Register allocation

Register allocation

Overview

- Variables may be stored in the main memory or in registers.
  - Main memory is much slower than registers.
  - The number of registers is strictly limited.
- The goal of register allocation is to decrease the number of memory accesses by keeping as many as possible variables in registers.
  - Decides which values to keep in registers and which in memory.
  - Assigns concrete registers for values which are kept there.

Register allocation

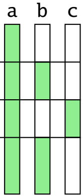
Observations

- Usually there are less registers than variables.
- Simultaneously alive variables cannot be allocated to the same register.
- Variables which life times do not overlap can be allocated to the same register.
- These constraints can be represented as an interference graph:
  - nodes are variables;
  - edges are between simultaneously alive variables.
- Register allocation can be stated as a graph coloring problem of the interference graph with k colors (Lavrov 1962, Chaitin 1981)
  - k = the number of registers.

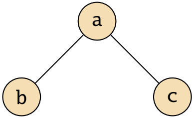
Register allocation

	{a}
`b = a + 2`	
	{a,b}
`c = b * b`	
	{a,c}
`b = c + 1`	
	{a,b}
`return b*a`	

Code — Live sets

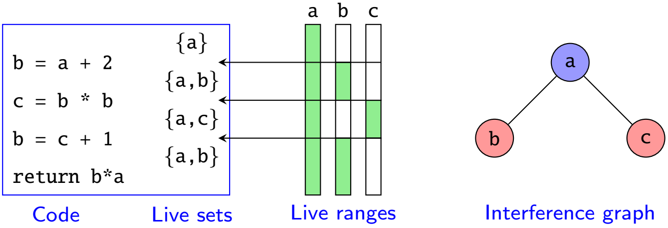


Live ranges



Interference graph

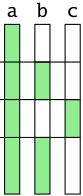
Register allocation



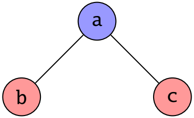
Register allocation

```
              {a}
b = a + 2
              {a,b}
c = b * b
              {a,c}
b = c + 1
              {a,b}
return b*a
```

Code Live sets



Live ranges



Interference graph

```
R2 = R1 + 2
R2 = R2 * R2
R2 = R2 + 1
return R2*R1
```

= Register 1 (R1)

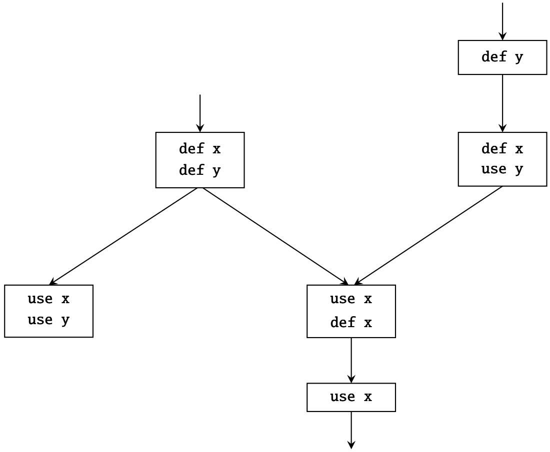
= Register 2 (R2)

Register allocation

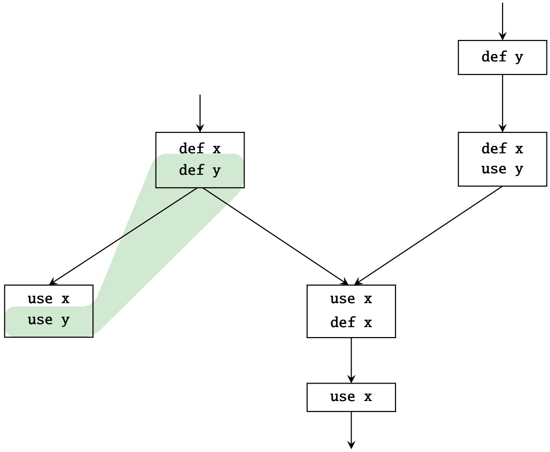
Construction of the interference graph

- To build the interference graph we need to determine live ranges of variables.
- In case of local register allocation inside basic blocks, all live ranges are linear.
  - ✔ Discovering live ranges and checking whether they overlap is very easy.
  - ✘ Variables have to be read from the memory before entering to the basic block, and to be stored to the memory when leaving.
- In case of global register allocation, live ranges form a web.
  - ✘ Discovering live ranges is more complex.
  - ✔ Allows more efficient usage of registers.

