Programmeerimine

15. loeng

Täna loengus

- Algoritmid ja keerukus
- Lineaarne vs. kahendotsimine
- Sorteerimine
  - Valikumeetodil sorteerimine
  - Ühildusmeetodil sorteerimine

Algoritmid ja keerukus

- Konkreetse programmi efektiivsus sõltub:
  - programmis kasutatud algoritmidest;
  - sisendandmete suurusest;
  - aga näiteks ka konkreetse masina riistvarast;
  - ...
- Põhiidee: ignoreerime masinast sõltuvaid konstante ja uurime asümptootilist keerukust:
  - millise funktsiooni $T(n)$ järgi kasvab algoritmis tehtavate sammude arv, kui sisendi suurus n kasvab lõpmatuks.
- Asümtootilist keerukust tähistame $\mathcal{O}(T(n))$.
- Keerukuse tähistamisel piisab kui näidata ainult kõige "kõrgemat järku" term.
  - Näiteks: $2n^3 + 5n^2 + 12n + 4 = \mathcal{O}(n^3)$.

Algoritmid ja keerukus

Ülesande lahendamise aeg

Kui palju aega kulutab kaasaegne arvuti aega ülesande lahendamiseks sõltuvalt algandmete hulgast?

	10	20	30	40	50	60
n	0,00001	0,00002	0,00003	0,00004	0,00005	0,00006
n^2	0,0001	0,0004	0,0009	0,0016	0,0025	0,0036
n^3	0,001	0,008	0,027	0,064	0,125	0,216
n^5	0,1	3,2	24,3	$1{,}7m$	$5{,}2m$	$13m$
2^n	0,001	1,0	$17{,}9m$	$12{,}7p$	$35{,}3a$	$366saj$
3^n	0,059	$58m$	$6{,}5a$	$3855saj$	$2 * 10^8 saj$	$10^{13} saj$

