Recording quantum queries – explained

Dominique Unruh

This draft was intended to give a formal treatment of Zhandry's results from [1], with all definitions and proofs worked out.

It is unfinished and there are currently no plans to finish it.

However, since the manuscript has been cited in some places, we provide this incomplete draft as-is for reference.

Contents

1 Some notation

We assume that $\perp$ is a fixed symbol different from any bitstring in $\{0,1\}^*$. Let $\{0,1\}^n_\perp := \{0,1\}^n \cup \{\perp\}$. In slight abuse of notation, we define $CNOT^{\otimes n}$ to be the unitary $CNOT^{\otimes n}|x,y\rangle := |x, y \oplus x\rangle$, and we write  for it (it will always be clear from the context that $CNOT^{\otimes n}$ is meant since $CNOT$ can only be applied to single qubit wires).

Given a unitary transformation U that operates on $\mathbb{C}^{\{0,1\}^n}$, we naturally extend it to $\mathbb{C}^{\{0,1\}^n_\perp}$ by setting $U|\perp\rangle := |\perp\rangle$. For example, when applied to a quantum register with space $\mathbb{C}^{\{0,1\}^n_\perp}$, $H^{\otimes n}$ is the following matrix: $H^{\otimes n}|x\rangle := \sum_y 2^{-n/2}(-1)^{x\cdot y}|y\rangle$, $H^{\otimes n}|\perp\rangle := |\perp\rangle$.

This generalizes directly to unitaries that operate on more than one quantum register. For example, $CNOT^{\otimes n}$ operates on $\mathbb{C}^{\{0,1\}^n_\perp} \otimes \mathbb{C}^{\{0,1\}^n}$ as $CNOT^{\otimes n}|x,y\rangle := |x, y\oplus x\rangle$, $CNOT^{\otimes n}|\perp,y\rangle := |\perp,y\rangle$ and on $\mathbb{C}^{\{0,1\}^n} \otimes \mathbb{C}^{\{0,1\}^n_\perp}$ as $CNOT^{\otimes n}|x,y\rangle := |x, y\oplus x\rangle$, $CNOT^{\otimes n}|x,\perp\rangle := |x,\perp\rangle$. That is, when one wire contains $|\perp\rangle$, the unitary operates as the identity on all other wires.

2 Oracles

In our setting, an *oracle* O consists of the following:

- A state register S_O (described by the underlying Hilbert space).
- One or more query registers $X_1, \ldots, X_n$ (described by the underlying Hilbert spaces).
- An initial state $|\Psi_O\rangle$ for the state register, or a probability distribution D_O^Ψ of initial states.
- A unitary operating U_O operating on $S_O, X_1, \ldots, X_n$.

An *oracle algorithm* A is an algorithm that can make queries to an oracle O. More specifically, an execution of A^O uses four registers, the state register S_A of A, the state register S_O of O, as well as the query registers $X_1, \ldots, X_n$ of O. S_O is initialized with the initial state $|\Psi_O\rangle$ (or with a state sampled according to D_O^Ψ). Then A can perform arbitrary operations on $S_A, X_1, \ldots, X_n$ but not on S_O. In addition, A can query O which means that the unitary U_O is applied to $S_O, X_1, \ldots, X_n$.

Definition 1: Perfectly indistinguishable

Two oracles O_1, O_2 are *perfectly indistinguishable* iff for any oracle algorithm A that outputs a classical bit b, $\Pr[b = 1 : b \leftarrow A^{O_1}] = \Pr[b = 1 : b \leftarrow A^{O_2}]$.

We say O_1, O_2 are *perfectly indistinguishable within q queries* if the above holds for every q-query oracle algorithm A.

2.1 Growing oracles

Definition 2: Growing core oracles

Let $O_{\mathfrak{core}}$ be an oracle with state register $S_{O_{\mathfrak{core}}}$ with Hilbert space $\mathcal{H}_{\mathfrak{core}}$ and query register Y with Hilbert space $\mathcal{H}_Y$, and with initial state $|\Psi_{O_{\mathfrak{core}}}\rangle$ (not a distribution).

