EXERCISES 6

In Exercises 1 and 2, make the given changes in the indicated examples of this section and then solve the resulting problem.

1. In Example 2, change u to $5 - 3x^2$.
2. In Example 5, change the numerator to $3x^2 - x$ and then find and evaluate the derivative at $x = 2$.

In Exercises 3–8, find the derivative of each function by using Eq. (12). Do not find the product before finding the derivative.

3. $y = 6x(3x^2 - 5x)$
4. $y = 2x^3(3x^4 + x)$
5. $s = (3t + 2)(2t - 5)$
6. $f(x) = (3x - 2)(4x^2 + 3)$
7. $y = (x^4 - 3x^2 + 3)(1 - 2x^3)$
8. $y = (x^3 - 6x)(2 - 4x^3)$

In Exercises 9–12, find the derivative of each function by using Eq. (12). Then multiply out each function and find the derivative by treating it as a polynomial. Compare the results.

9. $y = (2x - 7)(5 - 2x)$
10. $f(s) = (5s^2 + 2)(2s^2 - 1)$
11. $V = (h^3 - 1)(2h^2 - h - 1)$
12. $y = (3x^2 - 4x + 1)(5 - 6x^2)$

In Exercises 13–24, find the derivative of each function by using Eq. (13).

13. $y = \dfrac{x}{8x + 3}$
14. $u = \dfrac{4}{v^2}$
15. $y = \dfrac{\pi}{2x^2 + 1}$
16. $R = \dfrac{5i + 9}{6i + 3}$
17. $y = \dfrac{6x^2}{3 - 2x}$
18. $y = \dfrac{e^2}{x(3x - 5)}$
19. $y = \dfrac{2x - 1}{3x^2 + 2}$
20. $P = \dfrac{8i^2}{4 - 3i}$
21. $f(x) = \dfrac{3x + 8}{x^2 + 4x + 2}$
22. $y = \dfrac{33x}{4x^5 - 3x - 4}$
23. $y = \dfrac{2x^2 - x - 1}{x^2(x + 2)}$
24. $y = \dfrac{x(3x^2 - 1)}{2x^2 - 5x + 4}$

In Exercises 25–32, evaluate the derivatives of the given functions for the given values of x. In Exercises 25–28, use Eq. (12). In Exercises 27, 28, 31, and 32, check your results using the derivative evaluation feature of a calculator.

25. $y = (3x - 1)(4 - 7x), x = 3$
26. $y = (3x^2 - 5)(2x^2 - 1), x = -1$
27. $y = (2x^2 - x + 1)(4 - 2x - x^2), x = -3$
28. $y = (4x^4 + 0.5x^2 + 1)(3x - 2x^2), x = 0.5$
29. $y = \dfrac{3x - 5}{2x + 3}, x = -2$
30. $y = \dfrac{2x^2 - 5x}{3x + 2}, x = 2$
31. $S = \dfrac{2n^3 - 3n + 8}{2n - 3n^4}, n = -1$
32. $y = \dfrac{2x^3 - x^2 - 2}{4x + 3}, x = 0.5$

In Exercises 33–58, solve the given problems by finding the appropriate derivatives.

33. What text equation from Section 5 is equivalent to the product rule if one of the functions u and v is a constant?
34. By use of the quotient rule, derive a formula for the derivative of the function $1/v(x)$.
35. Using the product rule, find the point(s) on the curve of $y = (2x^2 - 1)(1 - 4x)$ for which the tangent line is $y = 4x - 1$.
(W) 36. Do the curves of $y = x^2$ and $y = 1/x^2$ cross at right angles? Explain.
37. If $f(x)$ is a differentiable function, find an expression for the derivative of $y = x^2 f(x)$.
38. If $f(x)$ is a differentiable function, find an expression for the derivative of $y = f(x)/x^2$.
39. Find the derivative of $y = \dfrac{x^2 - 1}{x - 1}$ by (a) the quotient rule, and (b) by first simplifying the function.
40. Find the derivative of $y = [(3x - 1)(3x + 1)(x^2 - 4)]$ by use of Eq. (12), and not first multiplying the factors. Check by first multiplying the factors.
41. Find the derivative of $y = \dfrac{x^2(1 - 2x)}{3x - 7}$ in each of the following two ways. (1) Do not multiply out the numerator before finding the derivative. (2) Multiply out the numerator before finding the derivative. Compare the results.
42. Find the derivative of $y = 4x^2 - \dfrac{1}{x - 1}$ in each of the following two ways. (1) Do not combine the terms over a common denominator before finding the derivative. (2) Combine the terms over a common denominator before finding the derivative. Compare the results.
43. Find the slope of a line tangent to the curve of the function $y = (4x + 1)(x^4 - 1)$ at the point $(-1, 0)$. Do not multiply the factors together before taking the derivative. Use the derivative evaluation feature of a calculator to check your result.
44. Find the slope of a line tangent to the curve of the function $y = (3x + 4)(1 - 4x)$ at the point $(2, -70)$. Do not multiply the factors together before taking the derivative. Use the derivative evaluation feature of a calculator to check your result.
45. For what value(s) of x is the slope of a tangent to the curve of $y = \dfrac{x}{x^2 + 1}$ equal to zero? View the graph on a calculator to verify the values found.

50. The number n of dollars saved by increasing the fuel efficiency of e mi/gal to $e + 6$ mi/gal for a car driven 10,000 mi/year is $n = \dfrac{195{,}000}{e(e + 6)}$, if the cost of gas is \$3.25/gal. Find dn/de.
51. During each cycle, the vertical displacement s of the end of a robot arm is given by $s = (t^2 - 8t)(2t^2 + t + 1)$, where t is the time. Find the expression for the instantaneous velocity of the end of the robot arm. See Fig. 36.

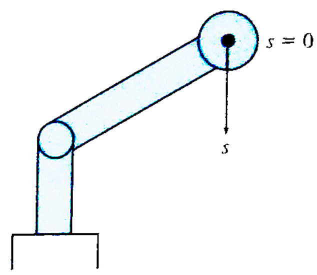

Fig. 36

52. The concentration c (in mg/L) of a certain drug in the bloodstream is found to be $c = 25t/(t^2 + 5)$, where t is the time (in h) after the drug is taken. Find dc/dt.
53. A computer, using data from a refrigeration plant, estimated that in the event of a power failure the temperature T (in °C) in the freezers would be given by $T = \dfrac{2t}{0.05t + 1} - 20$, where t is the number of hours after the power failure. Find the time rate of change of temperature after 6.0 h.
54. During a television commercial, a rectangular image appeared, increased in size, and then disappeared. If the length l (in cm) of the image, as a function of time t (in s) was $l = 6 - t$, and the width w (in cm) of the image was $w = t^2 + 4$, find the time rate of change of the area of the rectangle when $t = 5.00$ s.
55. The frictional radius r_f of a disc clutch is given by the equation $r_f = \dfrac{2(R^2 + Rr + r^2)}{3(R + r)}$, where R and r are the outer radius and the inner radius of the clutch, respectively. Find the derivative of r_f with respect to R with r constant.