

Applications of the Derivative

13. An alpha particle moves through a magnetic field along the parabolic path $y = x^2 - 4$. Determine the closest that the particle comes to the origin.
14. The electric potential V on the line $3x + 2y = 6$ is given by $V = 3x^2 + 2y^2$. At what point on this line is the potential a minimum?
15. In deep water, the velocity of a wave is $v = k\sqrt{\frac{l}{a} + \frac{a}{l}}$, where a and k are constants, and l is the length of the wave. What is the length of the wave that results in the minimum velocity?
16. The sum of the length l and width w of a rectangular table top is to be 240 cm. Determine l and w if the area of the table top is to be a maximum.
17. A rectangular hole is to be cut in a wall for a vent. If the perimeter of the hole is 48 in. and the length of the diagonal is a minimum, what are the dimensions of the hole?
18. When two electric resistors R_1 and R_2 are in series, their total resistance (the sum) is 32Ω . If the same resistors are in parallel, their total resistance (the reciprocal of which equals the sum of the reciprocals of the individual resistances) is the maximum possible for two such resistors. What is the resistance of each?
19. A microprocessor chip is being designed with a given rectangular area A . Show that the chip with the minimum perimeter should be a square.
20. The rectangular animal display area in a zoo is enclosed by chain-link fencing and divided into two areas by internal fencing parallel to one of the sides. What dimensions will give the maximum area for the display if a total of 240 m of fencing are used?
21. What are the dimensions of the largest rectangular piece that can be cut from a semi-circular metal sheet of diameter 14.0 cm?
22. A rectangular storage area is to be constructed along side of a building. A security fence is required along the remaining three sides of the area. See Fig. 58. What is the maximum area that can be enclosed with 800 ft of fencing?

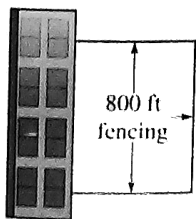


Fig. 58

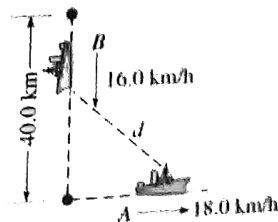


Fig. 59

23. Ship A is traveling due east at 18.0 km/h as it passes a point 40.0 km due south of ship B , which is traveling due south at 16.0 km/h . See Fig. 59. How much later are the ships nearest each other?
24. An architect is designing a rectangular building in which the front wall costs twice as much per linear meter as the other three walls. The building is to cover 1350 m^2 . What dimensions must it have such that the cost of the walls is a minimum?
25. A computer is programmed to display a slowly changing right triangle with its hypotenuse always equal to 12.0 cm . What are the legs of the triangle when it has its maximum area?

26. U.S. Postal Service regulations require that the length plus the girth (distance around) of a package not exceed 108 in. What are the dimensions of the largest rectangular box with square ends that can be mailed?
27. Referring to Exercise 26, what are the radius, length, and volume of the largest cylindrical package that may be sent through the mail?
28. An architect designs a window in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is to be 6.00 m , what dimensions of the rectangle give the window the largest area?
29. The printed area of a rectangular poster is 384 cm^2 , with margins of 4.00 cm on each side and margins of 6.00 cm at the top and bottom. Find the dimensions of the poster with the smallest area.
30. A conical funnel, with a very small opening, is being designed such that the slant height of the cone is 4.00 cm . What is the maximum volume of liquid that the funnel will be able to hold?
31. A culvert designed with a semicircular cross section of diameter 6.00 ft is redesigned to have an isosceles trapezoidal cross section by inscribing the trapezoid in the semicircle. See Fig. 60. What is the length of the bottom base b of the trapezoid if its area is to be maximum?

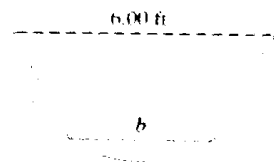


Fig. 60

32. A 36-cm -wide sheet of metal is bent into a rectangular trough as shown in Fig. 61. What dimensions give the maximum water flow?

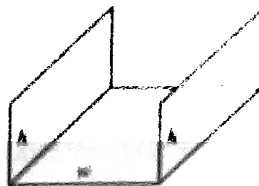


Fig. 61

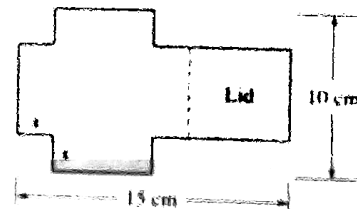


Fig. 62