Fix some length n.

Then $\mathbf{Grow}(O_{\mathfrak{core}})$ is the following oracle:

- Its state register $S_{\mathbf{Grow}(O_{\mathfrak{core}})}$ consists of registers $(S_x)_{x\in\{0,1\}^n}$, each with Hilbert space $\mathcal{H}_{\mathfrak{core}}$.
- It has query registers X with Hilbert space $\mathbb{C}^{\{0,1\}^n}$ and Y with Hilbert space $\mathcal{H}_Y$.
- It has initial state $|\Psi_{\mathbf{Grow}(O_{\mathfrak{core}})}\rangle := \bigotimes_{x\in\{0,1\}^n} |\Psi_{O_{\mathfrak{core}}}\rangle$.
- Its unitary is $U_{\mathbf{Grow}(O_{\mathfrak{core}})} := \sum_{x\in\{0,1\}^n} U_x \otimes |x\rangle\langle x|$ where U_x stands for $U_{O_{\mathfrak{core}}}$ applied to S_x, Y.

Definition 3: Efficiently growing core oracles

Let $O_{\mathfrak{core}}$, n be as in Definition 2. Let q be an integer (query number). Then $\mathbf{FastGrow}_q(O_{\mathfrak{core}})$ is defined as .

Lemma 4

$\mathbf{Grow}(O_{\mathfrak{core}})$ and $\mathbf{FastGrow}_q(O_{\mathfrak{core}})$ are perfectly indistinguishable within q queries.

2.2 Random oracle

For this and the following subsections, fix two integers n, m (denoting the input / output size of the random oracle).

Definition 5: Random oracle

The *random oracle* RO has state register S_{RO} with Hilbert space $\mathbb{C}^{Fun}$ where Fun is the set of all functions $\{0,1\}^n \to \{0,1\}^m$. It has query registers X and Y with Hilbert spaces $\mathbb{C}^{\{0,1\}^n}$ and $\mathbb{C}^{\{0,1\}^m}$, respectively. Its unitary is $U_{\mathsf{RO}} : |H\rangle|x\rangle|y\rangle \mapsto |H\rangle|x\rangle|y \oplus H(x)\rangle$. The initial state distribution D_{RO}^Ψ returns $|H\rangle$ for uniformly random $H \in Fun$.

2.3 Standard oracle

Definition 6: Standard oracle

The standard oracle StdO has state register S with Hilbert space $\bigotimes_{x\in\{0,1\}^n} \mathbb{C}^{\{0,1\}^m_\perp}$, query registers X and Y with Hilbert spaces $\mathbb{C}^{\{0,1\}^n}, \mathbb{C}^{\{0,1\}^m}$, respectively. The initial state is $\bigotimes_{x\in\{0,1\}^n} |0^m\rangle$ (i.e., $|0^{2^n m}\rangle$). The unitary operation is:

$$U_{\mathsf{StdO}} : |D\rangle|x\rangle|y\rangle := \begin{cases} |D\rangle|x\rangle|y \oplus D_x\rangle & (\text{if } D_x \neq \perp) \\ |D\rangle|x\rangle|y\rangle & (\text{if } D_x = \perp) \end{cases}$$

for $D \in \prod_{x\in\{0,1\}^n} \{0,1\}^m_\perp$.

Note: we could have easily defined the standard oracle to use state space $\bigotimes_{x\in\{0,1\}^n} \mathbb{C}^{\{0,1\}^m}$ (no $\perp$). This would be more natural. However, defining it this way makes it easier to derive the "compressed" oracles below.

Lemma 7

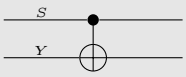
StdO and RO are perfectly indistinguishable.

