29. $y = \frac{1}{1 + 4x^2} - \tan^{-1} 2x$

30. $y = \sin^{-1} x - \sqrt{1 - x^2}$

31. $y = 3(4 - \cos^{-1} 2x)^3$

32. $\sin^{-1}(x + y) + y = x^2$

33. $2 \tan^{-1} xy + x = 3$

34. $y = \sqrt{2\pi - \sin^{-1} 4x}$

In Exercises 35–54, solve the given problems.

(W) 35. On a calculator, find the values of (a) $1/\sqrt{1 - 0.5^2}$ and (b) $(\sin^{-1} 0.5001 - \sin^{-1} 0.5000)/0.0001$. Compare the values and give the meaning of each in relation to the derivative of $\sin^{-1} x$ where $x = 0.5$.

(W) 36. Display the graphs of $y_1 = \frac{1}{\sqrt{1 - x^2}}$ and $y_n = \frac{\sin^{-1}(x + h) - \sin^{-1} x}{h}$ on the same screen of a calculator for $-1.2 < x < 1.2$. For $n = 2$, let $h = 0.5$; for $n = 3$, let $h = 0.1$. (You might try even smaller values for h.) Use a heavier curve for y_3. What do these curves show?

37. Find the differential of the function $y = (\sin^{-1} x)^3$.

38. Find the linearization $L(x)$ of the function $f(x) = 2x \cos^{-1} x$ for $a = 0$.

39. Find the slope of a line tangent to the curve of $y = x/\tan^{-1} x$ at $x = 0.80$. Verify the result by using the *numerical derivative* feature of a calculator.

(W) 40. Explain what is wrong with a problem that requires finding the derivative of $y = \sin^{-1}(x^2 + 1)$.

41. Find the second derivative of $y = x \tan^{-1} x$.

42. Find the point(s) at which the line normal to $y = 2 \sin^{-1} 0.5x$ is parallel to the line $y = 1 - x$.

43. Use a calculator to display the graphs of $y = \sin^{-1} x$ and $y = 1/\sqrt{1 - x^2}$. By roughly estimating slopes of tangent lines of $y = \sin^{-1} x$, note that $y = 1/\sqrt{1 - x^2}$ gives reasonable values for the derivative of $y = \sin^{-1} x$.

44. Use a calculator to display the graphs of $y = \tan^{-1} x$ and $y = 1/(1 + x^2)$. By roughly estimating slopes of tangent lines of $y = \tan^{-1} x$, note that $y = 1/(1 + x^2)$ gives reasonable values for the derivative of $y = \tan^{-1} x$.

45. Find the second derivative of the function $y = \tan^{-1} 2x$.

46. Show that $\frac{d(\cot^{-1} u)}{dx} = -\frac{1}{1 + u^2}\frac{du}{dx}$.

47. Show that $\frac{d(\sec^{-1} u)}{dx} = \frac{1}{\sqrt{u^2(u^2 - 1)}}\frac{du}{dx}$.

48. Show that $\frac{d(\csc^{-1} u)}{dx} = -\frac{1}{\sqrt{u^2(u^2 - 1)}}\frac{du}{dx}$.

49. In the analysis of the waveform of an AM radio wave, the equation $t = \frac{1}{\omega}\sin^{-1}\frac{A - E}{mE}$ arises. Find dt/dm, assuming that the other quantities are constant.

50. An equation that arises in the theory of solar collectors is $\alpha = \cos^{-1}\frac{2f - r}{r}$. Find the expression for $d\alpha/dr$ if f is constant.

51. When an alternating current passes through a series RLC circuit, the voltage and current are out of phase by angle θ. Here $\theta = \tan^{-1}[(X_L - X_C)/R]$, where X_L and X_C are the reactances of the inductor and capacitor, respectively, and R is the resistance. Find $d\theta/dX_C$ for constant X_L and R.

52. When passing through glass, a light ray is refracted (bent) such that the angle of refraction r is given by $r = \sin^{-1}[(\sin i)/\mu]$. Here, i is the angle of incidence, and μ is the index of refraction of the glass (see Fig. 13). For different types of glass, μ differs. Find the expression for dr for a constant value of i.

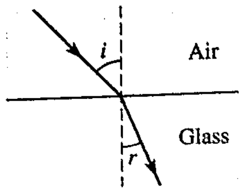

Fig. 13

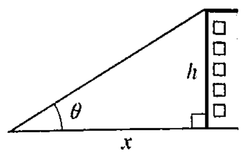

Fig. 14

53. As a person approaches a building of height h, the angle of elevation of the top of the building is a function of the person's distance from the building. Express the angle of elevation θ in terms of h and the distance x from the building and then find $d\theta/dx$. Assume the person's height is negligible to that of the building. See Fig. 14.

54. A triangular metal frame is designed as shown in Fig. 15. Express angle A as a function of x and evaluate dA/dx for $x = 6$ cm.

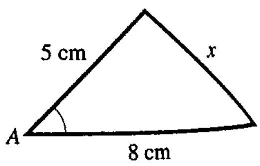

Fig. 15

Answers to Practice Exercises

1. $y' = 10x/\sqrt{1 - x^4}$ 2. $y' = (6 \tan^{-1} 3x)/(1 + 9x^2)$