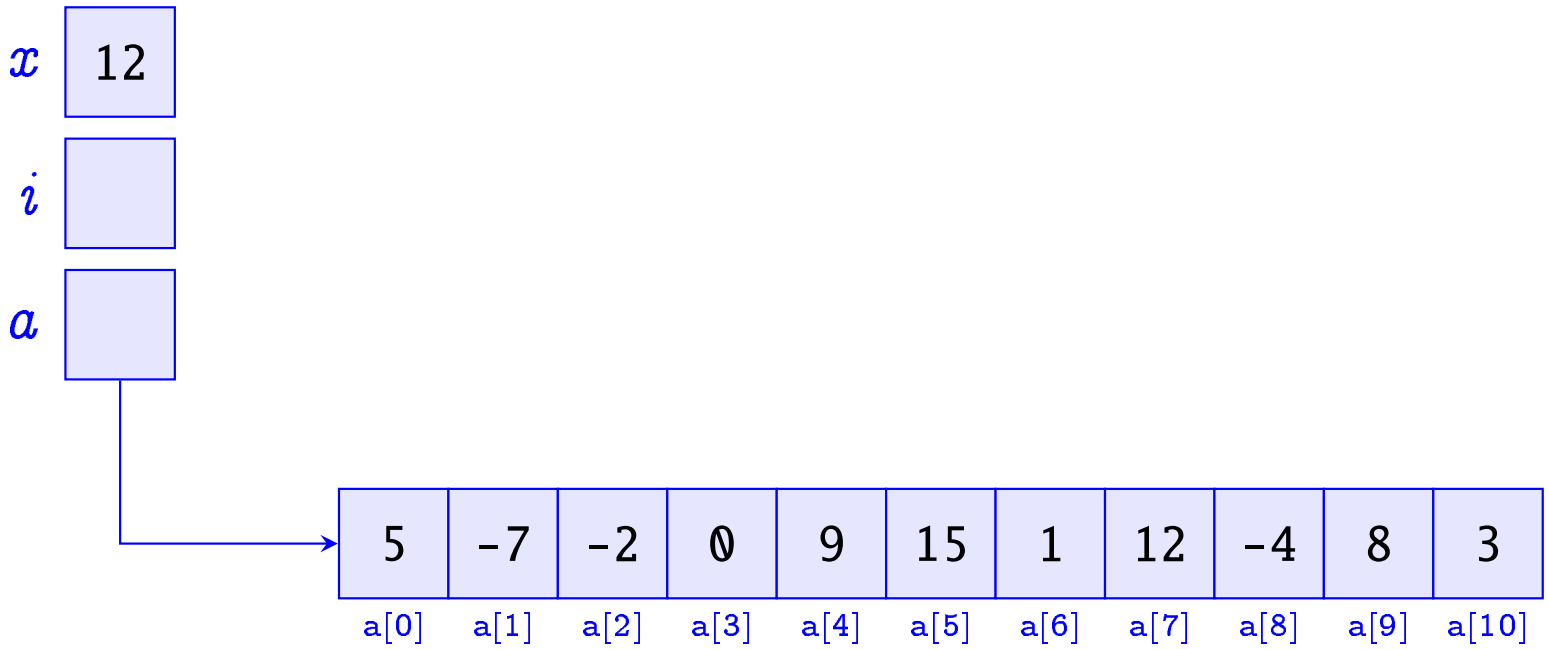
Massiivist elemendi otsimine

- Lihtsaim meetod massiivist elemendi otsimiseks vaatab massiivi elemente ükshaaval järjestikku läbi.
- Kui jõutakse otsitava elemendini, siis väljastatakse vastava elemendi indeks.
- Kui otsitavat elementi ei leidu, siis väljastatakse -1.

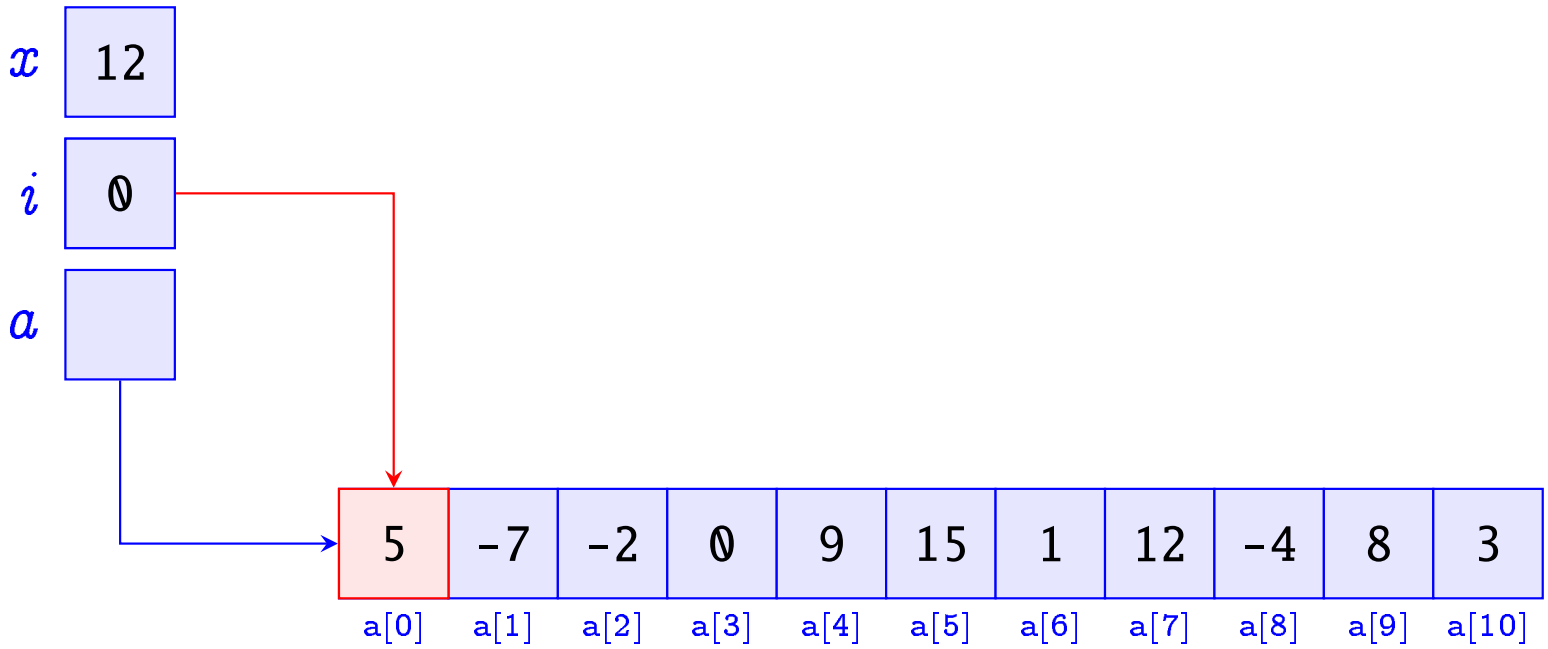
Lineaarne otsimine

```
def linearSearch(x, a):
    for i in range(len(a)):
        if a[i] == x: return i
    return -1
```

