Differentiation of Transcendental Functions

29. \( y = \frac{1}{1 + 4x^2} - \tan^{-1}2x \)
30. \( y = \sin^{-1}x - \sqrt{1 - x^2} \)
31. \( y = 3(4 - \cos^{-1}2x)^3 \)
32. \( \sin^{-1}(x + y) + y = x^2 \)
33. \( 2\tan^{-1}xy + x = 3 \)
34. \( y = \sqrt{2\pi} - \sin^{-1}4x \)

In Exercises 35–54, solve the given problems.

W 35. On a calculator, find the values of (a) \( 1/\sqrt{1 - 0.5^2} \) and (b) \( (\sin^{-1}0.5001 - \sin^{-1}0.5000)/0.0001 \). Compare the values and give the meaning of each in relation to the derivative of \( \sin^{-1}x \) where \( x = 0.5 \).

W 36. Display the graphs of \( y = \frac{1}{\sqrt{1-x^2}} \)
and
\[ y_n = \frac{\sin^{-1}(x + h) - \sin^{-1}x}{h} \]
on the same screen of a calculator for 
-1.2 < x < 1.2. For \( n = 2 \), let \( h = 0.5 \); for \( n = 3 \), let \( h = 0.1 \). (You might try even smaller values for \( h \).) Use a heavier curve for \( y_3 \). What do these curves show?

37. Find the differential of the function \( y = (\sin^{-1}x)^3 \).

38. Find the linearization \( L(x) \) of the function \( f(x) = 2x \cos^{-1}x \) for \( a = 0 \).

39. Find the slope of a line tangent to the curve of \( y = x/\tan^{-1}x \) at \( x = 0.80 \). Verify the result by using the numerical derivative feature of a calculator.

W 40. Explain what is wrong with a problem that requires finding the derivative of \( y = \sin^{-1}(x^2 + 1) \).

41. Find the second derivative of \( y = x \tan^{-1}x \).

42. Find the point(s) at which the line normal to \( y = 2 \sin^{-1}0.5x \) is parallel to the line \( y = 1 - x \).

43. Use a calculator to display the graphs of \( y = \sin^{-1}x \) and \( y = 1/\sqrt{1 - x^2} \). By roughly estimating slopes of tangent lines of \( y = \sin^{-1}x \), note that \( y = 1/\sqrt{1 - x^2} \) gives reasonable values for the derivative of \( y = \sin^{-1}x \).

44. Use a calculator to display the graphs of \( y = \tan^{-1}x \) and \( y = 1/(1 + x^2) \). By roughly estimating slopes of tangent lines of \( y = \tan^{-1}x \), note that \( y = 1/(1 + x^2) \) gives reasonable values for the derivative of \( y = \tan^{-1}x \).

45. Find the second derivative of the function \( y = \tan^{-1}2x \).

46. Show that \( \frac{d(\cot^{-1}u)}{dx} = \frac{-1}{1 + u^2} \frac{du}{dx} \)

47. Show that \( \frac{d(\sec^{-1}u)}{dx} = \frac{1}{\sqrt{u^2(u^2 - 1)}} \frac{du}{dx} \)

48. Show that \( \frac{d(\csc^{-1}u)}{dx} = \frac{-1}{\sqrt{u^2(u^2 - 1)}} \frac{du}{dx} \)

49. In the analysis of the waveform of an AM radio wave, the equation \( t = -\sin^{-1} \frac{A - E}{mE} \) arises. Find \( dt/dm \), assuming that the other quantities are constant.

50. An equation that arises in the theory of solar collection is \( \alpha = \cos^{-1} \frac{2f - r}{r} \). Find the expression for \( d\alpha/dr \) if \( f \) is constant.

51. When an alternating current passes through a series RLC circuit, the voltage and current are out of phase by angle \( \theta \). Here \( \theta = \tan^{-1}[(X_L - X_C)/R] \), where \( X_L \) and \( X_C \) are the reactances of the inductor and capacitor, respectively, and \( R \) is the resistance. Find \( d\theta/dX_C \) for constant \( X_L \) and \( R \).

52. When passing through glass, a light ray is refracted (bent) such that the angle of refraction \( r \) is given by \( r = \sin^{-1}(\sin \theta)/\mu \). Here, \( i \) is the angle of incidence, and \( \mu \) is the index of refraction of the glass (see Fig. 13). For different types of glass, \( \mu \) differs. Find the expression for \( dr/d\theta \) for a constant value of \( i \).

![Fig. 13](image1)

![Fig. 14](image2)

53. As a person approaches a building of height \( h \), the angle of elevation of the top of the building is a function of the person's distance from the building. Express the angle of elevation \( \theta \) in terms of \( h \) and the distance \( x \) from the building and then find \( d\theta/dx \). Assume the person's height is negligible to that of the building. See Fig. 14.

54. A triangular metal frame is designed as shown in Fig. 15. Express angle \( A \) as a function of \( x \) and evaluate \( dA/dx \) for \( x = 6 \) cm.

![Fig. 15](image3)

**Answers to Practice Exercises**

1. \( y' = 10x/\sqrt{1 - x^3} \)
2. \( y' = (6 \tan^{-1}3x)/(1 + 9x^2) \)