EXERCISES 5

In Exercises 1 and 2, make the given changes in the indicated examples of this section and then find the derivatives.

1. In Example 3, in the given function, change tan x to cos x.
2. In Example 4, in the given function, change $x - 1$ to $x^2$.

In Exercises 3–36, find the derivatives of the given functions.

3. $y = \log x^3$
4. $y = \log x^6$
5. $y = 4 \log(3 - x)$
6. $y = \log(x^2 + 4)$
7. $u = \ln(3x^4)$
8. $y = 2 \ln(3x^2 - 1)$
9. $y = 2 \ln \tan 2x$
10. $s = 3 \sin^2 \sqrt{t}$
11. $R = \ln \sqrt{4T + 1}$
12. $y = \ln (4x - 3)^3$
13. $y = \ln(x - x^2)^3$
14. $s = 3 \ln^2(7x^3 - 1)$
15. $t = 3(t + \ln x)^2$
16. $y = 6x^2 \ln 5x$
17. $y = 3x \ln(6 - x)^2$
18. $y = \frac{8 \ln(2x + 1)}{x}$
19. $y = \ln \ln x$
20. $y = \ln \frac{2x}{x + 1}$
21. $r = 0.5 \ln \cos(x^2)$
22. $y = \ln(x \sqrt{x + 1})$
23. $y = \ln \sin x$
24. $y = \tan^{-1}(\ln(2x + \ln x))$
25. $u = 3e \ln^2 2v$
26. $h = 0.15 \ln^4 s$
27. $y = \ln(x \tan x)$
28. $y = \ln(x + \sqrt{x^2 - 1})$
29. $r = \frac{\ln v^2}{2}$
30. $y = \sqrt{x} + \ln 3 + \ln x$
31. $y = \ln \sin 2x$
32. $3 \ln xy + \sin y = x^2$
33. $y = x - \ln^2(x + y)$
34. $y = \ln(x + \ln x)$
35. $y = \ln(x + 4)$
36. $y = \ln \frac{1 + \sqrt{x^2 + 1}}{x}$

In Exercises 37–38, solve the given problems.

37. On a calculator, find the value of $(\ln 2000) - (\ln 20000) (0.0001)$ and compare it with 0.5. Give the meaning of the value found and 0.5 in relation to the derivative of $\ln x$, where $e < 2$.

38. Display the graphs of $y_1 = \frac{1}{x}$ and $y_2 = \ln(x + h) - \ln x$ on the same calculator screen for $0 < x < 10$. For $n = 2$, let $h = 0.5$; for $n = 3$, let $h = 0.1$. (You might try smaller values of $h$.) Use a heavier curve for $y_2$. What do these curves show?

39. Using a calculator, (a) display the graph of $y = (1 + x)^{1.1}$ to verify that $(1 + x)^{1.1} \rightarrow 2.718$ as $x \rightarrow 0$ and (b) verify the values for $(1 + x)^{1.1}$ in the tables.

40. (a) Display the graph of $y = \ln x$ on a calculator, and using the derivative feature, evaluate $dy/dx$ for $x = 2$. (b) Display the graph of $y = 1/x$, and evaluate $y$ for $x = 2$. (c) Compare the values in parts (a) and (b).

41. Given that $\ln(\sin 45^\circ) = -0.3466$, use differentials to approximate $\ln 44^\circ$.

42. Find the second derivative of the function $y = x^3 \ln x$.
43. Evaluate the derivative of $y = \sin^{-1} 2x + \sqrt{1 - 4x^2}$, where $x = 0.250$.
44. Evaluate the derivative of $y = \ln \left(\frac{2x + 1}{3x + 1}\right)$, where $x = 2.75$.

45. Find the linearization $L(x)$ for the function $f(x) = 2 \ln \tan x$ for $a = \pi/4$.
46. Find the differential of the function $y = 6 \log_{10} 2$.
47. Find the slope of a line tangent to the curve of $y = \tan^{-1} 4x + \ln(4x^2 + 1)$, where $x = 0.625$. Verify the result by using the numerical derivative feature of a calculator.
48. Find the slope of a line tangent to the curve of $y = x \ln 3x$ at $x = 4$. Verify the result by using the numerical derivative feature of a calculator.

49. Find the derivative of $y = x^4$ by first taking logarithms of each side of the equation. Explain why Eq. (23.15) cannot be used to find the derivative of this function.

50. Find the derivative of $y = (\sin x)^4$ by first taking logarithms of each side of the equation. Explain why Eq. (23.15) cannot be used to find the derivative of this function.

51. Find the derivatives of $y_1 = \ln(x^2)$ and $y_2 = 2 \ln x$, and evaluate these derivatives for $x = -1$. Explain your results.

52. The inductance $L$ (in $\mu$H) of a coaxial cable is given by $L = 0.032 + 0.15 \log(a/x)$, where $a$ and $x$ are the radii of the outer and inner conductors, respectively. For constant $a$, find $dL/dx$.

53. If the loudness $b$ (in decibels) of a sound of intensity $I$ is given by $b = 10 \log(I/I_0)$, where $I_0$ is a constant, find the expression for $db/dt$ in terms of $dt/dt$.

54. The time $t$ for a particular computer system to process $N$ bits of data is directly proportional to $N \ln N$. Find the expression for $dt/dN$.

55. When a tractor-trailer turns a right-angle corner, the rear wheels follow a curve known as a tractrix, the equation for which is $y = \ln \left(\frac{1 + \sqrt{1 + x^2}}{x}\right) - \sqrt{1 - x^2}$. Find $dy/dx$.

56. When designing a computer to sort files on a hard disk, the equation $y = x \log_{10} A$ arises. If $A$ is constant, find $dy/dx$.

57. When a frictional object is considered, the time $t$ (in s) it takes a certain falling object to attain a velocity $v$ (in ft/s) is given by $t = 5 \ln \frac{10}{16 - 0.1v}$. Find $dt/dv$ for $v = 100$ ft/s.

58. The electric potential $V$ at a point $P$ at a distance $x$ from an electric charge distributed along a wire of length $2a$ (see Fig. 34) is

$$V = k \ln \frac{\sqrt{x^2 + a^2} + a}{\sqrt{x^2 + a^2} - a},$$

where $k$ is a constant. Find the expression for the electric field $E$, where $E = -dV/dx$.

Answers to Practice Exercises

1. $y' = 4x$; 2. $y' = 4\left(\frac{x}{x^2 + 4x}\right)$; 3. $y'' = 12x$; 4. $y'' = -12x^2$. 

Fig. 34