Massiivist elemendi otsimine



Massiivist elemendi otsimine



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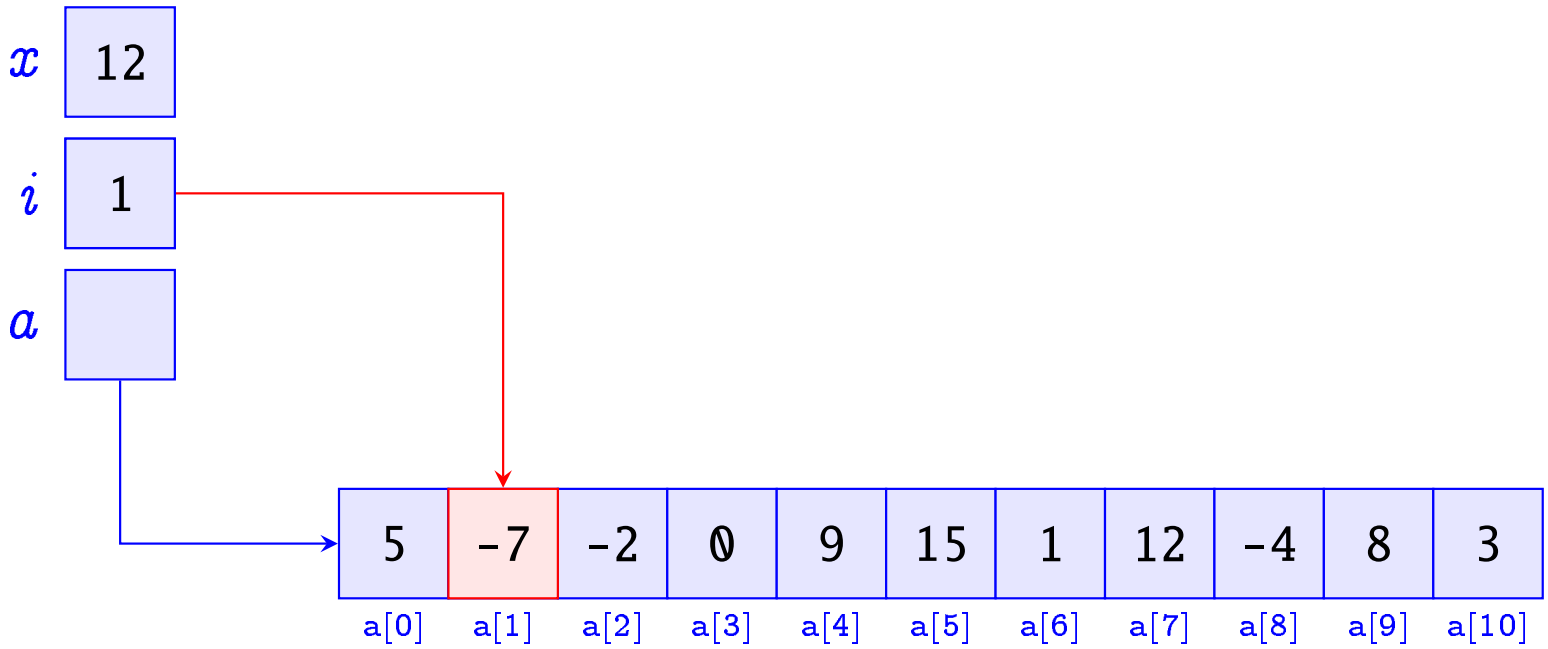

