EXERCISES 9

In Exercises 1 and 2, make the given changes in the indicated examples of this section and then solve the resulting problem.

1. In Example 1, change $2x$ to $2x^2$.
2. In Example 3, in the denominator change $1 - x$ to $1 + 2x$.

In Exercises 3–10, find all the higher derivatives of the given functions.

3. $y = x^3 + 7x^2$
4. $f(x) = 3x - x^4$
5. $f(x) = x^3 - 6x^4$
6. $s = 8t^5 + 5t^4$
7. $y = (1 - 2x)^4$
8. $f(x) = 4(3x + 2)^3$
9. $f(r) = r(4r + 9)^3$
10. $y = x(5x - 1)^3$

In Exercises 11–30, find the second derivative of each of the given functions.

11. $y = 2x^7 - x^6 - 3x$
12. $y = 6x - 2x^5$
13. $y = 5x + 8\sqrt{x}$
14. $r = 3\theta^2 - \dfrac{20}{\sqrt{\theta}}$
15. $f(x) = \sqrt[4]{8x - 3}$
16. $f(x) = 3\sqrt[3]{6x + 5}$
17. $f(p) = \dfrac{4.8\pi}{\sqrt{1 + 2p}}$
18. $f(x) = \dfrac{7.5}{\sqrt{3 - 4x}}$
19. $y = 2(2 - 5x)^4$
20. $y = \dfrac{1}{3}(4x + 1)^6$
21. $y = (3x^2 - 1)^5$
22. $y = 3(2x^3 + 3)^4$
23. $f(x) = \dfrac{2\pi^2}{6 - x}$
24. $f(R) = \dfrac{1 - 3R}{1 + 3R}$
25. $u = \dfrac{v^2}{4v + 15}$
26. $y = \dfrac{8x}{\sqrt{9 - x^2}}$
27. $x^2 - 4y^2 = 9$
28. $xy + y^2 = 4$
29. $x^2 - xy = 1 - y^2$
30. $4xy = y^2 + 2e^3$

In Exercises 31–36, evaluate the second derivative of the given function for the given value of x.

31. $f(x) = \sqrt{x^2 + 9}, x = 4$
32. $f(x) = x - \dfrac{2}{x^3}, x = -1$
33. $y = 3x^{2/3} - \dfrac{2}{x}, x = -8$
34. $y = 3(1 + 2x)^4, x = \dfrac{1}{2}$
35. $v = t(8 - t)^5, t = 2$
36. $y = \dfrac{9x}{2 - 3x}; x = -\dfrac{1}{3}$

In Exercises 37–40, find the acceleration of an object for which the displacement s (in m) is given as a function of the time t (in s) for the given value of t.

37. $s = 26t - 4.9t^2, t = 3.0$ s
38. $s = 3(1 + 2t)^4, t = 0.500$ s
39. $s = \dfrac{16}{0.5t^2 + 1}, t = 2$ s
40. $s = 250\sqrt{6t + 7}, t = 7.0$ s

In Exercises 41–54, solve the given problems by finding the appropriate derivatives.

41. Show that $\dfrac{d^2}{dx^2}(uv) = u\dfrac{d^2v}{dx^2} + 2\dfrac{du}{dx}\dfrac{dv}{dx} + \dfrac{d^2u}{dx^2}v$.
42. Show that $\dfrac{d^6(x^6)}{dx^6} = 6!$
43. What is the instantaneous rate of change of the first derivative of y with respect to x for $y = (1 - 2x)^4$ for $x = 1$?
44. What is the instantaneous rate of change of the first derivative of y with respect to x for $2xy + y = 1$ for $x = 0.5$?
45. If the population of a city is $P(t) = 8000(1 + 0.02t + 0.005t^2)$ (t is in years from 2010), what is the acceleration in the size of the population?
46. Find a second degree polynomial such that $f(2) = 6, f'(2) = 3$, and $f''(2) = 2$.
47. Find a third degree polynomial such that $f(-1) = 9, f'(-1) = 8$, $f''(-1) = -14$, and $f'''(-1) = 12$.
48. The potential V (in V) of a certain electric charge is given by $V = 6/(2t + 1)$, where t is the time (in s). Find d^2V/dt^2.
49. A bullet is fired vertically upward. Its distance s (in ft) above the ground is given by $s = 2250t - 16.1t^2$, where t is the time (in s). Find the acceleration of the bullet.
50. In testing the brakes on a new model car, it was found that the distance s (in ft) it traveled after the brakes were applied was given by $s = 57.6 - 1.20t^3$, where t is the time (in s). What were the velocity and acceleration for $t = 4.00$ s?
51. The voltage V induced in an inductor in an electric circuit is given by $V = L(d^2q/dt^2)$, where L is the inductance (in H). Find the expression for the voltage induced in a 1.60-H inductor if $q = \sqrt{2t + 1} - 1$.
52. How fast is the rate of change of solar radiation changing on the surface in Exercise 39 of Section 4 at 3 P.M.?
53. The deflection y (in m) of a 5.00-m beam as a function of the distance x (in m) from one end is $y = 0.0001(x^5 - 25x^2)$. Find the value of d^2y/dx^2 (the rate of change at which the slope of the beam changes) where $x = 3.00$ m.
54. The force F (in N) on an object is $F = 12\,dv/dt + 2.0v + 5.0$, where v is the velocity (in m/s) and t is the time (in s). If the displacement is $s = 25t^{0.60}$, find F for $t = 3.5$ s.

Answers to Practice Exercises

1. $y'' = 48x^2 - 12$
2. $y'' = \dfrac{18x^2 - 24}{(x^2 + 4)^3}$