EXERCISES 6

In Exercises 1 and 2, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 3, in the given function, change cos to sin.
2. In Example 6, in the given function, change $x^2 + 1$ to $x + 1$.

In Exercises 3–32, find the derivatives of the given functions.

3. $y = 4^{6x}$
4. $y = 10^{x^2}$
5. $y = 6e^{\sqrt{x}}$
6. $r = 0.3e^{\theta^2}$
7. $y = 4e^t(e^{2t} - e^t)$
8. $y = 0.6\ln(e^{5x} + 3^4)$
9. $R = Te^{-3T}$
10. $y = 5x^2e^{2x}$
11. $y = xe^{\tan 2x}$
12. $y = 4e^x \sin\frac{1}{2}x$
13. $r = \dfrac{2(e^{2s} - e^{-2s})}{e^{2s}}$
14. $u = \dfrac{e^{0.5v}}{2v}$
15. $y = e^{-3x}\sec 4x$
16. $y = (\cos^{-1} 2x)(e^{x^2-1})$
17. $y = \dfrac{2e^{3x}}{4x + 3}$
18. $y = \dfrac{7\ln 3x}{e^{2x} + 8}$
19. $y = 0.5\ln(e^{t^2} + 4)$
20. $p = (3e^{2n} + e^2)^3$
21. $y = (2e^{2x})^3 \sin x^2$
22. $y = (e^{3/x}\cos x)^2$
23. $u = 4\sqrt{\ln 2t + e^{2t}}$
24. $y = (2e^{x^2} + x^2)^3$
25. $y = xe^{xy} + \cot y$
26. $y = 4e^{-2/x}\ln y + 1$
27. $y = e^{2x}\ln x^3$
28. $r = 0.4e^{2\theta}\ln\cos\theta$
29. $I = \ln\sin 2e^{6t}$
30. $y = 6\tan e^{x+1}$
31. $y = 2\sin^{-1} e^{2x}$
32. $w = 5\tan^{-1} e^{3x}$

In Exercises 33–54, solve the given problems.

(W) 33. On a calculator, find the values of (a) e and (b) $(e^{1.0001} - e^{1.0000})/0.0001$. Compare the values and give the meaning of each in relation to the derivative of e^x, where $x = 1$.

(W) 34. Display the graphs of $y_1 = e^x$ and $y_n = \dfrac{e^{x+h} - e^x}{h}$ on the same calculator screen for $0 < x < 3$. For $n = 2$, let $h = 0.5$; for $n = 3$, let $h = 0.1$. (You might try smaller values of h.) What do these curves show?

35. Display the graph of $y = e^x$ on a calculator. Using the *derivative* feature, evaluate dy/dx for $x = 2$ and compare with the value of y for $x = 2$.

36. Find a formula for the nth derivative of $y = ae^{bx}$.

37. Find the slope of a line tangent to the curve of $y = e^{-x/2}\cos 4x$ for $x = 0.625$. Verify the result by using the *numerical derivative* feature of a calculator.

38. Find the slope of a line tangent to the curve of $y = \dfrac{e^{-x}}{1 + \ln 4x}$ for $x = 1.842$. Verify the result by using the *numerical derivative* feature of a calculator.

39. Find the differential of the function $y = \dfrac{12e^{4x}}{x + 6}$.

40. Find the linearization of the function $f(x) = \dfrac{6e^{4x}}{2x + 3}$ for $a = 0$.

41. Use a calculator to display the graph of $y = e^x$. By roughly estimating slopes of tangent lines, note that it is reasonable that these values are equal to the y-coordinates of the points at which these estimates are made. (*Remember:* For $y = e^x$, $dy/dx = e^x$ also.)

42. Use a calculator to display the graphs of $y = e^{-x}$ and $y = -e^{-x}$. By roughly estimating slopes of tangent lines of $y = e^{-x}$, note that $y = -e^{-x}$ gives reasonable values for the derivative of $y = e^{-x}$.

43. Show that $y = xe^{-x}$ satisfies the equation $(dy/dx) + y = e^{-x}$.

44. Show that $y = e^{-x}\sin x$ satisfies the equation
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0.$$

45. For $y = \dfrac{e^{2x} - 1}{e^{2x} + 1}$, show that $\dfrac{dy}{dx} = 1 - y^2$.

46. If $e^x + e^y = e^{x+y}$, show that $dy/dx = -e^{y-x}$.

47. For what values of m does the function $y = ae^{mx}$ satisfy the equation $y'' + y' - 6y = 0$?

48. For what values of m does the function $y = ae^{mx}$ satisfy the equation $y'' + 4y = 0$?

49. If $y = Ae^{kx} + Be^{-kx}$, show that $y'' = k^2y$.

50. The average energy consumption C (in MJ/year) of a certain model of refrigerator-freezer is approximately $C = 5350e^{-0.0748t} + 1800$, where t is measured in years, with $t = 0$ corresponding to 1990, and a newer model is produced each year. Assuming the function is continuous, use differentials to estimate the reduction of the 2012 model from that of the 2011 model.

51. The reliability R $(0 \le R \le 1)$ of a certain computer system is given by $R = e^{-0.0002t}$, where t is the time of operation (in h). Find dR/dt for $t = 1000$ h.