

## Applications of the Derivative

16. A variable resistor  $R$  and an  $8\text{-}\Omega$  resistor in parallel have a combined resistance  $R_T$  given by  $R_T = \frac{8R}{8 + R}$ . If  $R$  is changing at  $0.30\ \Omega/\text{min}$ , find the rate at which  $R_T$  is changing when  $R = 6.0\ \Omega$ .
17. The radius  $r$  of a ring of a certain holograph (an image produced without using a lens) is given by  $r = \sqrt{0.4\lambda}$ , where  $\lambda$  is the wavelength of the light being used. If  $\lambda$  is changing at the rate of  $0.10 \times 10^{-7}\ \text{m/s}$  when  $\lambda = 6.0 \times 10^{-7}\ \text{m}$ , find the rate at which  $r$  is changing.
18. An earth satellite moves in a path that can be described by  $\frac{x^2}{28.0} + \frac{y^2}{27.6} = 1$ , where  $x$  and  $y$  are in thousands of miles. If  $\frac{dx}{dt} = 7750\ \text{mi/h}$  for  $x = 2020\ \text{mi}$  and  $y > 0$ , find  $\frac{dy}{dt}$ .
19. The magnetic field  $B$  due to a magnet of length  $l$  at a distance  $r$  is given by  $B = \frac{k}{[r^2 + (l/2)^2]^{3/2}}$ , where  $k$  is a constant for a given magnet. Find the expression for the time rate of change of  $B$  in terms of the time rate of change of  $r$ .
20. An approximate relationship between the pressure  $p$  and volume  $v$  of the vapor in a diesel engine cylinder is  $pv^{1.4} = k$ , where  $k$  is a constant. At a certain instant,  $p = 4200\ \text{kPa}$ ,  $v = 75\ \text{cm}^3$ , and the volume is increasing at the rate of  $850\ \text{cm}^3/\text{s}$ . What is the time rate of change of the pressure at this instant?
21. A swimming pool with a rectangular surface  $18.0\ \text{m}$  long and  $12.0\ \text{m}$  wide is being filled at the rate of  $0.80\ \text{m}^3/\text{min}$ . At one end it is  $1.0\ \text{m}$  deep, and at the other end it is  $2.5\ \text{m}$  deep, with a constant slope between ends. How fast is the height of water rising when the depth of water at the deep end is  $1.0\ \text{m}$ ?
22. An engine cylinder  $15.0\ \text{cm}$  deep is being bored such that the radius increases by  $0.100\ \text{mm}/\text{min}$ . How fast is the volume  $V$  of the cylinder changing when the diameter is  $9.50\ \text{cm}$ ?
23. Fatty deposits have decreased the circular cross-sectional opening of a person's artery. A test drug reduces these deposits such that the radius of the opening increases at the rate of  $0.020\ \text{mm}/\text{month}$ . Find the rate at which the area of the opening increases when  $r = 1.2\ \text{mm}$ .
24. A rectangular image  $4.00\ \text{in.}$  high on a computer screen is widening at the rate of  $0.25\ \text{in.}/\text{s}$ . Find the rate at which the diagonal is increasing when the width is  $6.50\ \text{in.}$
25. A metal cube dissolves in acid such that an edge of the cube decreases by  $0.50\ \text{mm}/\text{min}$ . How fast is the volume of the cube changing when the edge is  $8.20\ \text{mm}$ ?
26. A metal sphere is placed in seawater to study the corrosive effect of seawater. If the surface area decreases at  $35\ \text{cm}^2/\text{year}$  due to corrosion, how fast is the radius changing when it is  $12\ \text{cm}$ ?
27. A uniform layer of ice covers a spherical water-storage tank. As the ice melts, the volume  $V$  of ice decreases at a rate that varies directly as the surface area  $A$ . Show that the outside radius decreases at a constant rate.
28. A light in a garage is  $9.50\ \text{ft}$  above the floor and  $12.0\ \text{ft}$  behind the door. If the garage door descends vertically at  $1.50\ \text{ft/s}$ , how fast is the door's shadow moving toward the garage when the door is  $2.00\ \text{ft}$  above the floor?