We show how the standard oracle can be alternatively defined by just specifying its core:

Definition 8: Standard oracle core

The standard oracle core $\mathsf{StdO}_{\mathfrak{core}}$ has state register $S_{\mathsf{StdO}_{\mathfrak{core}}} =: S$ with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|\Psi_{\mathsf{StdO}_{\mathfrak{core}}}\rangle := |+\rangle^{\otimes m}$. The unitary operation is $U_{\mathsf{StdO}_{\mathfrak{core}}} := CNOT^{\otimes m}$, i.e.,

$U_{\mathsf{StdO}_{\mathfrak{core}}} \quad \equiv$



Lemma 9

$\mathsf{StdO} = \mathbf{Grow}(\mathsf{StdO}_{\mathfrak{core}})$.

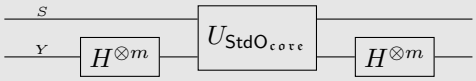
Since this definition is considerably more compact, we will define the following oracles simply by specifying their cores.

2.4 Phase oracle

Definition 10: Phase oracle core

The phase oracle core $\mathsf{PhO}_{\mathfrak{core}}$ has state register $S_{\mathsf{PhO}_{\mathfrak{core}}} =:$ with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|\Psi_{\mathsf{PhO}_{\mathfrak{core}}}\rangle := |+\rangle^{\otimes m}$. The unitary operation $U_{\mathsf{PhO}_{\mathfrak{core}}}$ is given by the following quantum circuit:

$U_{\mathsf{PhO}_{\mathfrak{core}}} \quad \equiv$

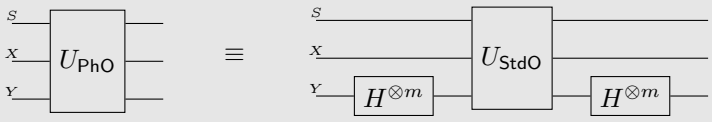


Lemma 11

$\mathsf{PhO} := \mathbf{Grow}(\mathsf{PhO}_{\mathfrak{core}})$.

Lemma 12

$|\Psi_{\mathsf{PhO}}\rangle = |\Psi_{\mathsf{StdO}}\rangle$ and

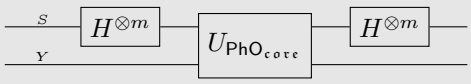


2.5 Fourier phase oracle

Definition 13: Fourier phase oracle core

The Fourier phase oracle core $\mathsf{FPhO}_{\mathfrak{core}}$ has state register $S_{\mathsf{FPhO}_{\mathfrak{core}}} =: S$ with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|\Psi_{\mathsf{FPhO}_{\mathfrak{core}}}\rangle := |0^m\rangle$. The unitary operation is given by the following quantum circuit:

$U_{\mathsf{FPhO}_{\mathfrak{core}}}$ ≡



Definition 14

$\mathsf{FPhO} := \mathbf{Grow}(\mathsf{FPhO}_{\mathfrak{core}})$.

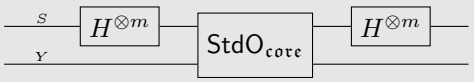
Lemma 15

2.6 Fourier standard oracle

Definition 16: Fourier standard oracle core

The Fourier standard oracle core $\mathsf{FStdO}_{\mathfrak{core}}$ has state register S with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|0^m\rangle$. The unitary operation is given by the following quantum circuit:

$\mathsf{FStdO}_{\mathfrak{core}}$ ≡



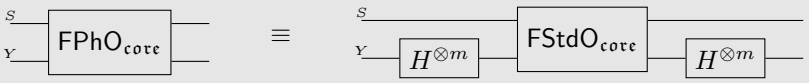
Definition 17

$\mathsf{FStdO} := \mathbf{Grow}(\mathsf{FStdO}_{\mathfrak{core}})$.

