Problem 1: One-time-pad

(a) In a brute force attack, one tries every possible key k and tries to decrypt the ciphertext c using k. When decrypting c using k yields a valid plaintext (e.g., an English sentence), one has found the key.

Given enough time, one can also enumerate all possible keys for the one-time pad. Thus, given unlimited computational power, one can apply the brute-force attack to the one-time pad. One the other hand, we have proven that the one-time pad has perfect secrecy. Thus is should not be possible to break the one-time pad.

Explain why a brute-force attack fails on the one-time pad (even if one has unlimited time).

(b) Write a program that achieves the following: It takes as input two ciphertexts $c_1$ and $c_2$ of the same length. Both are expected to be the encryption of a single word $m_1, m_2$ using the one-time pad. To produce the ciphertexts, the same key has been used. The program then finds $m_1$ and $m_2$.

Consider the following ciphertexts: $c_1 = 4A5C45492449552A$, $c_2 = 5A47534D35525F20$ (eight bytes each, presented in hex). Figure out the plaintexts using your program.

Note: On many Linux systems, you find a wordlist in /usr/share/dict/words. Or use the file wordlist.txt from the webpage. Please submit a printout of your source code and the plaintexts.
**Hint:** If you use python (version 3.x), you may find the following code snippets useful:
```
bytes.fromhex("5AC643BE8504E35E")
```
decodes a hex string. And the following XORs two string bitwise:
```
def xor_two_words(x,y):
    assert len(x)==len(y)
    assert isinstance(x,bytes)
    assert isinstance(y,bytes)
    return bytes([a^b for a,b in zip(x,y)])
```

(c) **[Bonus problem.]** Write a program that does the same as in [b], except that $m_1$, $m_2$ are now English sentences.

This is much more difficult, but if you enjoy the challenge, you can do it.

**Problem 2: Perfect secrecy**

Show that there is no encryption scheme that has perfect secrecy and allows us to reuse the key. More precisely, show that there is no encryption scheme $E$ that satisfies the following definition (and that can be decrypted):

**Definition 1 (Perfect secrecy with key reuse)** Let $K$ be the set of keys, let $M$ be the set of messages, and let $E$ be the encryption algorithm (possibly randomized) of an encryption scheme. We say the encryption scheme has perfect secrecy with key reuse iff for all $n$, and all $m_0^{(1)}, \ldots, m_0^{(n)}, m_1^{(1)}, \ldots, m_1^{(n)} \in M$ and for all $c_1, \ldots, c_n$, we have that

\[
\Pr[(c_1, \ldots, c_n) = (c_1', \ldots, c_n') : k \xleftarrow{\$} K, c_1' \leftarrow E(k, m_0^{(1)}), \ldots, c_n' \leftarrow E(k, m_0^{(n)})] = \Pr[(c_1, \ldots, c_n) = (c_1', \ldots, c_n') : k \xleftarrow{\$} K, c_1' \leftarrow E(k, m_1^{(1)}), \ldots, c_n' \leftarrow E(k, m_1^{(n)})]
\]

**Hint:** If you have an encryption scheme $E$ with perfect secrecy with key reuse, first construct from it a scheme $E'$ with perfect secrecy that has messages longer than keys. (Show that it indeed has perfect secrecy.) Then use **Theorem 1** in the lecture notes.