29. One statement of Boyle's law is that the pressure of a gas varies inversely as the volume for constant temperature. If a certain gas occupies  $650\ \text{cm}^3$  when the pressure is  $230\ \text{kPa}$  and the volume is increasing at the rate of  $20.0\ \text{cm}^3/\text{min}$ , how fast is the pressure changing when the volume is  $810\ \text{cm}^3$ ?
30. The tuning frequency  $f$  of an electronic tuner is inversely proportional to the square root of the capacitance  $C$  in the circuit. If  $f = 920\ \text{kHz}$  for  $C = 3.5\ \text{pF}$ , find how fast  $f$  is changing at this frequency if  $\frac{dC}{dt} = 0.3\ \text{pF/s}$ .
31. The shadow of a  $24\text{-m}$  high building is increasing at the rate of  $18\ \text{cm}/\text{min}$  when the shadow is  $18\ \text{m}$  long. How fast is the distance from the top of the building to the end of the shadow increasing?
32. The acceleration due to the gravity  $g$  on a spacecraft is inversely proportional to its distance from the center of the earth. At the surface of the earth,  $g = 32.2\ \text{ft/s}^2$ . Given that the radius of the earth is  $3960\ \text{mi}$ , how fast is  $g$  changing on a spacecraft approaching the earth at  $4500\ \text{ft/s}$  at a distance of  $25,500\ \text{mi}$  from the surface?
33. The intensity  $I$  of heat varies directly as the strength of the source and inversely as the square of the distance from the source. If an object approaches a heated object of strength  $8.00$  units at the rate of  $50.0\ \text{cm/s}$ , how fast is the intensity changing when it is  $100\ \text{cm}$  from the source?
34. The speed of sound  $v$  (in  $\text{m/s}$ ) is  $v = 331\sqrt{T/273}$ , where  $T$  is the temperature (in  $\text{K}$ ). If the temperature is  $303\ \text{K}$  ( $30^\circ\text{C}$ ) and is rising at  $2.0^\circ\text{C}/\text{h}$ , how fast is the speed of sound rising?
35. As a space shuttle moves into space, an astronaut's weight decreases. An astronaut weighing  $650\ \text{N}$  at sea level has a weight of  $w = 650\left(\frac{6400}{6400 + h}\right)$  at  $h$  kilometers above sea level. If the shuttle is moving away from Earth at  $6.0\ \text{km/s}$ , at what rate is  $w$  changing when  $h = 1200\ \text{km}$ ?
36. The oil reservoir for the lubricating mechanism of a machine is in the shape of an inverted pyramid. It is being filled at the rate of  $8.00\ \text{cm}^3/\text{s}$  and the top surface is increasing at the rate of  $6.00\ \text{cm}^2/\text{s}$ . When the depth of oil is  $6.50\ \text{cm}$  and the top surface area is  $22.5\ \text{cm}^2$ , how fast is the level increasing?
37. Coffee is draining through a conical filter into a coffee pot at the rate of  $18.0\ \text{cm}^3/\text{min}$ . If the filter is  $15.0\ \text{cm}$  in diameter and  $15.0\ \text{cm}$  deep, how fast is the level of coffee in the filter changing when the depth is  $10.0\ \text{cm}$ ?
38. The top of a ladder  $4.00\ \text{m}$  long is slipping down a vertical wall at a constant rate of  $0.75\ \text{m/s}$ . How fast is the bottom of the ladder moving along the ground away from the wall when it is  $2.50\ \text{m}$  from the wall? See Fig. 25.
39. A supersonic jet leaves an airfield traveling due east at  $1600\ \text{mi/h}$ . A second jet leaves the same airfield at the same time and travels  $1800\ \text{mi/h}$  along a line north of east such that it remains due north of the first jet. After a half-hour, how fast are the jets separating?

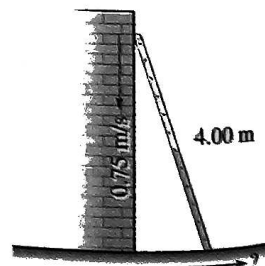


Fig. 25