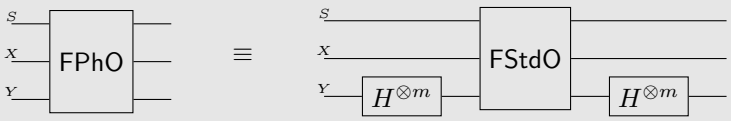
Lemma 18

$\mathsf{FStdO}_{\mathfrak{core}}$ is perfectly indistinguishable from $\mathsf{StdO}_{\mathfrak{core}}$. FStdO is perfectly indistinguishable from StdO.

Lemma 19



and



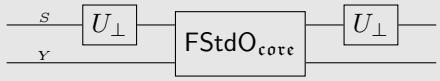
3 Compressed oracles

Let $U_\perp$ be the unitary on $\mathbb{C}^{\{0,1\}^m_\perp}$ defined by: $U_\perp|0^m\rangle := \perp$, $U_\perp|\perp\rangle := |0^m\rangle$, $U_\perp|x\rangle := |x\rangle$ for $x \in \{0,1\}^m$, $x \neq 0^m$.

3.1 Compressed Fourier standard oracle

Definition 20: Compressed Fourier standard oracle core

The compressed Fourier standard oracle core $\mathsf{CFStdO}_{\mathfrak{core}}$ has state register S with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|\perp\rangle$. The unitary operation is given by the following quantum circuit:

$\mathsf{CFStdO}_{\mathfrak{core}}$ $\equiv$ 

Definition 21: Compressed Fourier standard oracle

$\mathsf{CFStdO} := \mathbf{Grow}(\mathsf{CFStdO}_{\mathfrak{core}})$.

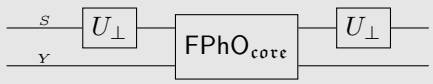
Lemma 22

$\mathsf{CFStdO}_{\mathfrak{core}}$, $\mathsf{FStdO}_{\mathfrak{core}}$, and $\mathsf{StdO}_{\mathfrak{core}}$ are perfectly indistinguishable. CFStdO, FStdO, and StdO are perfectly indistinguishable.

3.2 Compressed Fourier phase oracle

Definition 23: Compressed Fourier phase oracle core

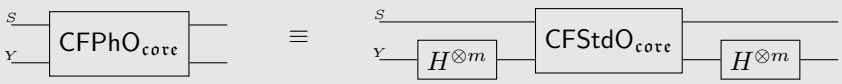
The compressed Fourier phase oracle core $\mathsf{CFPhO}_{\mathfrak{core}}$ has state register S with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|\perp\rangle$. The unitary operation is given by the following quantum circuit:

$\mathsf{CFPhO}_{\mathfrak{core}}$ $\equiv$ 

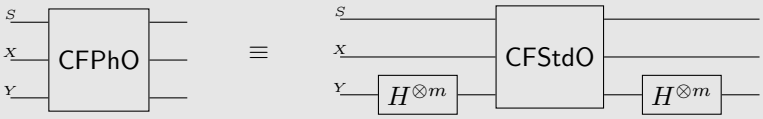
Definition 24: Compressed Fourier phase oracle

$\mathsf{CFPhO} := \mathbf{Grow}(\mathsf{CFPhO}_{\mathfrak{core}})$.

Lemma 25



and



Lemma 26

$\mathsf{CFPhO}_{\mathfrak{core}}$, $\mathsf{FPhO}_{\mathfrak{core}}$, and $\mathsf{PhO}_{\mathfrak{core}}$ are perfectly indistinguishable. CFPhO, FPhO, and PhO are perfectly indistinguishable.

