Connectors as Relations

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Two Views On Components

The OO Legacy

- components as (collections of) classes/objects
- method invocation as the kernel of component composition
- resort to middleware intervention to loosen tight-coupling

The Coordination Paradigm View

- temporal/spatial decoupling to support a looser inter-component dependency
- amenable to external control
- requires anonymous communication

Component Connectors

$$C : I_1 \times I_2 \times \ldots \times I_n \longrightarrow O_1 \times O_2 \times \ldots \times O_m \quad \triangleq \quad \langle ID, \mathbb{D}_C \rangle$$

where ID is an instance identifier and $\mathbb{D}_C$ is a specification of its dynamics

$$\overline{\mathbb{D}_C} : U \longrightarrow \mathcal{P}(U \times (\mathbb{O}_1 \times \ldots \times \mathbb{O}_m))^{\mathbb{I}_1 \times \ldots \times \mathbb{I}_n}$$

where $\mathbb{O}_i = O_i \times \mathbb{T}$ and $\mathbb{I}_i = I_i \times \mathbb{T}$ and $\mathbb{T}$ is the time tag domain:

- need weak notion of time to accommodate ordering of occcurrence
- simultaneity as atomicity
- every data item arrives stamped and is stamped on output

Component Connectors

Memory-Less Connectors are Relations

for $U = \mathbf{1}$ get

$$\mathbb{D}_C : \mathbb{I}_1 \times \ldots \times \mathbb{I}_n \longrightarrow \mathcal{P}(\mathbb{O}_1 \times \ldots \times \mathbb{O}_m)$$

which is isomorphic to relation $[\![C]\!]$:

$$[\![C]\!] = \in \cdot \mathbb{D}_C \text{ and (conversely) } \mathbb{D}_C = \Lambda[\![C]\!]$$

also define:

$$\begin{aligned} \mathsf{data}.[\![C]\!] &= \mathcal{P}(\pi_1 \cdot \mathsf{m})[\![C]\!] \\ \mathsf{time}.[\![C]\!] &= \mathcal{P}(\pi_2 \cdot \mathsf{m})[\![C]\!] \end{aligned}$$

Component Connectors

Synchronous Channel

$$[\![\bullet \overset{\sigma}{\longmapsto} \bullet]\!] = \mathsf{Id}_{I \times \mathbb{T}}$$

Synchronous Filter

$$\mathsf{data}.[\![\bullet \overset{\sigma_f}{\longmapsto} \bullet]\!] = \mathsf{Graph}(f)$$

$$\mathsf{time}.[\![\bullet \overset{\sigma_f}{\longmapsto} \bullet]\!] = \mathsf{Id}_{\mathbb{T}}$$

Lossy Channel

$$[\![\bullet \overset{\cdots}{\longmapsto} \bullet]\!] \subseteq \mathsf{Id}_{I \times \mathbb{T}}$$

Component Connectors

Synchronous Drain

$$\mathsf{data}.[\![\bullet \overset{\blacktriangledown}{\longmapsto} \bullet]\!] = I \times I$$

$$\mathsf{time}.[\![\bullet \overset{\blacktriangledown}{\longmapsto} \bullet]\!] = \mathsf{Id}_{\mathbb{T}}$$

Asynchronous Drain

$$\mathsf{data}.[\![\bullet \overset{\triangledown}{\longmapsto} \bullet]\!] = I \times I$$

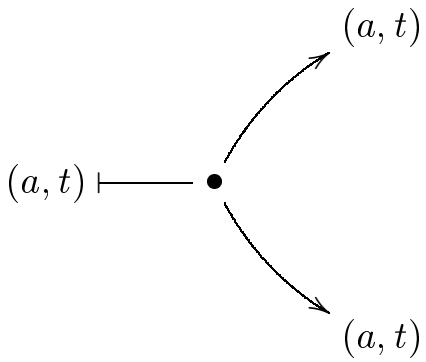
$$\mathsf{time}.[\![\bullet \overset{\triangledown}{\longmapsto} \bullet]\!] \cap \mathsf{Id}_{\mathbb{T}} = \emptyset$$

Component Connectors

Broadcast

$$[\![\triangleleft]\!] \;=\; \mathsf{Graph}(\triangle)$$

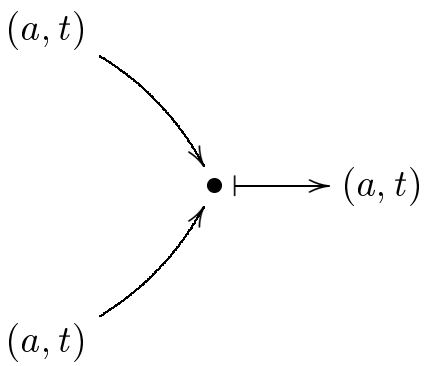
where $\triangle = \langle \mathsf{id}_{I\times\mathbb{T}}, \mathsf{id}_{I\times\mathbb{T}} \rangle$