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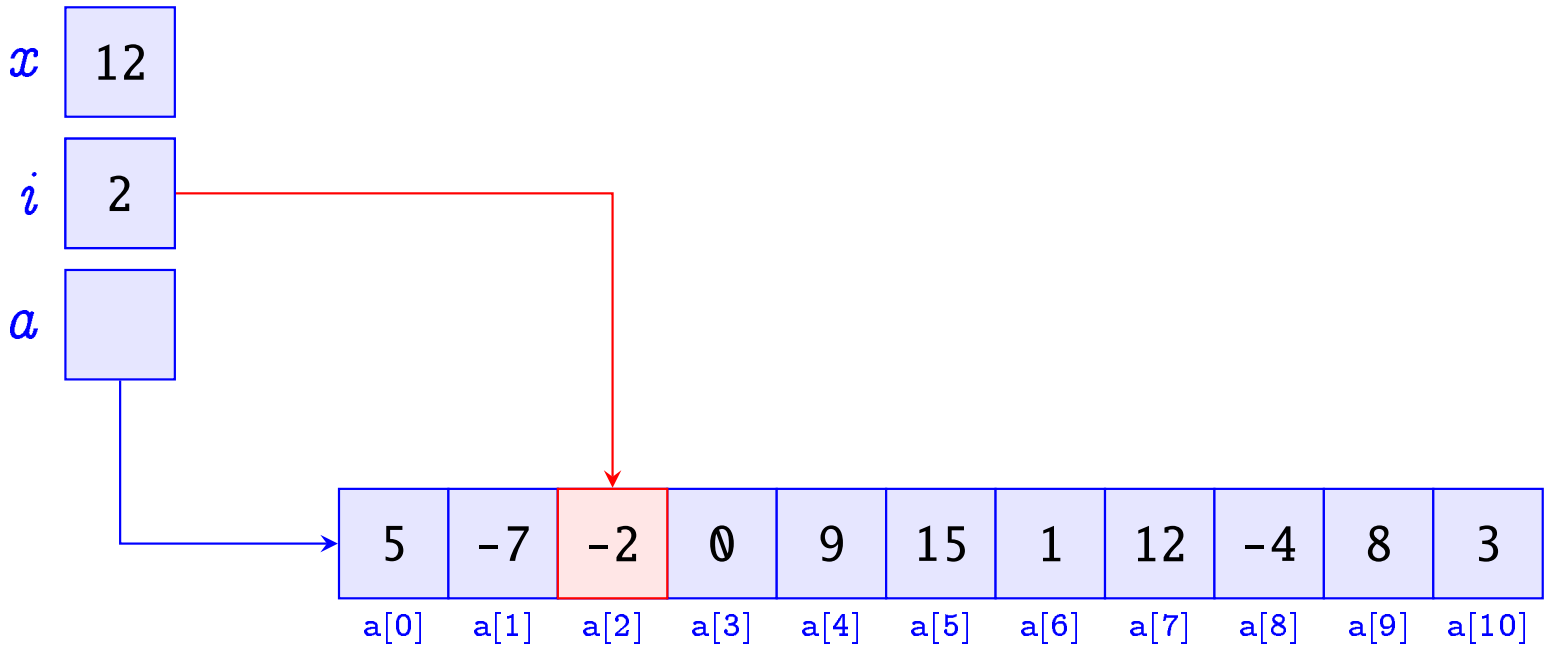