Lemma 27

For all $d \in \{0,1\}^m_\perp$, $y \in \{0,1\}^m$:

$$\begin{array}{lllll} \mathsf{CFPhO}_{\mathsf{core}}\colon & |\perp\rangle|y\rangle & \mapsto |y\rangle|y\rangle & (y \neq 0^m) \\ & |\perp\rangle|0^m\rangle & \mapsto |\perp\rangle|0^m\rangle & \\ & |d\rangle|y\rangle & \mapsto |d \oplus y\rangle|y\rangle & (d \neq 0^m, \perp,\ y \neq d) \\ & |y\rangle|y\rangle & \mapsto |\perp\rangle|y\rangle & (y \neq 0^m) \\ & |0\rangle|y\rangle & \mapsto |0\rangle|y\rangle & \end{array}$$

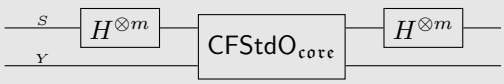
Note: this differs from Zhandry's description in the "impossible" case $d = 0^m$, $y \neq d$.

3.3 Compressed standard oracle

Definition 28: Compressed standard oracle core

The compressed standard oracle core $\mathsf{CStdO}_{\mathsf{core}}$ has state register S with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|\perp\rangle$. The unitary operation is given by the following quantum circuit:

$\mathsf{CStdO}_{\mathsf{core}} \quad \equiv$



Definition 29

$\mathsf{CStdO} := \mathbf{Grow}(\mathsf{CStdO}_{\mathsf{core}})$.

Lemma 30

$\mathsf{CStdO}_{\mathsf{core}}$, $\mathsf{CFStdO}_{\mathsf{core}}$, $\mathsf{FStdO}_{\mathsf{core}}$, and $\mathsf{StdO}_{\mathsf{core}}$ are perfectly indistinguishable. CStdO, CFStdO, FStdO, and StdO are perfectly indistinguishable.

Lemma 31: Some useful equations for working with CStdO

For clarity, the "error terms" are in gray.

$$H^{\otimes m} U_\perp H^{\otimes m}|d\rangle = |d\rangle - 2^{-m/2}|+^m\rangle + 2^{-m/2}|\perp\rangle \qquad (d \neq \perp)$$

$$H^{\otimes m} U_\perp H^{\otimes m}|\perp\rangle = |+^m\rangle$$

$$\mathsf{CStdO}_{\mathsf{core}}|d\rangle|y\rangle = |d\rangle|y \oplus d\rangle + 2^{-m/2}|\perp\rangle|y \oplus d\rangle - \sum_{e \in \{0,1\}^m} 2^{-m}|e\rangle|y \oplus e\rangle \qquad (d \neq \perp)$$
$$+ 2^{-m}|+^m\rangle|+^m\rangle - 2^{-m}|\perp\rangle|+^m\rangle$$

$$\mathsf{CStdO}_{\mathsf{core}}|\perp\rangle|y\rangle = \sum_{e \in \{0,1\}^m} 2^{-m/2}|e\rangle|y \oplus e\rangle - 2^{-m/2}|+^m\rangle|+^m\rangle + 2^{-m/2}|\perp\rangle|+^m\rangle$$

Lemma 32

Let ψ be a vector in $\mathbb{C}^{\{0,1\}^m} \otimes \mathbb{C}^{\{0,1\}^m} \otimes \mathcal{H}$. Let $P := \sum_{d \in M} |d\rangle\langle d| \otimes I \otimes I$ for some $M \subseteq \{0,1\}^m$. Then

$$\|P\,(\mathsf{CStdO}_{\mathsf{core}} \otimes I)\psi\| \ \leq\ 2^{-m/2+1}\sqrt{|M|}\,\|(1-P)\psi\| \ +\ \|P\psi\|$$

Lemma 33

Let ψ be a vector in . Fix a family $M_x \subseteq \{0,1\}^m$ with $x \in \{0,1\}^n$. Assume $|M_x| \leq B$ for all x. Let $P := 1 - \bigotimes_x (\sum_{d \notin M_x} |d\rangle\langle d|)$. Then

$$\|P\,(\mathsf{CStdO} \otimes I)\psi\| \;\leq\; 2^{-m/2+1}\sqrt{B}\,\|(1-P)\psi\| \;+\; \|P\psi\|$$

Can we generalize this? This only allows us to talk about properties like "for each x, $D(x) \notin M_x$." But not about properties like "D has no collision".

