EXERCISES 3

In Exercises 1 and 2, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 1, change $x = 3t^2$ to $x = 4t^2$.
2. In Example 4, change $y = x^2/3$ to $y = x^2/4$ and (2.00, 1.33) to (2.00, 1.00).

In Exercises 3–6, given that the x- and y-coordinates of a moving particle are given by the indicated parametric equations, find the magnitude and direction of the velocity for the specific value of t. Sketch the curves and show the velocity and its components.

3. $x = 3t, y = 1 - t, t = 4$
4. $x = \dfrac{5t}{2t + 1}, y = 0.1(t^2 + t), t = 2$
5. $x = t(2t + 1)^2, y = \dfrac{6}{\sqrt{4t + 3}}, t = 0.5$
6. $x = \sqrt{1 + 2t}, y = t - t^2, t = 4$

In Exercises 7–10, use the parametric equations and values of t of Exercises 3–6, to find the magnitude and direction of the acceleration in each case.

In Exercises 11–30, find the indicated velocities and accelerations.

11. A baseball is ejected horizontally toward home plate from a pitching machine on the mound with a velocity of 42.5 m/s. If y is the distance of the ball above the ground, and t is the time (in s) after being ejected, $y = 1.5 - 4.9t^2$. What are the height and velocity of the ball when it crosses home plate in 0.43 s?
12. A section of a bike trail can be described by $y = 0.0016x^2$. On this section of the trail a bike maintains a constant $v_x = 650$ m/min. What is the bike's velocity when $x = 100$ m?
13. The water from a fire hose follows a path described by $y = 2.0 + 0.80x - 0.20x^2$ (units are in meters). If v_x is constant at 5.0 m/s, find the resultant velocity at the point (4.0, 2.0).
14. A roller mechanism follows a path described by $y = \sqrt{4x + 1}$, where units are in feet. If $v_x = 2x$, find the resultant velocity (in ft/s) at the point (2.0, 3.0).
15. A float is used to test the flow pattern of a stream. It follows a path described by $x = 0.2t^2, y = -0.1t^3$ (x and y in ft, t in min). Find the acceleration of the float after 2.0 min.
16. A radio-controlled model car is operated in a parking lot. The coordinates (in m) of the car are given by $x = 3.5 + 2.0t^2$ and $y = 8.5 + 0.25t^3$, where t is the time (in s). Find the acceleration of the car after 2.5 s.
17. An astronaut on Mars drives a golf ball that moves according to the equations $x = 25t$ and $y = 15t - 3.7t^2$ (x and y in meters, t in seconds). Find the resultant velocity and acceleration of the golf ball for $t = 6.0s$.
18. A package of relief supplies is dropped and moves according to the parametric equations $x = 45t$ and $y = -4.9t^2$ (x and y in m, t in s). Find the velocity and acceleration when $t = 3.0$ s.
19. A spacecraft moves along a path described by the parametric equations $x = 10\left(\sqrt{1 + t^4} - 1\right), y = 40t^{3/2}$ for the first 100 s after launch. Here, x and y are measured in meters, and t is measured in seconds. Find the magnitude and direction of the velocity of the spacecraft 10.0 s and 100 s after launch.
20. An electron moves in an electric field according to the equations $x = 8.0/\sqrt{1 + t^2}$ and $y = 8.0t/\sqrt{1 + t^2}$ (x and y in Mm and t in s). Find the velocity of the electron when $t = 0.5$ s.
21. In a computer game, an airplane starts at (1.00, 4.00) (in cm) on the curve $y = 3.00 + x^{-1.50}$ and moves with a constant horizontal velocity of 1.20 cm/s. What is the plane's velocity after 0.500 s?
22. In an aerobic exercise machine, weights are lifted and a person's hands are constrained to move along arcs of the ellipse $16x^2 + 9y^2 = 9$ (in m). If the person's hands move upward at 0.100 m/s, and start at $y = 0$, at what velocity is each moving after 1.50 s?
23. Find the resultant acceleration of the spacecraft in Exercise 19 for the specified times.
24. A ski jump is designed to follow the path given by the equations $x = 3.50t^2$ and $y = 20.0 + 0.120t^4 - 3.00\sqrt{t^4 + 1}$ ($0 \le t \le 4.00$ s) (x and y in m, t in s). Find the velocity and acceleration of a skier when $t = 4.00$ s. See Fig. 19.

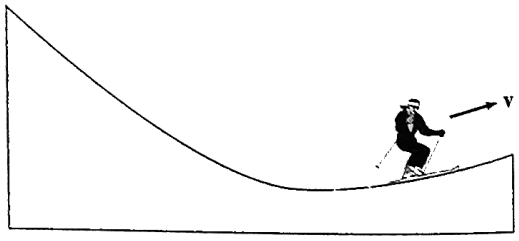

Fig. 19

25. A rocket follows a path given by $y = x - \frac{1}{90}x^3$ (distances in miles). If the horizontal velocity is given by $v_x = x$, find the magnitude and direction of the velocity when the rocket hits the ground (assume level terrain) if time is in minutes.
26. A ship is moving around an island on a route described by $y = 3x^2 - 0.2x^3$. If $v_x = 1.2$ km/h, find the velocity of the ship where $x = 3.5$ km.
27. A computer's hard disk is 3.50 in. in diameter and rotates at 3600 r/min. With the center of the disk at the origin, find the velocity components of a point on the rim for $x = 1.20$ in., if $y > 0$ and $v_x > 0$.
28. A robot arm joint moves in an elliptical path (horizontal major axis 8.0 cm, minor axis 4.0 cm, center at origin). For $y > 0$ and $-2\text{ cm} < x < 2\text{ cm}$, the joint moves such that $v_x = 2.5$ cm/s. Find its velocity for $x = -1.5$ cm.
29. An airplane ascends such that its gain h in altitude is proportional to the square root of the change x in horizontal distance traveled. If $h = 280$ m for $x = 400$ m and v_x is constant at 350 m/s, find the velocity at this point.
30. A meteor traveling toward the earth has a velocity inversely proportional to the square root of the distance from the earth's center. State how its acceleration is related to its distance from the center of the earth.