EXERCISES 8

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 2, change the t^2 to t^3.
2. In Example 3, change the $-$ before $2x$ to $+$.
3. In Example 5, change 0.020 to 0.025.
4. In Example 6, change $x = 4$ to $x = 12$; approximate $\sqrt{25.06}$.

In Exercises 5–16, find the differential of each of the given functions.

5. $y = x^5 + 4x$
6. $y = 3x^2 + 6$
7. $V = \frac{2}{r^5} + 3\pi^2$
8. $y = 2\sqrt{x} - \frac{1}{8x}$
9. $s = 2(3t^2 - 5)^4$
10. $y = 5(4 + 3x)^{1/3}$
11. $y = \frac{12}{3x^2 + 1}$
12. $R = \sqrt{\frac{6u}{1 + 2u}}$
13. $y = x^2(1 - x)^3$
14. $y = 6x\sqrt{1 - 4x}$
15. $y = \frac{x}{5x + 2}$
16. $y = \frac{3x + 1}{\sqrt{2x - 1}}$

In Exercises 17–20, find the values of Δy and dy for the given values of x and dx.

17. $y = 7x^2 + 4x, x = 4, \Delta x = 0.2$
18. $y = (x^2 + 2x)^3, x = 7, \Delta x = 0.02$
19. $y = x\sqrt{1 + 4x}, x = 12, \Delta x = 0.06$
20. $y = \frac{x}{\sqrt{6x - 1}}, x = 3.5, \Delta x = 0.025$

In Exercises 21–24, find the linearization $L(x)$ of the given functions for the given values of a. Display $f(x)$ and $L(x)$ on the same calculator screen.

21. $f(x) = x^2 + 2x, a = 0$
22. $f(x) = 6\sqrt[3]{x}, a = 8$
23. $f(x) = \frac{1}{2x + 1}, a = -1$
24. $g(x) = x\sqrt{2x + 8}, a = -2$

In Exercises 25–38, solve the given problems by finding the appropriate differential.

25. If a spacecraft circles the earth at an altitude of 250 km, how much farther does it travel in one orbit than an airplane that circles the earth at a low altitude? The radius of the earth is 6370 km.
26. Approximate the amount of paint needed to apply one coat of paint 0.50 mm thick on a hemispherical dome 55 m in diameter.
27. The radius of a circular manhole cover is measured to be 40.6 ± 0.05 cm (this means the possible error in the radius is 0.05 cm). Estimate the possible relative error in the area of the top of the cover.
28. The side of a square microprocessor chip is measured as 0.950 cm, and later it is measured as 0.952 cm. What is the difference in the calculations of the area due to the difference in the measurements of the side? See Fig. 76.

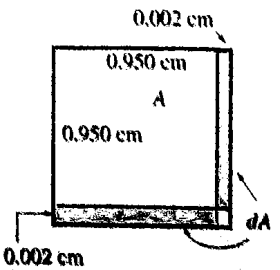

Fig. 76

29. The wavelength λ of light is inversely proportional to its frequency f. If $\lambda = 685$ nm for $f = 4.38 \times 10^{14}$ Hz, find the change in λ if f increases by 0.20×10^{14} Hz. (These values are for red light.)
30. The velocity of an object rolling down a certain inclined plane is given by $v = \sqrt{100 + 16h}$, where h is the distance (in ft) traveled along the plane by the object. What is the increase in velocity (in ft/s) of an object in moving from 20.0 ft to 20.5 ft along the plane? What is the relative change in the velocity?
31. If the diameter equals the height, what is the volume of the plastic in a closed cylindrical container for which the radius is 18.0 cm and the thickness is 2.00 mm?
32. The volume V of blood flowing through an artery is proportional to the fourth power of the radius r of the artery. Find how much a 5% increase in r affects V.
33. The radius r of a holograph is directly proportional to the square root of the wavelength λ of the light used. Show that $dr/r = \frac{1}{2}d\lambda/\lambda$.
34. The gravitational force F of the earth on an object is inversely proportional to the square of the distance r of the object from the center of the earth. Show that $dF/F = -2dr/r$.
35. Show that an error of 2% in the measurement of the radius of a DVD results in an error of approximately 4% in the calculation of the area.
36. Show that an error of 2% in the measurement of the radius of a ball bearing results in an error of approximately 6% in the calculation of the volume.
37. Calculate $\sqrt{4.05}$, using differentials.
38. (W) Explain how to evaluate 2.03^4.

In Exercises 39–44, solve the given linearization problems.

39. Show that the linearization of $f(x) = (1 + x)^k$ at $x = 0$ is $L(x) = 1 + kx$.
40. Use the result shown in Exercise 39 to approximate the value of $f(x) = \frac{1}{\sqrt{1 + x}}$ near zero.
41. Linearize $f(x) = \sqrt{2 - x}$ for $a = 1$ and use it to approximate the value of $\sqrt{1.9}$.
42. (W) Explain how to evaluate $\sqrt[3]{8.03}$, using linearization.
43. The capacitance C (in μF) in an element of an electronic tuner is given by $C = \frac{3.6}{\sqrt{1 + 2V}}$, where V is the voltage. Linearize C for $V = 4.0$ V.
44. A 16-Ω resistor is put in parallel with a variable resistor of resistance R. The combined resistance of the two resistors is $R_T = \frac{16R}{16 + R}$. Linearize R_T for $R = 4.0\ \Omega$.

Answers to Practice Exercises

1. $dy = 8(2x - 1)^3 dx$ 2. $dA = 0.92\ \text{cm}^2$