

Differentiation of Transcendental Functions

25. $y = \frac{2 \cos 4x}{1 + \cot 3x}$

27. $y = \frac{1}{3} \tan^3 x - \tan x$

29. $r = \tan(\sin 2\pi\theta)$

31. $y = \sqrt{2x + \tan 4x}$

33. $x \sec y - 2y = \sin 2x$

26. $y = \frac{\tan^2 3x}{2 + \sin(x^2 + 1)}$

28. $y = 4 \csc 4x - 2 \cot 4x$

30. $y = x \tan x + \sec^2 2x$

32. $V = (4 - \csc^2 3r)^3$

34. $3 \cot(x + y) = \cos y^2$

In Exercises 35–38, find the differentials of the given functions.

35. $y = 4 \tan^2 3x$

36. $y = 2.5 \sec^3 2t$

37. $y = \tan 4x \sec 4x$

38. $y = 2x \cot 3x$

In Exercises 39–54, solve the given problems.

39. On a calculator, find the values of (a) $\sec^2 1.0000$ and (b) $(\tan 1.0001 - \tan 1.0000)/0.0001$. Compare the values and give the meaning of each in relation to the derivative of $\tan x$ where $x = 1$.

40. Display the graphs of $y_1 = \sec^2 x$ and $y_2 = [\tan(x + h) - \tan x]/h$ on the same screen of a calculator for $-\pi/2 < x < \pi/2$. For $n = 2$, let $h = 0.5$, for $n = 3$, let $h = 0.1$. (You might try some even smaller values of h .) What do these curves show?

41. (a) Display the graph of $y = \tan x$ on a calculator, and using the derivative feature, evaluate dy/dx for $x = 1$. (b) Display the graph of $y = \sec^2 x$ and evaluate y for $x = 1$. (Compare the values in parts (a) and (b).)

42. Follow the instructions in Exercise 41, using the graphs of $y = \sec x$ and $y = \sec x \tan x$.

43. Find the derivative of each member of the identity $1 + \tan^2 x = \sec^2 x$ and show that the results are equal.

44. Find the points where a tangent to the curve of $y = \tan x$ is parallel to the line $y = 2x$ if $0 < x < 2\pi$.

45. Find the slope of a line tangent to the curve of $y = 2 \cot 3x$ where $x = \pi/12$. Verify the result by using the numerical derivative feature of a calculator.

46. Find the slope of a line normal to the curve of $y = \csc \sqrt{2x + 1}$ where $x = 0.45$. Verify the result by using the numerical derivative feature of a calculator.

47. Show that $y = 2 \tan x - \sec x$ satisfies $\frac{dy}{dx} = \frac{2 - \sin x}{\cos^2 x}$.

48. For the spring mechanism in Exercise 29 of Section 10.4, find db/dA . (Note that a and angle B are constants.)

49. For the cantilever column in Exercise 30 of Section 10.4, find x' ($= dx/dL$). (We use x' because of the constant d in the equation.)

50. A helicopter takes off such that its height h (in ft) above the ground is $h = 25 \sec 0.16t$ for the first 8.0 s of flight. What is its vertical velocity after 6.0 s?

51. The vertical displacement y (in cm) of the end of an industrial robot arm for each cycle is $y = 2t^{1.5} - \tan 0.1t$, where t is the time (in s). Find its vertical velocity for $t = 15$ s.

52. The electric charge q (in C) passing a given point in a circuit is given by $q = t \sec \sqrt{0.2t^2 + 1}$, where t is the time (in s). Find the current i (in A) for $t = 0.80$ s. ($i = dq/dt$.)

53. An observer to a rocket launch was 1000 ft from the takeoff position. The observer found the angle of elevation of the rocket as a function of time to be $\theta = 3t/(2t + 10)$. Therefore, the height h (in ft) of the rocket was $h = 1000 \tan \frac{3t}{2t + 10}$. Find the time rate of change of height after 5.0 s. See Fig. 8.

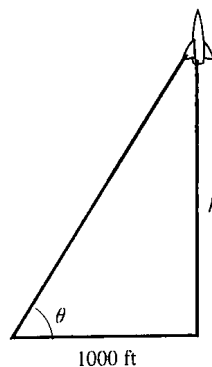


Fig. 8

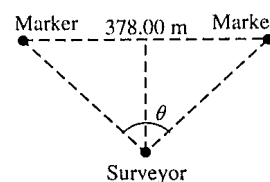


Fig. 9

54. A surveyor measures the distance between two markers to be 378.00 m. Then, moving along a line equidistant from the markers, the distance d from the surveyor to each marker is $d = 189.00 \csc \frac{1}{2}\theta$, where θ is the angle between the lines of sight to the markers. See Fig. 9. By using differentials, find the change in d if θ changes from 98.20° to 98.45° .

Answers to Practice Exercises

1. $y' = 24 \sec^2 8x$ 2. $y' = -30 \csc 3x (\cot 3x + \csc 3x)^2$