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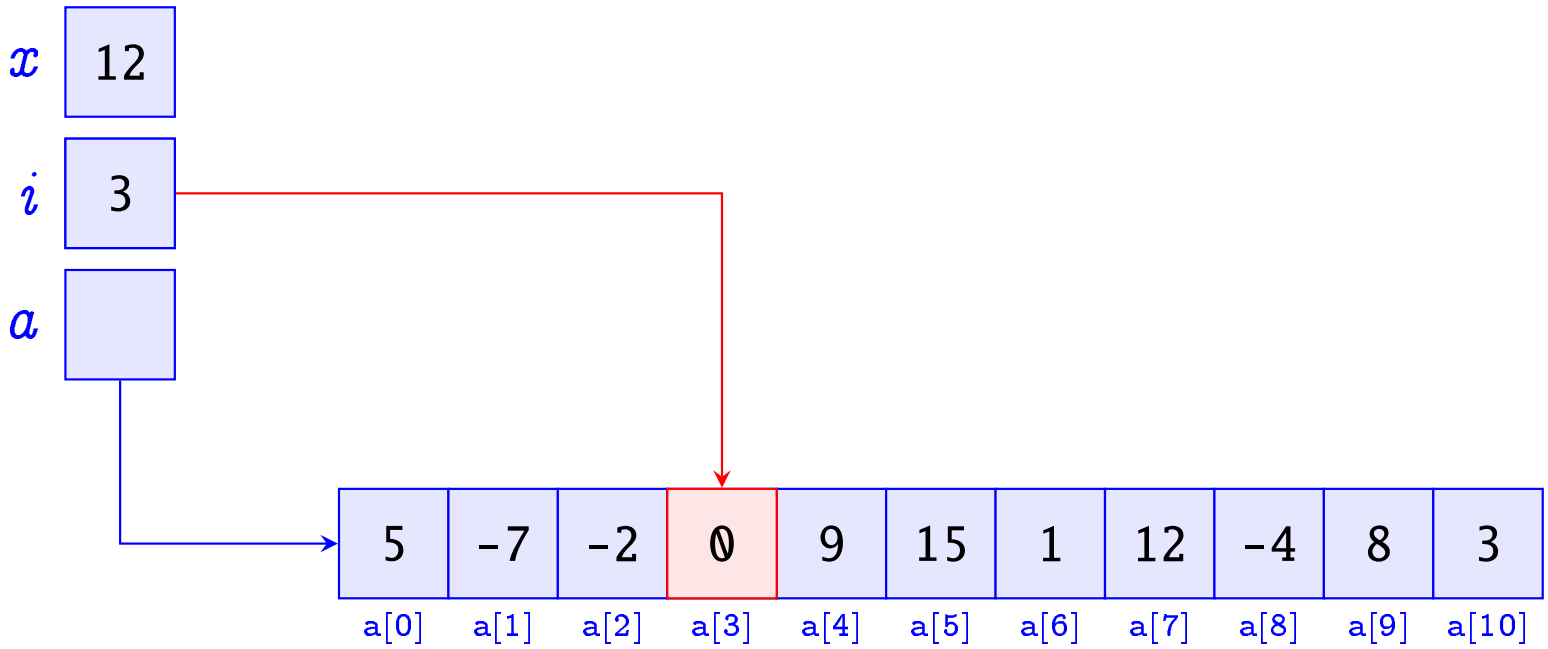

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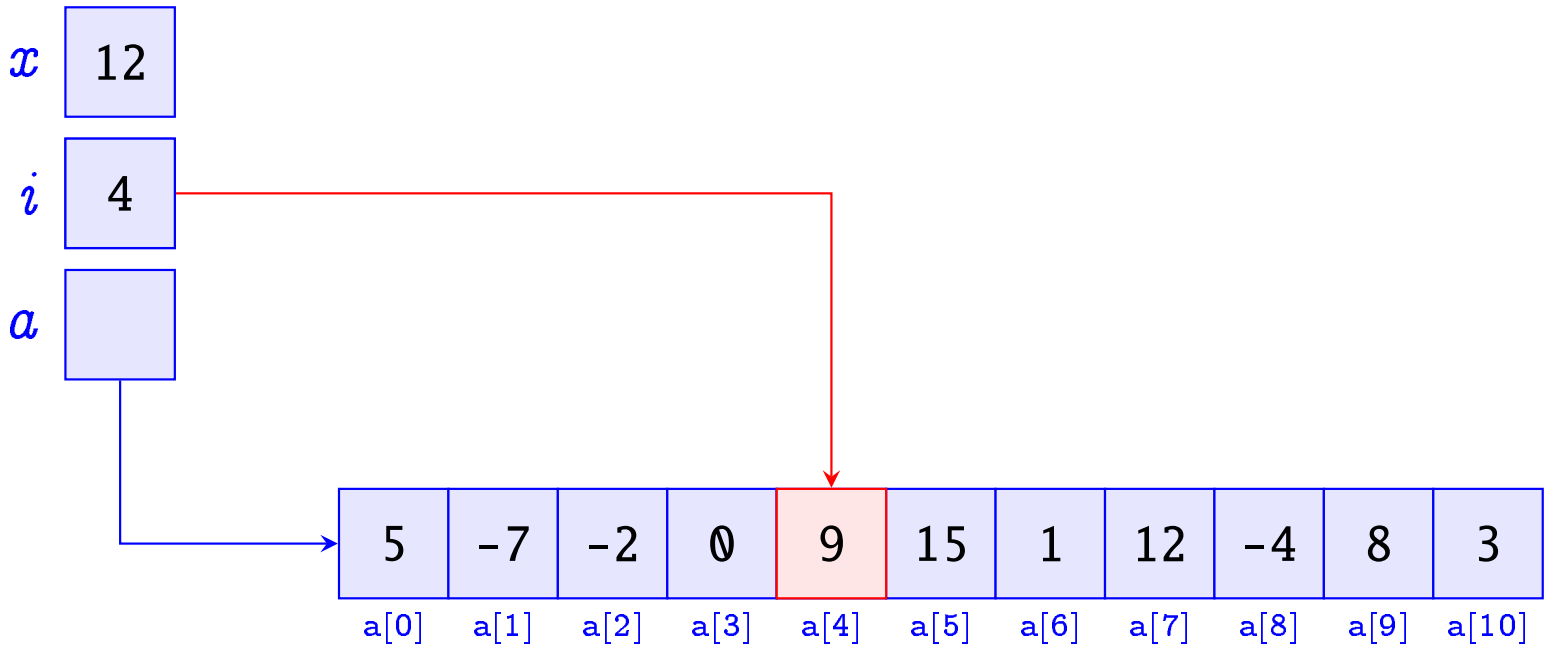

