30. Find $\lim_{x\to\infty} \frac{1-x^2}{8x^2+5}$.

x	10	100	1000
$f(x)$			

In Exercises 31–50, evaluate the indicated limits by direct evaluation as in Examples 10–14. Change the form of the function where necessary.

31. $\lim_{x\to3}(3x-2)$
32. $\lim_{x\to4}\sqrt{x^2-7}$
33. $\lim_{x\to0}\frac{6x^2+x}{x}$
34. $\lim_{v\to2}\frac{4v^2-8v}{v-2}$
35. $\lim_{x\to-1}\frac{x^2-1}{3x+3}$
36. $\lim_{x\to3}\frac{x^2-2x-3}{3-x}$
37. $\lim_{h\to3}\frac{h^3-27}{h-3}$
38. $\lim_{x\to1/3}\frac{9x-3}{3x^2+5x-2}$
39. $\lim_{x\to1}\frac{(2x-1)^2-1}{2x-2}$
40. $\lim_{x\to4}\frac{|x-4|}{x-4}$
41. $\lim_{p\to-1}\sqrt{p}(p+1.3)$
42. $\lim_{x\to1}(x-1)\sqrt{x^2-4}$
43. $\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}$
44. $\lim_{x\to8}\frac{x-8}{\sqrt[3]{x}-2}$
45. $\lim_{h\to0}\frac{\sqrt{9+h}-3}{h}$
46. $\lim_{h\to0}\frac{(4+h)^2-16}{h}$
47. $\lim_{x\to\infty}\frac{3x^2+4.5}{x^2-1.5}$
48. $\lim_{x\to\infty}\frac{x-27}{7x+4}$
49. $\lim_{t\to\infty}\frac{\sqrt{t^2+16}}{t-1}$
50. $\lim_{x\to\infty}\frac{1-2x^2}{(4x+3)^2}$

In Exercises 51 and 52, evaluate the function at 0.1, 0.01, and 0.001 from both sides of the value it approaches. In Exercises 53 and 54, evaluate the function for values of x of 10, 100, and 1000. From these values, determine the limit. Then, by using an appropriate change of algebraic form, evaluate the limit directly and compare values.

51. $\lim_{x\to0}\frac{x^2-3x}{x}$
52. $\lim_{x\to3}\frac{2x^2-6x}{x-3}$
53. $\lim_{x\to\infty}\frac{2x^2-x}{x^2-3}$
54. $\lim_{x\to\infty}\frac{x^2+5}{\sqrt{64x^4+1}}$

In Exercises 55–60, solve the given problems involving limits.

55. For a quadratic equation $ax^2+bx+c=0$, the solutions are $x_1=\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$. Show that, as $a\to0$, $x_1\to-b/c$ and $x_2\to\infty$. (Hint: rationalize numerators.) (Note that $-b/c$ is the root of the equation $bx+c=0$.)

56. Draw the graph of a function that is discontinuous at $x=2$, has a limit of 2 as $x\to2$, and has a value of 3 at $x=2$.

57. A certain object, after being heated, cools at such a rate that its temperature T (in °C) decreases 10% each minute. If the object is originally heated to 100°C, find $\lim_{t\to10}T$ and $\lim_{t\to\infty}T$, where t is the time (in min).

58. The area A (in mm²) of the pupil of a certain person's eye is given by $A=\frac{36+24b^3}{1+4b^3}$, where b is the brightness (in lumens) of the light source. Between what values does A vary?

59. Velocity can be found by dividing the displacement s of an object by the elapsed time t in moving through the displacement. In a certain experiment, the following values were measured for the displacements and elapsed times for the motion of an object. Determine the limiting value of the velocity.

s (cm)	0.480000	0.280000	0.029800	0.0029980	0.00029998
t (s)	0.200000	0.100000	0.010000	0.0010000	0.00010000

60. A 5-Ω resistor and a variable resistor of resistance R are placed in parallel. The expression for the resulting resistance R_T is given by $R_T=\frac{5R}{5+R}$. Determine the limiting value of R_T as $R\to\infty$.

In Exercises 61–64, use a calculator to evaluate the indicated limits.

61. Approximate $\lim_{x\to2}\frac{2^x-4}{x-2}$
62. Approximate $\lim_{x\to1}\frac{4^x-4}{x-1}$
63. $\lim_{x\to0}(1+x)^{1/x}$ (Do you recognize the limiting value?)
64. $\lim_{x\to0}\frac{\sin x}{x}$ (Use radian mode.)

In Exercises 65–72, $\lim_{x\to a^-}f(x)$ means to find the limit as x approaches a from the left only, and $\lim_{x\to a^+}f(x)$ means to find the limit as x approaches a from the right only. These are called ***one-sided*** *limits. Solve the following problems.*

65. For the function displayed in Exercise 13, find:
(a) $\lim_{x\to2^-}f(x)$ (b) $\lim_{x\to2^+}f(x)$ (c) $\lim_{x\to2}f(x)$

66. For the function displayed in Exercise 16, find:
(a) $\lim_{x\to-2^-}f(x)$ (b) $\lim_{x\to-2^+}f(x)$ (c) $\lim_{x\to-2}f(x)$

67. Find $\lim_{x\to4^-}x\sqrt{16-x^2}$

(W) 68. Explain why $\lim_{x\to0^+}2^{1/x}\neq\lim_{x\to0^-}2^{1/x}$.

(W) 69. For $f(x)=\frac{x}{|x|}$, find $\lim_{x\to0^-}f(x)$ and $\lim_{x\to0^+}f(x)$. Is $f(x)$ continuous at $x=0$? Explain.

(W) 70. In Einstein's theory of relativity, the length L of an object moving at a velocity v is $L=L_0\sqrt{1-\frac{v^2}{c^2}}$, where c is the speed of light and L_0 is the length of the object at rest. Find $\lim_{v\to c^-}L$ and explain why a limit from the left is used.

71. Is there a difference between $\lim_{x\to2^-}\frac{1}{x-2}$ and $\lim_{x\to2^+}\frac{1}{x-2}$?

72. Is there a difference between $\lim_{x\to2^-}\frac{1}{\sqrt{x-2}}$ and $\lim_{x\to2^+}\frac{1}{\sqrt{x-2}}$?