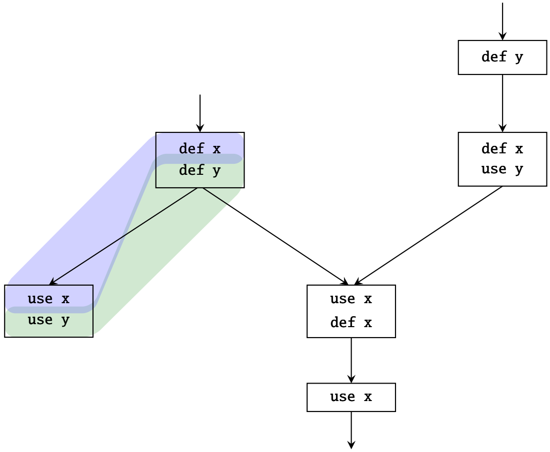
Register allocation



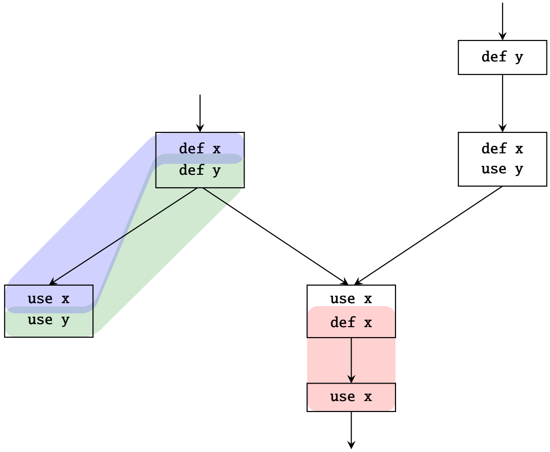
Register allocation



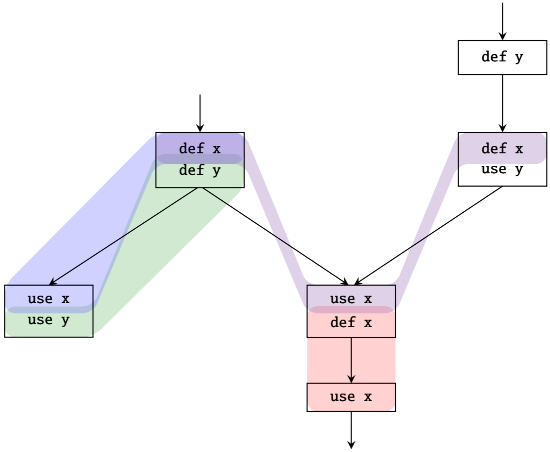
Register allocation



Register allocation



Register allocation



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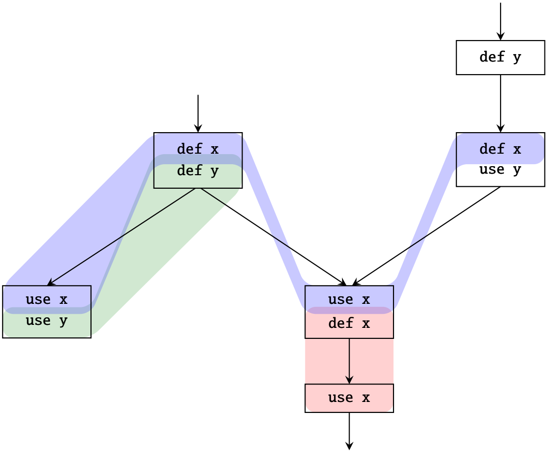
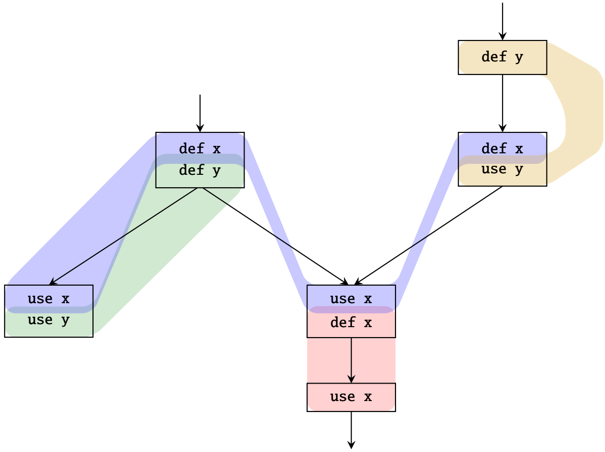


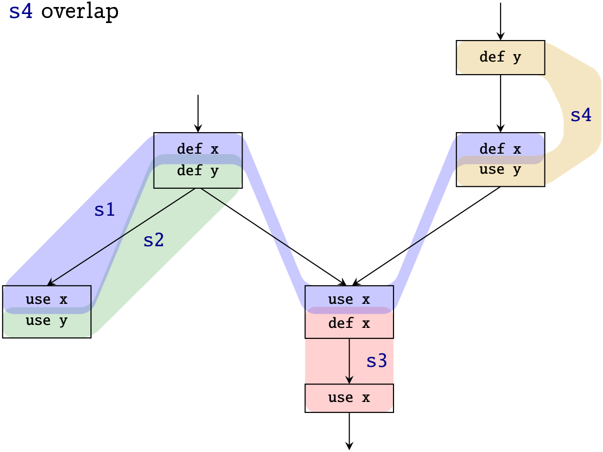
Register allocation



Register allocation

Webs s1 and s2 overlap

Webs s1 and s4 overlap



Graph coloring

Definition

A graph G is k-colorable iff its nodes can be labeled with integers $1 \dots k$ so that no edge in G connects two nodes with the same label.