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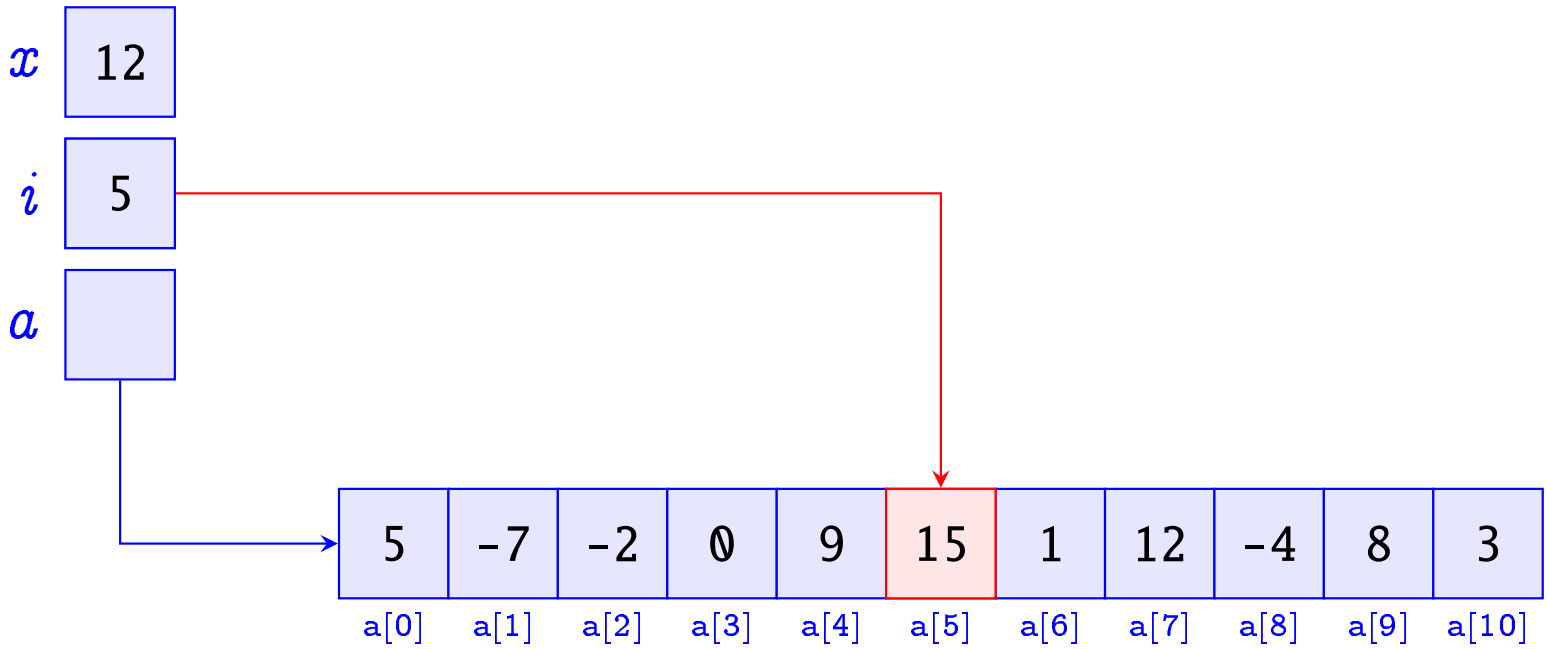

