EXERCISES 5

In Exercises 1 and 2, make the given changes in the indicated examples of this section and then find the derivatives.

1. In Example 3, in the given function, change tan to cos.
2. In Example 4, in the given function, change $x - 1$ to x^2.

In Exercises 3–36, find the derivatives of the given functions.

3. $y = \log x^2$
4. $y = \log_2 6x$
5. $y = 4\log_5(3 - x)$
6. $y = \log_7(x^2 + 4)$
7. $u = 2\ln(3 - x)^4$
8. $y = 2\ln(3x^2 - 1)$
9. $y = 2\ln\tan 2x$
10. $s = 3\ln\sin^2 t$
11. $R = \ln\sqrt{4T + 1}$
12. $y = \ln(4x - 3)^3$
13. $y = \ln(x - x^2)^3$
14. $s = 3\ln^2(7t^3 - 1)$
15. $v = 3(t + \ln t^2)^2$
16. $y = 6x^2 \ln 5x$
17. $y = 3x\ln(6 - x)^2$
18. $y = \dfrac{8\ln(2x + 1)}{x}$
19. $y = \ln(\ln x)$
20. $y = \ln\dfrac{2x}{1 + x}$
21. $r = 0.5\ln\cos(\pi\theta^2)$
22. $y = \ln(x\sqrt{x + 1})$
23. $y = \sin\ln x$
24. $y = \tan^{-1}(\ln 2x + \ln x)$
25. $u = 3v\ln^2 2v$
26. $h = 0.1s\ln^4 s$
27. $y = \ln(x\tan x)$
28. $y = \ln(x + \sqrt{x^2 - 1})$
29. $r = \ln\dfrac{v^2}{v + 2}$
30. $y = \sqrt{x + \ln 3 + \ln x}$
31. $y = \dfrac{\ln\sin 2x}{\ln\tan x}$
32. $3\ln xy + \sin y = x^2$
33. $y = x - \ln^2(x + y)$
34. $y = \ln(x + \ln x)$
35. $y = \dfrac{\ln(x + 4)}{\ln x^2}$
36. $y = \sqrt{x^2 + 1} - \ln\dfrac{1 + \sqrt{x^2 + 1}}{x}$

In Exercises 37–58, solve the given problems.

37. (W) On a calculator, find the value of $(\ln 2.0001 - \ln 2.0000)/0.0001$ and compare it with 0.5. Give the meanings of the value found and 0.5 in relation to the derivative of $\ln x$, where $x = 2$.

38. (W) Display the graphs of $y_1 = \dfrac{1}{x}$ and $y_n = \dfrac{\ln(x + h) - \ln x}{h}$ on the same calculator screen for $0 < x < 10$. For $n = 2$, let $h = 0.5$; for $n = 3$, let $h = 0.1$. (You might try smaller values of h.) Use a heavier curve for y_3. What do these curves show?

39. Using a calculator, (a) display the graph of $y = (1 + x)^{1/x}$ to verify that $(1 + x)^{1/x} \to 2.718$ as $x \to 0$ and (b) verify the values for $(1 + x)^{1/x}$ in the tables.

40. (a) Display the graph of $y = \ln x$ on a calculator, and using the derivative feature, evaluate dy/dx for $x = 2$. (b) Display the graph of $y = 1/x$, and evaluate y for $x = 2$. (c) Compare the values in parts (a) and (b).

41. Given that $\ln\sin 45^\circ = -0.3466$, use differentials to approximate $\ln\sin 44^\circ$.

42. Find the second derivative of the function $y = x^2\ln x$.

43. Evaluate the derivative of $y = \sin^{-1}2x + \sqrt{1 - 4x^2}$, where $x = 0.250$.

44. Evaluate the derivative of $y = \ln\sqrt{\dfrac{2x + 1}{3x + 1}}$, where $x = 2.75$.

45. Find the linearization $L(x)$ for the function $f(x) = 2\ln\tan x$ for $a = \pi/4$.

46. Find the differential of the function $y = 6\log_x 2$.

47. Find the slope of a line tangent to the curve of $y = \tan^{-1}2x + \ln(4x^2 + 1)$, where $x = 0.625$. Verify the result by using the *numerical derivative* feature of a calculator.

48. Find the slope of a line tangent to the curve of $y = x\ln 3x$ at $x = 4$. Verify the result by using the *numerical derivative* feature of a calculator.

49. (W) Find the derivative of $y = x^x$ by first taking logarithms of each side of the equation. Explain why Eq. (23.15) cannot be used to find the derivative of this function.

50. (W) Find the derivative of $y = (\sin x)^x$ by first taking logarithms of each side of the equation. Explain why Eq. (23.15) cannot be used to find the derivative of this function.

51. (W) Find the derivatives of $y_1 = \ln(x^2)$ and $y_2 = 2\ln x$, and evaluate these derivatives for $x = -1$. Explain your results.

52. The inductance L (in μH) of a coaxial cable is given by $L = 0.032 + 0.15\log(a/x)$, where a and x are the radii of the outer and inner conductors, respectively. For constant a, find dL/dx.

53. If the loudness b (in decibels) of a sound of intensity I is given by $b = 10\log(I/I_0)$, where I_0 is a constant, find the expression for db/dt in terms of dI/dt.

54. The time t for a particular computer system to process N bits of data is directly proportional to $N\ln N$. Find the expression for dt/dN.

55. When a tractor-trailer turns a right-angle corner, the rear wheels follow a curve known as a *tractrix*, the equation for which is $y = \ln\left(\dfrac{1 + \sqrt{1 + x^2}}{x}\right) - \sqrt{1 - x^2}$. Find dy/dx.

56. When designing a computer to sort files on a hard disk, the equation $t = xA\log_2 A$ arises. If A is constant, find dy/dx.

57. When air friction is considered, the time t (in s) it takes a certain falling object to attain a velocity v (in ft/s) is given by $t = 5\ln\dfrac{16}{16 - 0.1v}$. Find dt/dv for $v = 100$ ft/s.

58. The electric potential V at a point P at a distance x from an electric charge distributed along a wire of length $2a$ (see Fig. 34) is $V = k\ln\dfrac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}$, where k is a constant. Find the expression for the electric field E, where $E = -dV/dx$.

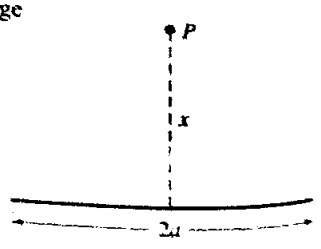

Fig. 34

Answers to Practice Exercises

1. $y' = 4/x$ 2. $y' = 4/(x^2 + 4x)$