39. A weight is suspended 50.0 cm below a ceiling by cables as shown in Fig. 63. Find the total minimum possible length of the cables.

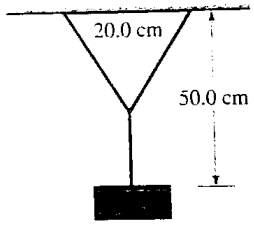

Fig. 63

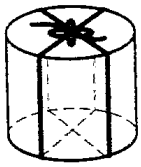

Fig. 64

40. A cylindrical container is to be tied twice around vertically (crossed at the top and bottom) with 108 cm of string plus 8 cm for the knot on top. See Fig. 64. What is the maximum volume the container can have?

41. An airline requires that a carry-on bag has dimensions (length + width + height) that do not exceed 45 in. If a carry-on has a length 2.4 times the width, find the dimensions (to the nearest inch) of this type of carry-on that has the greatest volume.

42. A window is designed in the shape of a rectangle surmounted by a trapezoid, each of whose legs and upper base are one-half of the base of the rectangle. See Fig. 65. If the perimeter of the window is 5.70 m, find the width w and height h such that it lets in the maximum amount of light.

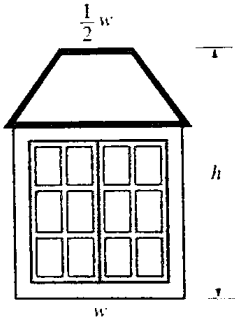

Fig. 65

43. For raising a load, the efficiency E (in %) of a screw with square threads is $E = \dfrac{100T(1 - fT)}{T + f}$, where f is the coefficient of friction and T is the tangent of the pitch angle of the screw. If $f = 0.25$, what acute angle makes E the greatest?

44. Computer simulation shows that the drag F (in N) on a certain airplane is $F = 0.00500v^2 + 3.00 \times 10^8/v^2$, where v is the velocity (in km/h) of the plane. For what velocity is the drag the least?

45. Factories A and B are 8.0 km apart, with factory B emitting eight times the pollutants into the air as factory A. If the number n of particles of pollutants is inversely proportional to the square of the distance from a factory, at what point between A and B is the pollution the least?

46. The potential energy E of an electric charge q due to another charge q_1 at a distance of r_1 is proportional to q_1 and inversely proportional to r_1. If charge q is placed directly between two charges of 2.00 nC and 1.00 nC that are separated by 10.0 mm, find the point at which the total potential energy (the sum due to the other two charges) of q is a minimum.

47. An open box is to be made from a square piece of cardboard whose sides are 8.00 in. long, by cutting equal squares from the corners and bending up the sides. Determine the side of the square that is to be cut out so that the volume of the box may be a maximum. See Fig. 66.

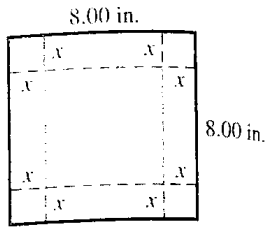

Fig. 66

48. A cone-shaped paper cup is to hold 100 cm^3 of water. Find the height and radius of the cup that can be made from the least amount of paper.

49. A race track 400 m long is to be built around an area that is a rectangle with a semicircle at each end. Find the open side of the rectangle if the area of the rectangle is to be a maximum. See Fig. 67.

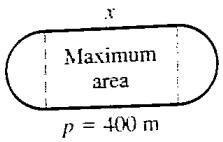

Fig. 67

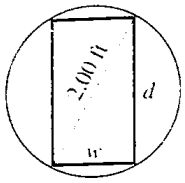

Fig. 68

50. A beam of rectangular cross section is to be cut from a log 2.00 ft in diameter. The stiffness of the beam varies directly as the width and the cube of the depth. What dimensions will give the beam maximum stiffness? See Fig. 68.

51. An oil pipeline is to be built from a refinery to a tanker loading area. The loading area is 10.0 mi downstream from the refinery and on the opposite side of a river 2.5 mi wide. The pipeline is to run along the river and then cross to the loading area. If the pipeline costs $50,000 per mile alongside the river and $80,000 per mile across the river, find the point P (see Fig. 69) at which the pipeline should be turned to cross the river if construction costs are to be a minimum.

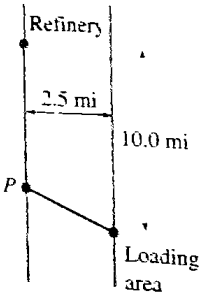

Fig. 69

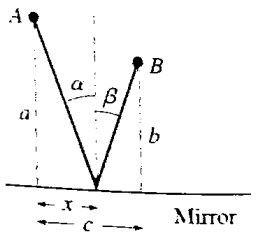

Fig. 70

52. A light ray follows a path of least time. If a ray starts at point A (see Fig. 70) and is reflected off a plane mirror to point B, show that the angle of incidence α equals the angle of reflection β. (*Hint*: Set up the expression in terms of x, which will lead to $\sin\alpha = \sin\beta$.)