Lemma 34

Let ψ be a vector in . Fix $M, N \subseteq (\{0,1\}^n \to \{0,1\}^m_\perp)$. Assume $N \subseteq M$. Assume that for all $x \in \{0,1\}^n$ and all $D \notin M$, we have that

$$\left|\left\{d : d \in \{0,1\}^m_\perp, D(x := d) \in N\right\}\right| \leq B.$$

Let $P_M := \sum_{D \in M} |D\rangle\langle D| \otimes I \otimes I$ and P_N analogous.
Then

$$\|P_N\,(\mathsf{CStdO}_{\mathfrak{core}} \otimes I)\psi\| \;\leq\; 2^{-m/2+1}\sqrt{B}\,\|(1-P_M)\psi\| \;+\; \|P_M\psi\|$$

Example: For collision resistance, in the i-th query, M is the set of all D that have a collision or more than $i-1$ non-$\perp$, and N is the set of all D that have a collision or more than i non-$\perp$. Then $B = i-1$. Total success probability: $\left(2^{-m/2+1}\sum_{i=0}^{q-1}\sqrt{i-1}\right)^2 \leq 2^{-m+2}(q\sqrt{q})^2 = 4q^3/2^m$.

Lemma 35

Let A be an algorithm with oracle access to CStdO that outputs a list L of input/output pairs (i.e., a list $L = \{(x_1,y_1),\ldots,(x_n,y_n)\}$). Assume that if $(x,y) \in L$, then A has made a classical query with input x to CStdO and measured the output and gotten the result y.

Then, conditioned on output $L = \{(x_1,y_1),\ldots,(x_n,y_n)\}$, the final state of CStdO in register is of the form $\sum_D \alpha_D |D\rangle\langle D|$ ranging only over values D with $D(x_i) = y_i \forall i$.

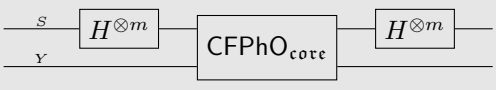
For example, for analyzing Grover, we transform a search algorithm B into A which queries the final output of B and outputs the result. If B is successful, then A will have a zero-value in the D-register, and thus happens with small probability by analysis via Lemma 34. For collision-finder B, we let A query the collision and output the result. This reduces it to the probability that D contains a collision.

3.4 Compressed phase oracle

Definition 36: Compressed phase oracle core

The compressed phase oracle core $\mathsf{CPhO}_{\mathfrak{core}}$ has state register S with Hilbert space $\mathbb{C}^{\{0,1\}^m_\perp}$, and query register Y with Hilbert space $\mathbb{C}^{\{0,1\}^m}$. The initial state is $|\perp\rangle$. The unitary operation is given by the following quantum circuit:

$\mathsf{CPhO}_{\mathfrak{core}} \quad \equiv$



Definition 37

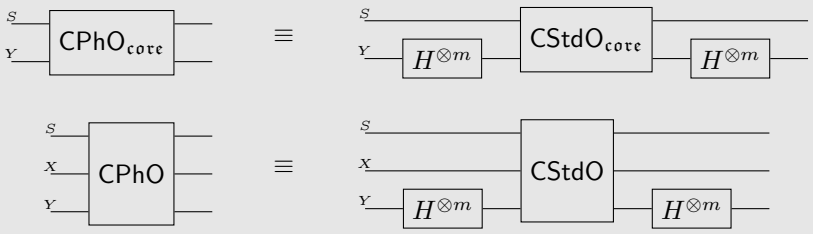
$\mathsf{CPhO} := \mathbf{Grow}(\mathsf{CPhO}_{\mathfrak{core}})$.

