^\nu - 2Z_{AB} \nabla^2 \phi^B = 0.
\]

- Richer cosmology, can be further generalized [MH ‘18], [MH, Pfeifer ‘18] & [MH ‘18].
A more general torsion scalar: new general relativity

- Action depends on three parameters $c_i$ [Hayashi, Shirafuji ’79]:

$$ S = \frac{1}{2\kappa^2} \int_M \left( c_1 T^{\mu\nu\rho} T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho} T_{\rho\nu\mu} + c_3 T^{\mu}_{\mu\rho} T^{\nu\rho} \right) \theta^4 x + S_m[\theta^a, \chi^l] $$
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  - II$_6$ - 6 polarizations.
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Outline

1. Overview
2. Teleparallel gravity and cosmology
3. Symmetric teleparallel gravity and cosmology
4. Conclusion
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Terms appearing in the action and field equations:

- Superpotential: \( P_{\mu\nu}^\alpha = -\frac{1}{4} Q_{\mu\nu}^\alpha + \frac{1}{2} Q_{(\mu}^\alpha \nu) + \frac{1}{4} Q^\alpha g_{\mu\nu} - \frac{1}{4} (\tilde{Q}^\alpha g_{\mu\nu} + \delta_{(\mu}^\alpha Q_{\nu)} \).
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Modified teleparallel gravity: $f(Q)$ theory action and field equations

- **Gravitational action** [Beltran Jimenez, Heisenberg, Koivisto ’17]:

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- Terms appearing in the action and field equations:

  - **Superpotential:**
    \[ P^\alpha_{\mu\nu} = -\frac{1}{4} Q^\alpha_{\mu\nu} + \frac{1}{2} Q_{(\mu}^\alpha {\nu)} + \frac{1}{4} Q^\alpha g_{\mu\nu} - \frac{1}{4} \left( \tilde{Q}^\alpha g_{\mu\nu} + \delta^\alpha_{(\mu} Q_{\nu)} \right) . \]

  - **Non-metricity scalar:**
    \[ Q = Q^\rho_{\mu\nu} P^\rho_{\mu\nu} \] and vectors \[ Q_\mu = Q^\nu_{\nu\mu} \] & \[ \tilde{Q}_\mu = Q_{\mu\nu} \nu . \]

  - **Energy-momentum tensor** \[ \Theta_{\mu\nu} \] derived from the matter part \[ S_m \] of the action.
Choose *coincident gauge* $\Gamma_{\nu\rho}^\mu = 0$ and $k = 0$ FLRW metric [Beltran Jimenez, Heisenberg, Koivisto '17]

\[ g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \delta_{ij} dx^i dx^j. \]
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$$g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \delta_{ij} dx^i dx^j.$$ 

Cosmological field equations:

$$12H^2 f_Q + f = 2\kappa^2 \rho,$$

$$48H^2 \dot{H} f_{QQ} - (12H^2 + 4\dot{H}) f_Q - f = 2\kappa^2 p,$$
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Cosmological dynamics essentially equivalent to $f(T)$ cosmology.
Action involves in addition also scalar field $\phi$ [Järv, Rünkla, Saal, Vilson ’18]:

$$S = \frac{1}{2\kappa^2} \int_M \left[ A(\phi) Q - B(\phi) g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - 2\mathcal{V}(\phi) \right] \sqrt{-g} \, d^4x + S_m[g_{\mu\nu}, \chi^I].$$
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- Cosmological dynamics for $k = 0$ FLRW metric:

$$ H^2 = \frac{1}{3A} \left( \kappa^2 \rho + \frac{1}{2} B \dot{\phi}^2 + \mathcal{V} \right), $$

$$ 2\dot{H} + 3H^2 = \frac{1}{A} \left( -2A' H \dot{\phi} - \frac{1}{2} B \dot{\phi}^2 + \mathcal{V} - \kappa^2 p \right), $$

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- Rich cosmology that deserves further studies (dynamical system).
A more general non-metricity scalar: newer general relativity

- Action depends on five parameters $c_i$ [Beltran Jimenez, Heisenberg, Koivisto ‘17]:

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- Reduces to STEGR for

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[MH, Pfeifer, Said, Ualikhanova '18]
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Conclusion

**Summary:**
- Teleparallel and symmetric teleparallel gravity use geometries without curvature.
- Gravity is mediated by torsion in teleparallel gravity.
- Gravity is mediated by non-metricity in symmetric teleparallel gravity.
- Rich cosmology in $f(T)$ and $f(Q)$ theories.
- Even richer cosmology when adding scalar fields, different from scalar-curvature.
- Possible ambiguity in cosmological evolution for certain cases.
- Certain theories can be distinguished using gravitational waves.

Outlook:
- Enhance analysis of cosmology by, e.g., cosmological perturbations.
- Resolve ambiguity in cosmological solutions and evolution.
- Obtain constraints on shown theories, chart the landscape of parameters.

Describe the Universe as a whole in teleparallel gravity!
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