Register allocation
Register allocation

Overview

- Variables may be stored in the main memory or in registers.
  - Main memory is much slower than registers.
  - The number of registers is strictly limited.
- The goal of register allocation is to decrease the number of memory accesses by keeping as many as possible variables in registers.
  - Decides which values to keep in registers and which in memory.
  - Assigns concrete registers for values which are kept there.
### Register allocation

**Observations**

- Usually there are less registers than variables.
- Simultaneously alive variables cannot be allocated to the same register.
- Variables which life times do not overlap can be allocated to the same register.
- These constraints can be represented as an **interference graph**:
  - nodes are variables;
  - edges are between simultaneously alive variables.
- Register allocation can be stated as a graph coloring problem of the interference graph with $k$ colors (Lavrov 1962, Chaitin 1981)
  - $k = \text{the number of registers.}$
Register allocation

\[
\begin{align*}
    b &= a + 2 \\
    c &= b \times b \\
    b &= c + 1 \\
    \text{return } b \times a
\end{align*}
\]

**Live sets**

- \{a\}
- \{a, b\}
- \{a, c\}
- \{a, b\}

**Live ranges**

**Interference graph**
Register allocation

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Register allocation

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Live sets:
- \{a\}
- \{a, b\}
- \{a, c\}
- \{a, b\}

Live ranges:

Interference graph:
- \(a\)
- \(b\)
- \(c\)

= Register 1 (R1)

= Register 2 (R2)
Register allocation

Construction of the interference graph

- To build the interference graph we need to determine live ranges of variables.
- In case of local register allocation inside basic blocks, all live ranges are linear.
  - ✔ Discovering live ranges and checking whether they overlap is very easy.
  - ✗ Variables have to be read from the memory before entering to the basic block, and to be stored to the memory when leaving.
- In case of global register allocation, live ranges form a **web**.
  - ✗ Discovering live ranges is more complex.
  - ✔ Allows more efficient usage of registers.
Register allocation
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Register allocation

Webs s1 and s2 overlap
Webs s1 and s4 overlap
Graph coloring

**Definition**
A graph $G$ is $k$-colorable iff its nodes can be labeled with integers $1 \ldots k$ so that no edge in $G$ connects two nodes with the same label.

**Main questions**
- How to find efficiently $k$-coloring of a graph?
- Whether and how to find an optimal coloring (ie. a coloring with the minimum number of colors)?
- What to do when there are not enough colors (ie. registers)?
Problem

The graph coloring problem is NP-complete.

Observations

- Optimal algorithm works with all graphs.
  - The "worst case graph" doesn't appear in practice.
- It always finds a minimal coloring.
  - Often, an approximate coloring is enough.
<table>
<thead>
<tr>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>What to do if the graph is not $k$-colorable?</td>
</tr>
<tr>
<td>- I.e. there is not enough registers?</td>
</tr>
<tr>
<td>- Happens very often.</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Spilling</th>
</tr>
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<tbody>
<tr>
<td>Choose a variable and keep its values in the memory (i.e. in the stack) instead of an register.</td>
</tr>
<tr>
<td>- The process is called <strong>spilling</strong>.</td>
</tr>
<tr>
<td>Places where the variable is accessed, generate an extra code for reading from and storing to the memory.</td>
</tr>
</tbody>
</table>
Graph coloring

Idea

- Pick a node with degree $< k$.
  - This node is $k$-colorable!
- Remove the node (and all its edges) from the graph.
  - All its neighbours have now degree decremented by one.
  - May result to new nodes with degree $< k$.
- If all nodes have degree $\geq k$, then pick a node, spill it to the memory, and continue.
Graph coloring

Chaitin’s algorithm

1. Until there are nodes with degree $< k$:
   - choose such node and push it into the stack;
   - delete the node and all its edges from the graph.

2. If the graph is non-empty (and all nodes have degree $\geq k$), then:
   - choose a node (using some heuristics) and spill it to the memory;
   - delete the node and all its edges from the graph.
   - if this results to some nodes with degree $< k$, then go to the step 1;
   - otherwise continue with the step 2.

3. Successively pop nodes off the stack and color them in the lowest color not used by some neighbor.
Chaitin’s algorithm

Example:

Stack
Chaitin’s algorithm

Example:

Stack

1

1

2

3

4

5
Chaitin’s algorithm

Example:

Stack

\[ \begin{array}{c|c}
2 & 1 \\
\end{array} \]

Graph:

```
1 -- 2 -- 4 -- 5
|     |     |
|     |     |
|     |     |
1     3
```
Chaitin’s algorithm

Example:

Stack

```
4
2
1
```
Chaitin’s algorithm

Example:

Stack

3
4
2
1

Diagram with nodes and edges representing the algorithm's structure.
Chaitin’s algorithm

Example:

Stack

5
3
4
2
1

Colors

Green
Red
Blue
Chaitin’s algorithm

Example:

Stack

Colors
Chaitin’s algorithm

Example:

Stack

Colors
Chaitin’s algorithm

Example:

Stack

Colors
Chaitin’s algorithm

Example:

```
Stack
1

Colors
```

![Graph with nodes and lines connecting them, along with colors for each node.](image-url)
Chaitin’s algorithm

Example:

Stack

Colors
Graph coloring

Optimistic coloring (Briggs et al)

- If all nodes have a degree $\geq k$, then instead of spilling order the nodes and push them into stack.
  - When taking nodes back from the stack they may still be colorable!
- The following graph is 2-colorable:

![Graph Diagram](image)
Optimistic coloring (Briggs et al)

- If all nodes have a degree $\geq k$, then instead of spilling order the nodes and push them into stack.
  - When taking nodes back from the stack they may still be colorable!

- The following graph is 2-colorable:
Graph coloring

Chaitin-Briggs’i algoritm

1. Until there are nodes with degree $< k$:
   - choose such node and push it into the stack;
   - delete the node and all its edges from the graph.

2. If the graph is non-empty (and all nodes have degree $\geq k$), then:
   - choose a node, push it into the stack, and delete it (together with edges) from the graph;
   - if this results to some nodes with degree $< k$, then go to the step 1;
   - otherwise continue with the step 2.

3. Pop a node from the stack and color it by the least free color.
   - If there is no free colors, then choose an uncolored node, spill it into the memory, and go to the step 1.
Graph coloring

Spilling heuristics

Choosing a node for spilling is a critical for efficiency.

Chaitin’s heuristics:
- to minimize the value of $\frac{\text{cost}}{\text{degree}}$, where cost is a spilling cost and degree is a current degree of the node;
- i.e. choose for spilling a ”cheapest” possible node which decreases the degree of most other nodes.

Alternative popular metrics: $\frac{\text{cost}}{\text{degree}^2}$.

Variations:
- spilling of interference regions;
- partitioning of live ranges;
- rematerialization.