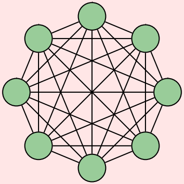
Main questions

- How to find efficiently k-coloring of a graph?
- Whether and how to find an optimal coloring (ie. a coloring with the minimum number of colors)?
- What to do when there are not enough colors (ie. registers)?

Graph coloring

Problem

The graph coloring problem is NP-complete.



Observations

- Optimal algorithm works with all graphs.
  - The "worst case graph" doesn't appear in practice.
- It always finds a minimal coloring.
  - Often, an approximate coloring is enough.

Graph coloring

Problem

What to do if the graph is not k-colorable?

- Ie. there is not enough registers?
- Happens very often.

Spilling

- Choose a variable and keep its values in the memory (ie. in the stack) instead of an register.
  - The process is called spilling.
- Places where the variable is accessed, generate an extra code for reading from and storing to the memory.

Graph coloring

Idea

- Pick a node witch has degree $< k$.
  - This node is k-colorable!
- Remove the node (and all its edges) from the graph.
  - All its neighbours have now degree decremented by one.
  - May result to new nodes with degree $< k$.
- If all nodes have degree $\geq k$, then pick a node, spill it to the memory, and continue.

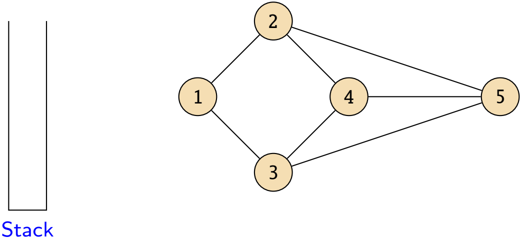
Graph coloring

Chaitin's algorithm

1. Until there are nodes with degree $< k$:
   - choose such node and push it into the stack;
   - delete the node and all its edges from the graph.
2. If the graph is non-empty (and all nodes have degree $\geq k$), then:
   - choose a node (using some heuristics) and spill it to the memory;
   - delete the node and all its edges from the graph.
   - if this results to some nodes with degree $< k$, then go to the step 1;
   - otherwise continue with the step 2.
3. Successively pop nodes off the stack and color them in the lowest color not used by some neighbor.

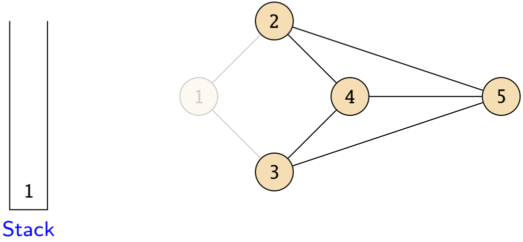
Chaitin's algorithm

Example:



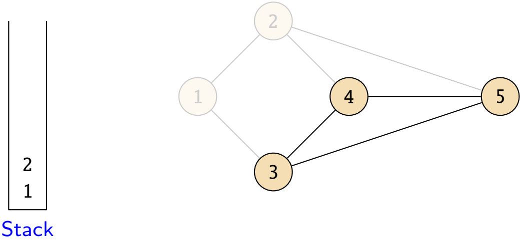
Chaitin's algorithm

Example:



Chaitin's algorithm

Example:



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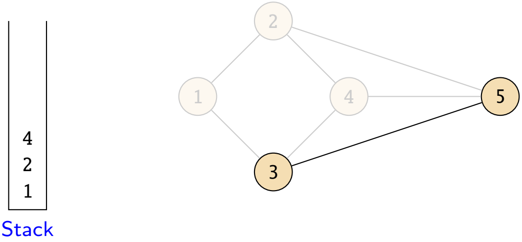

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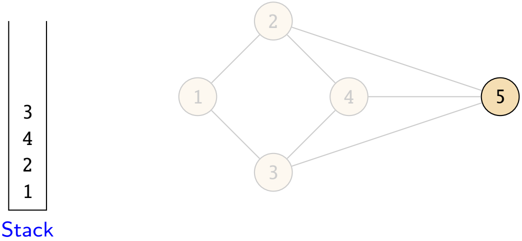

