Exercises

(for the Tartu course)

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1. Exercise I (Processes). Consider the following law which establishes the distributivity of synchronous product over *choice*:

$$p \otimes (q + r) \;=\; (p \otimes q) + (p \otimes r) \tag{1}$$

1. Give a *pointfree* version of this law.
2. Prove it.
3. Explain why a similar distribution law for *interleaving* does not hold in general.
4. Do you think a similar law holds for *parallel composition*? Why?

2. Exercise II (Processes). Prove that *renaming* commutes with *interleaving* and preserves both identity and composition of *Act*-homomorphisms, as expressed by the following equations:

$$[\mathsf{id}] \;=\; \mathsf{id} \tag{2}$$

$$[f] \cdot [f'] \;=\; [f \cdot f'] \tag{3}$$

$$[f] \cdot ||| \;=\; ||| \cdot ([f] \times [f]) \tag{4}$$

for any $f, f' : Act \longrightarrow Act$.

3. Exercise III (Components). Recall the definition of $\boxtimes$ given in the lectures. Prove that, for any functions f and g,

$$\ulcorner f \urcorner \boxtimes \ulcorner g \urcorner \;\sim\; \ulcorner f \times g \urcorner \tag{5}$$

Prove also that $\boxtimes$ is associative.

4. Exercise IV (Components). Recall the calculus of components framed as generalized Mealy machines, parametric on a strong monad B, introduced in this course. Suppose you want to study how such a calculus could be modified to allow the combination of components with different behaviour models represented by different choices for B. As part of this study

1. Re-work the definitions of **;**, $\boxtimes$ and $\boxplus$ so that the first and second arguments are components over, respectively, B_1 and B_2. What conditions, on B_1 and B_2, must be imposed, in each case, to make the combinator well defined?
2. Identify the identity elements for this three operators in the new setting and prove they are actually so.

5. Exercise V (Components). Consider an *objects-as-components* calculus in which components are defined as

$$p \;=\; \langle \gamma_p : U_p \longrightarrow \mathbf{2}, \langle o_p : U_p \longrightarrow O, \overline{a_p} : U_p \longrightarrow \mathsf{B}U_p^I \rangle\rangle \tag{6}$$

where U_p is a state space, o_p an attribute, $\overline{a_p}$ a method and γ_p a predicate over U_p intended to characterise all possible initial states for p (therefore replacing the usual seed value $u_p \in U_p$ by a more general requirement).

1. Identify the functor for which this kind of components are coalgebras.
2. Define a notion of morphism $h : p \longrightarrow q$ between two components p and q and express such a definition in terms of equations for o_p, o_q, a_p, a_q and γ_p, γ_q.
3. Define *choice* ($\boxplus$) and *parallel* ($\boxtimes$) for this kind of components and prove both of them to be commutative.