Component Connectors

Concentrator

$$[\![\triangleright]\!] \;=\; [\![\triangleleft]\!]^{\circ}$$



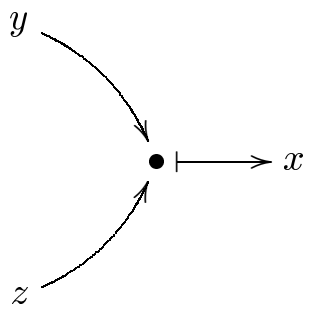
Forbidden

$$[\![|]\!] \;=\; \emptyset$$

Component Connectors

Merge

$$x \, [\![\blacktriangleright]\!] \, (y, z) \;\equiv\; (x = y \vee x = z) \;\wedge\; \pi_2 \; y \neq \pi_2 \; z$$



Component Connectors

Postponer

$$\mathsf{data}.[\![\bullet \overset{\delta}{\longmapsto} \bullet]\!] \;=\; \mathsf{Id}_I$$

$$\mathsf{time}.[\![\bullet \overset{\delta}{\longmapsto} \bullet]\!] \;=\; <$$

Antecipator

$$\mathsf{time}.[\![\bullet \overset{\alpha}{\longmapsto} \bullet]\!] \;=\; (\mathsf{time}.[\![\delta]\!])^{\circ}$$

Composing Connectors

Sequential Composition

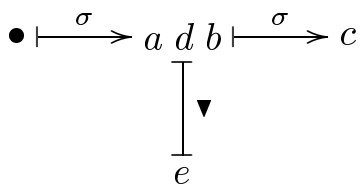
$$[\![C_1 \,;\, C_2]\!] \;=\; [\![C_2]\!] \cdot [\![C_1]\!]$$

Parallel Composition

$$[\![C_1 \boxtimes C_2]\!] \;=\; [\![C_1]\!] \times [\![C_1]\!]$$

New Connectors From Old

External Control Flow



because from the semantics of ; and

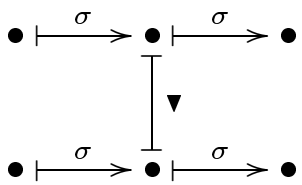
$$\begin{aligned} (b,d)\,[\![\triangleleft]\!]\,a &\;\Rightarrow\; a = b = d \\ d\,[\![\blacktriangledown]\!]\,e &\;\Rightarrow\; \pi_2\; d = \pi_2\; e \\ b\,[\![\sigma]\!]\,c &\;\Rightarrow\; b = c \end{aligned}$$

conclude $\pi_2\; c \;=\; \pi_2\; e$

New Connectors From Old

External Control Flow

$$\begin{tikzcd}
\bullet \arrow[r, mapsto, "\sigma"] & \bullet \arrow[r, mapsto, "\sigma"] \arrow[d, mapsto, "\blacktriangledown"] & \bullet \\
\bullet \arrow[r, mapsto, "\sigma"] & \bullet \arrow[r, mapsto, "\sigma"] & \bullet
\end{tikzcd}$$



New Connectors From Old

Laws

$$[\![C \;;\; \bullet \xmapsto{\sigma} \bullet]\!] \;=\; C \;=\; [\![\bullet \xmapsto{\sigma} \bullet \;;\; C]\!]$$

$$[\![\bullet \xmapsto{\sigma_f} \bullet \;;\; \bullet \xmapsto{\sigma_g} \bullet]\!] \;=\; [\![\bullet \xmapsto{\sigma_{g \cdot f}} \bullet]\!]$$

$$[\![\bullet \xmapsto{\cdots} \bullet \;;\; \bullet \xmapsto{\cdots} \bullet]\!] \;=\; [\![\bullet \xmapsto{\cdots} \bullet]\!]$$

$$[\![\triangleleft \;;\; \triangleright]\!] \;=\; [\![\bullet \xmapsto{\sigma} \bullet]\!]$$

$$[\![\triangleleft \;;\; \blacktriangleright]\!] \;=\; [\![|]\!]$$

$$[\![\blacktriangleright \;;\; \bullet \xmapsto{\blacktriangledown} \bullet]\!] \;=\; [\![\bullet \xmapsto{\triangledown} \bullet]\!]$$

To Follow

- connectors generalized to connector operators: flexible pipe, merge and broadcast
- state-based connectors: buffers, ...
- specification of architectural patterns
- comparasion with stream-based formalisms (eg, [Broy01] and [Arbab&Rutten03])