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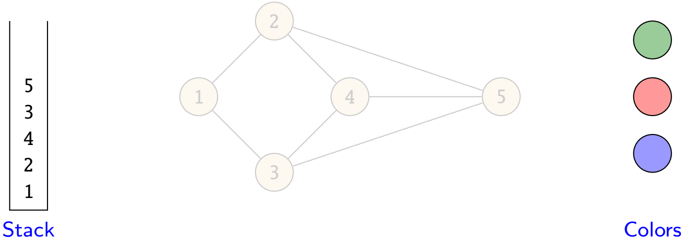

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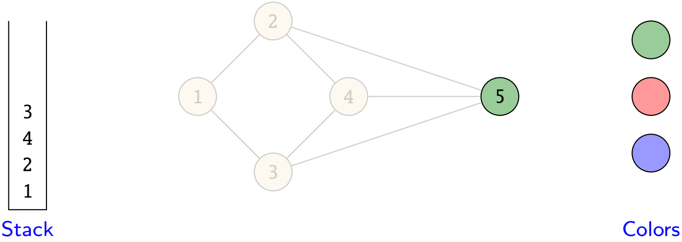

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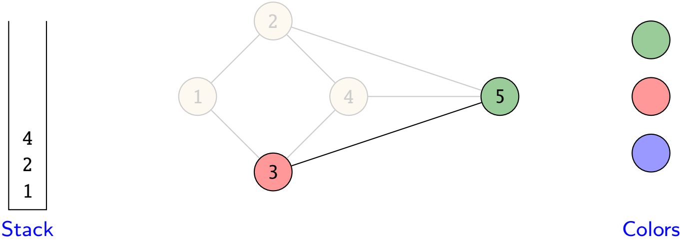

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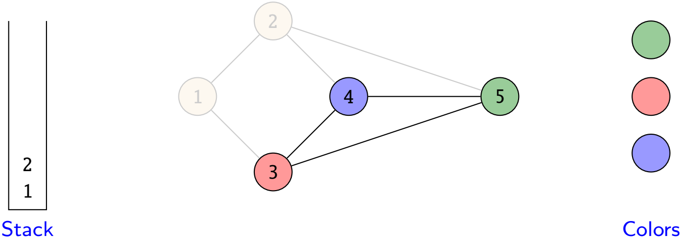

Chaitin's algorithm

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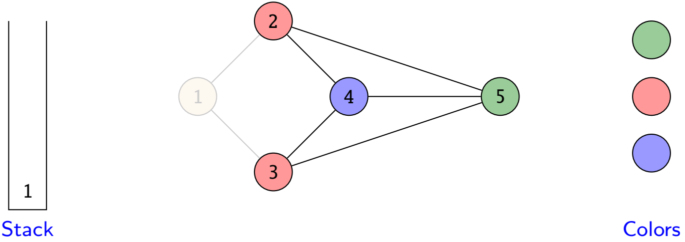

Chaitin's algorithm

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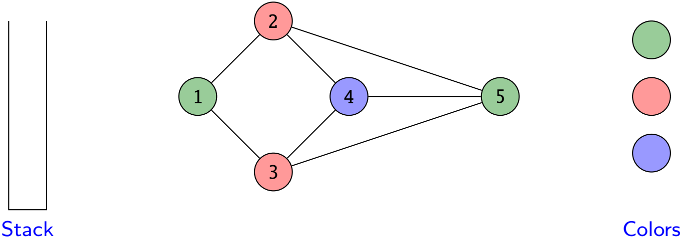
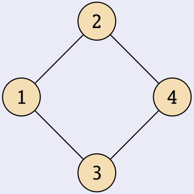


Graph coloring

Optimistic coloring (Briggs et al)

- If all nodes have a degree $\geq k$, then instead of spilling order the nodes and push them into stack.
  - When taking nodes back from the stack they may still be colorable!
- The following graph is 2-colorable:



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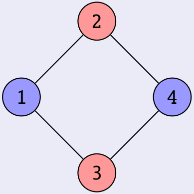

Graph coloring

Chaitin-Briggs'i algorithm

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2. If the graph is non-empty (and all nodes have degree $\geq k$), then:
   - choose a node, push it into the stack, and delete it (together with edges) from the graph;
   - if this results to some nodes with degree $< k$, then go to the step 1;
   - otherwise continue with the step 2.
3. Pop a node from the stack and color it by the least free color.
   - If the is no free colors, then choose an uncolored node, spill it into the memory, and go to the step 1.

Graph coloring

Spilling heuristics

- Choosing a node for spilling is a critical for efficiency.
- Chaitin's heuristics:
  - to minimize the value of $\frac{cost}{degree}$, where *cost* is a spilling cost and *degree* is a current degree of the node;
  - ie. choose for spilling a "cheapest" possible node which decreases the degree of most other nodes.
- Alternative popular metrics: $\frac{cost}{degree^2}$.
- Variations:
  - spilling of interference regions;
  - partitioning of live ranges;
  - rematerialization.