Lemma 38

$\mathsf{CPhO}_{\mathfrak{core}}$, $\mathsf{CFPhO}_{\mathfrak{core}}$, $\mathsf{FPhO}_{\mathfrak{core}}$, and $\mathsf{PhO}_{\mathfrak{core}}$ are perfectly indistinguishable. CPhO, CFPhO, FPhO,

and PhO are perfectly indistinguishable.

Lemma 39



and

4 Efficient compressed oracles

5 Example: Hardness of finding collisions

Let A be an oracle quantum algorithm making at most q queries to a random oracle $H : \{0,1\}^n \to \{0,1\}^m$. Let $\varepsilon := \Pr[x \neq x' \wedge H(x) = H(x') : (x, x') \leftarrow A^H]$.

Let B^H do: Run $(x, x') \leftarrow A^H$, query $y \leftarrow H(x)$, $y' \leftarrow H(x')$. Return $(x, y), (x', y')$. We call the output of B *good* iff $x \neq x'$ and $y = y'$. Then $\Pr[out \text{ good} : out \leftarrow B^H] = \varepsilon$.

By , $\Pr[out \text{ good} : out \leftarrow B^{\mathsf{CStdO}}] = \varepsilon$.

By Lemma 35, this implies that measuring the oracle's state register using P_M where M is the set of all D that contains a collision will succeed with probability $\geq \varepsilon$. (P_M is as in Lemma 34.)

Let ψ_i be the quantum state before the i-th query, and ψ'_i after the i-th query. Let M_i be the set of all D such that D contains a collision or contains $\geq i$ entries.

Note that for all $i \leq q+2$ and $D \notin M_{i-1}$, we have

$$\left|\left\{d : d \in \{0,1\}^m_\perp, D(x := d) \in M_i\right\}\right| \leq q.$$

Since ψ_1 contains $D = \perp$, we have $\|P_{M_0}\psi_1\| = 0$.

By Lemma 34, $\|P_{M_i}\psi'_i\| \leq 2^{-m/2+1}\sqrt{q} + \|P_{M_{i-1}}\psi_i\|$. Furthermore, since P_{M_i} operates only on the state register, $\|P_{M_i}\psi'_i\| = \|P_{M_i}\psi_{i+1}\|$. By induction, $\|P_{M_{q+2}}\psi'_{q+2}\| \leq (q+2)2^{-m/2+1}\sqrt{q}$.

Then

$$\varepsilon = \|P_M\psi'_{q+2}\|^2 \leq \|P_{M_{q+2}}\psi'_{q+2}\|^2 \leq (q+2)^2 2^{-m+2} q.$$

Symbol index

$\mathbb{C}$	Complex numbers	
H	Hadamard matrix	
$\langle n\|$	Adjoing of basis vector n	
RO	Random oracle	
$CNOT$	CNOT matrix	1
$\mathsf{StdO}_{\mathfrak{core}}$	Standard oracle core	3
StdO	Standard oracle	3
$\mathsf{PhO}_{\mathfrak{core}}$	Phase oracle core	3
PhO	Phase oracle	3
$\mathsf{CFPhO}_{\mathfrak{core}}$	Compressed Fourier phase oracle core	5
CFPhO	Compressed Fourier phase oracle	5
$\mathsf{CFStdO}_{\mathfrak{core}}$	Compressed Fourier standard oracle core	5
CFStdO	Compressed Fourier standard oracle	5
$\|\psi\|$	(Hilbert space-)norm of vector ψ	
$\mathbf{Grow}(q)U_{\mathfrak{core}}$	"Growing" an oracle efficiently for q queries	
$\|x\|$	Absolute value / cardinality	

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References

[1] Mark Zhandry. "How to Record Quantum Queries, and Applications to Quantum Indifferentiability". In: *Advances in Cryptology – CRYPTO 2019*. Ed. by Alexandra Boldyreva and Daniele Micciancio. Eprint is IACR ePrint 2018/276. Cham: Springer International Publishing, 2019, pp. 239–268. ISBN: 978-3-030-26951-7.