In Exercises 7–10, find the equations of the lines normal to the indicated curves at the given points. In Exercises 7 and 10, sketch the curve and normal line. In Exercises 8 and 9, use a calculator to view the curve and normal line.

7. $y = 6x - 2x^2$ at $(2, 4)$
8. $y = 8 - x^3$ at $(-1, 9)$
9. $y^2(2 - x) = x^3$ at $(1, 1)$
10. $4x^2 - y^2 = 20$ at $(-3, -4)$.

In Exercises 11–14, find the equations of the lines tangent or normal to the given curves and with the given slopes. View the curves and lines on a calculator.

11. $y = x^2 - 2x$, tangent line with slope 2
12. $y = \sqrt{2x - 9}$, tangent line with slope 1
13. $y = (2x - 1)^3$, normal line with slope $-\frac{1}{24}$, $x > 0$
14. $y = \frac{1}{2}x^4 + 1$, normal line with slope 4

In Exercises 15–30, solve the given problems involving tangent and normal lines.

15. Find the equations of the tangent and normal lines to the parabola with vertex at $(0, 3)$ and focus at $(0, 0)$, where $x = -1$. Use a calculator to view the curve and lines.
16. Find the equations of the tangent and normal lines to the ellipse with focus at $(4, 0)$, vertex at $(5, 0)$, and center at the origin, where $x = 2$. Use a calculator to view the curve and lines.
17. Show that the line tangent to the graph of $y = x + 2x^2 - x^4$ at $(1, 2)$ is also tangent at $(-1, 0)$.
18. Show that the graphs of $y^2 = 4x + 4$ and $y^2 = 4 - 4x$ cross at right angles.

(W) 19. Without actually finding the points of intersection, explain why the parabola $y^2 = 4x$ and the ellipse $2x^2 + y^2 = 6$ intersect at right angles. (*Hint:* Call a point of intersection (a, b).)

20. Show that the curve $y = x^3 + 4x - 5$ has no normal line with a slope of $-1/3$.
21. Show that the equation of the tangent line to the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1) is $x_1x + y_1y = a^2$.
22. Where does the normal line to the parabola $y = x - x^2$ at $(1, 0)$ intersect the parabola other than at $(1, 0)$?
23. Heat flows normal to isotherms, curves along which the temperature is constant. Find the line along which heat flows through the point $(2, 1)$ and the isotherm is along the graph of $2x^2 + y^2 = 9$.
24. The sparks from an emery wheel to sharpen blades fly off tangent to the wheel. Find the equation along which sparks fly from a wheel described by $x^2 + y^2 = 25$, at $(3, 4)$.
25. A certain suspension cable with supports on the same level is closely approximated as being parabolic in shape. If the supports are 200 ft apart and the sag at the center is 30 ft, what is the equation of the line along which the tension acts (tangentially) at the right support? (Choose the origin of the coordinate system at the lowest point of the cable.)
26. In a video game, airplanes move from left to right along the path described by $y = 2 + 1/x$. They can shoot rockets tangent to the direction of flight at targets on the x-axis located at $x = 1, 2, 3,$ and 4. Will a rocket fired from $(1, 3)$ hit a target?
27. In an electric field, the lines of force are perpendicular to the curves of equal electric potential. In a certain electric field, a curve of equal potential is $y = \sqrt{2x^2 + 8}$. If the line along which the force acts on an electron has an inclination of 135°, find its equation.
28. A radio wave reflects from a reflecting surface in the same way as a light wave (see Example 4). A certain horizontal radio wave reflects off a parabolic reflector such that the reflected wave is 43.60° below the horizontal, as shown in Fig. 7. If the equation of the parabola is $y^2 = 8x$, what is the equation of the normal line through the point of reflection?

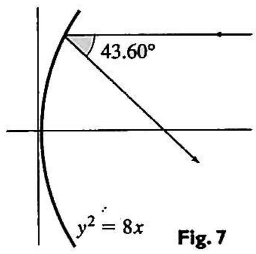

Fig. 7

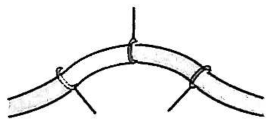

Fig. 8

29. In designing a flexible tubing system, the supports for the tubing must be perpendicular to the tubing. If a section of the tubing follows the curve $y = \dfrac{4}{x^2 + 1}$ $(-2\text{ dm} < x < 2\text{ dm})$, along which lines must the supports be directed if they are located at $x = -1$, $x = 0$, and $x = 1$? See Fig. 8.
30. On a particular drawing, a pulley wheel can be described by the equation $x^2 + y^2 = 100$ (units in cm). The pulley belt is directed along the lines $y = -10$ and $4y - 3x - 50 = 0$ when first and last making contact with the wheel. What are the first and last points on the wheel where the belt makes contact?

Answers to Practice Exercises

1. $6x + y = 13$ 2. $x - 6y = 33$