

EXERCISES 8

In Exercises 1 and 2, make the given changes in the indicated examples of this section and then find dy/dx .

- In Example 2, change y^2 to y^3 .
- In Example 3, change xy^2 to x^2y .

In Exercises 3–6, terms that might appear in an implicit function are shown. Differentiate each term with respect to x .

- x^2y
- $2xy^3$
- $\frac{3}{x^4y}$
- $\frac{y^2}{\sqrt{4x+1}}$

In Exercises 7–26, find dy/dx by differentiating implicitly. When applicable, express the result in terms of x and y .

- $3x + 2y = 5$
- $6x - 3y = 4$
- $4y - 3x^2 = x$
- $x^5 - 5y = 6 - 4x^{3/2}$
- $x^2 - 4y^2 - 9 = 0$
- $\sqrt{x^2 + 2y^2} - 11 = 0$
- $y^5 = x^2 - 1$
- $x^{2/3} + y^{2/3} = 5$
- $6y^{2/3} + y = x^2 - 4$
- $2y^3 - y = 7 - x^4$
- $y + 3xy - 4 = 0$
- $xy^3 + 3y + x^2 = 2\pi^2$
- $x^2 = \frac{x-y}{x+y}$
- $y^2x - \frac{5y}{x+1} + 3x = 4$
- $\frac{3x^2}{y^2+1} + y = 3x + 1$
- $\sqrt{xy} = \frac{x}{4} - \frac{1}{y^2}$
- $(2y-x)^4 + x^2 = y + 3$
- $(y^2+2)^3 = x^4y + e^2$
- $2(x^2+1)^3 + \sqrt{y^2+1} = 17$
- $(2x+1)(1-3y) + y^2 = 13$

In Exercises 27–32, evaluate the derivatives of the given functions at the given points.

- $3x^3y^2 - 2y^3 = -4$; $(1, 2)$
- $2y + 5 - x^2 - y^3 = 0$; $(2, -1)$
- $5y^4 + 7 = x^4 - 3y$; $(3, -2)$
- $(xy - y^2)^{3/2} = 5y^2 + 3$; $(5, 1)$
- $xy^2 + 3x^2 - y^2 + 15 = 0$; $(-1, 3)$
- $2(x+y)^3 - y^2/x = 15$; $(4, -2)$

In Exercises 33–48, solve the given problems by using implicit differentiation.

- At what point(s) does the graph of $x^2 + y^2 = 4x$ have a horizontal tangent?
- Show that if $P(x, y)$ is any point on the circle $x^2 + y^2 = a^2$, then a tangent line at P is perpendicular to a line through P and the origin.
- Show that two tangents to the curve $x^2 + xy + y^2 = 7$ at the points where it crosses the x -axis are parallel.
- At what point(s) is the tangent to the curve $y^2 = 2x^3$ perpendicular to the line $4x - 3y + 1 = 0$?
- In an RLC circuit, the angular frequency ω at which the circuit resonates is given by $\omega^2 = 1/LC - R^2/L^2$. Find $d\omega/dL$.
- Show that the graphs of $2x^2 + y^2 = 24$ and $y^2 = 8x$ are perpendicular at the point $(2, 4)$. Display the graphs on a calculator.

- Find the slope of a line tangent to the curve of the implicit function $xy + y^2 + 2 = 0$ at the point $(-3, 1)$. Use the derivative evaluation feature of a calculator to check your result.

- A lens is formed by cutting a cap (with a flat base) from a spherical piece of glass. The volume V is the lens is $V = \frac{1}{3}\pi h^2(3r - h)$, where r is the radius of the sphere and h is the thickness of the lens. See Fig. 43. If $V = 8\pi/3$, find dr/dh .

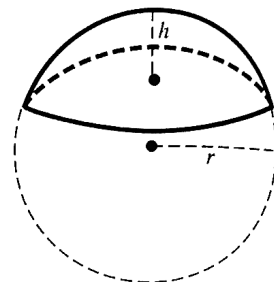


Fig. 43

- The pressure P , volume V , and temperature T of a gas are related by $PV = n(RT + aP - bP/T)$, where a , b , n , and R are constants. For constant V , find dP/dT .

- Oil moves through a pipeline such that the distance s it moves and the time t are related by $s^3 - t^2 = 7t$. Find the velocity of the oil for $s = 4.01$ m and $t = 5.25$ s.

- The shelf support shown in Fig. 44 is 2.38 ft long. Find the expression for dy/dx in terms of x and y .

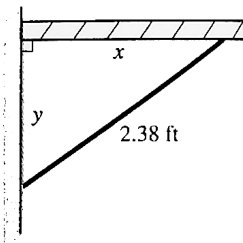


Fig. 44

- An open (no top) right circular cylindrical container of radius r and height h has a total surface area of 940 cm^2 . Find dr/dh in terms of r and h .

- Two resistors, with resistances r and $r + 2$, are connected in parallel. Their combined resistance R is related to r by the equation $r^2 = 2rR + 2R - 2r$. Find dR/dr .

- The polar moment of inertia I of a rectangular slab of concrete is given by $I = \frac{1}{12}(b^3h + bh^3)$, where b and h are the base and the height, respectively, of the slab. If I is constant, find the expression for db/dh .

- A formula relating the length L and radius of gyration r of a steel column is $24C^3Sr^3 = 40C^3r^3 + 9LC^2r^2 - 3L^3$, where C and S are constants. Find dL/dr .

- A computer is programmed to draw the graph of the implicit function $(x^2 + y^2)^3 = 64x^2y^2$ (see Fig. 45 and Example 7). Find the slope of a line tangent to this curve at $(2.00, 0.56)$ and at $(2.00, 3.07)$.

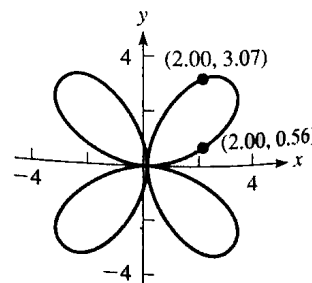


Fig. 45

Answer to Practice Exercise

- $dy/dx = -y/(6y^2 + x)$