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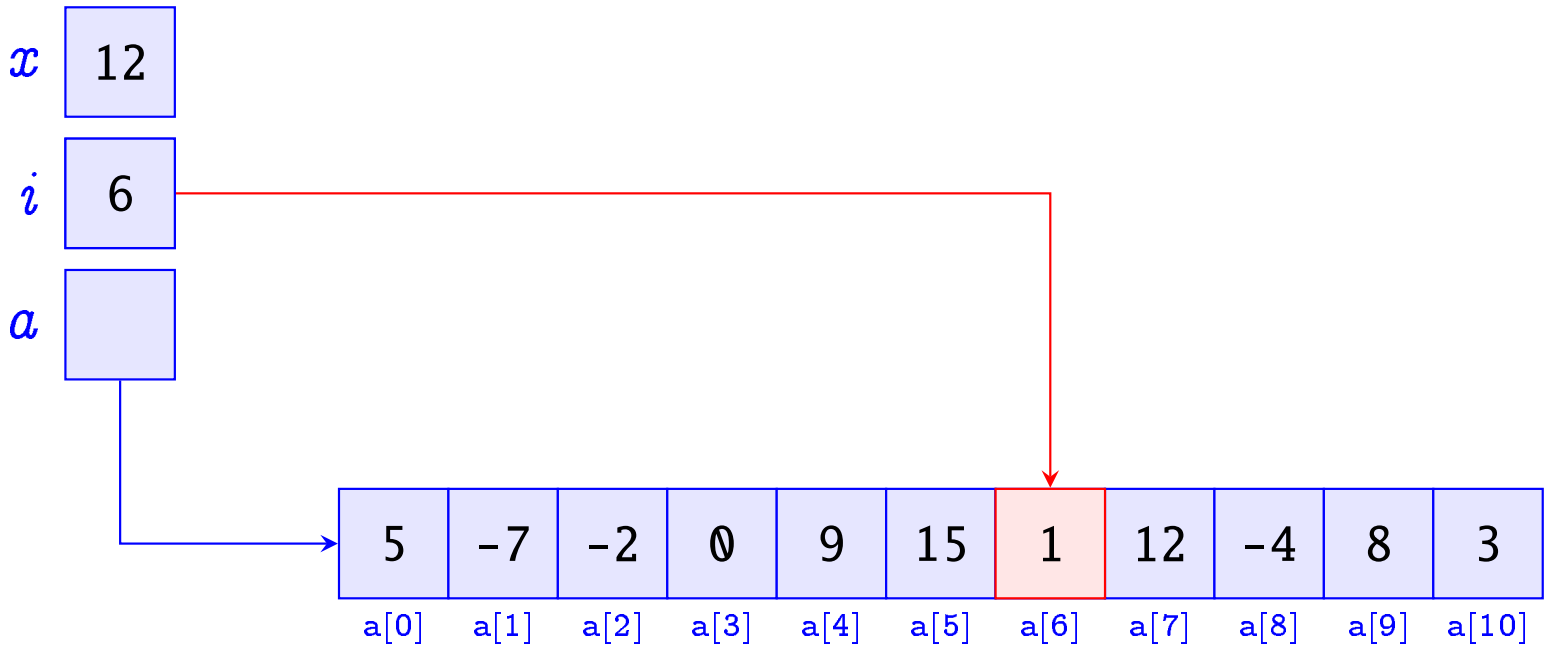
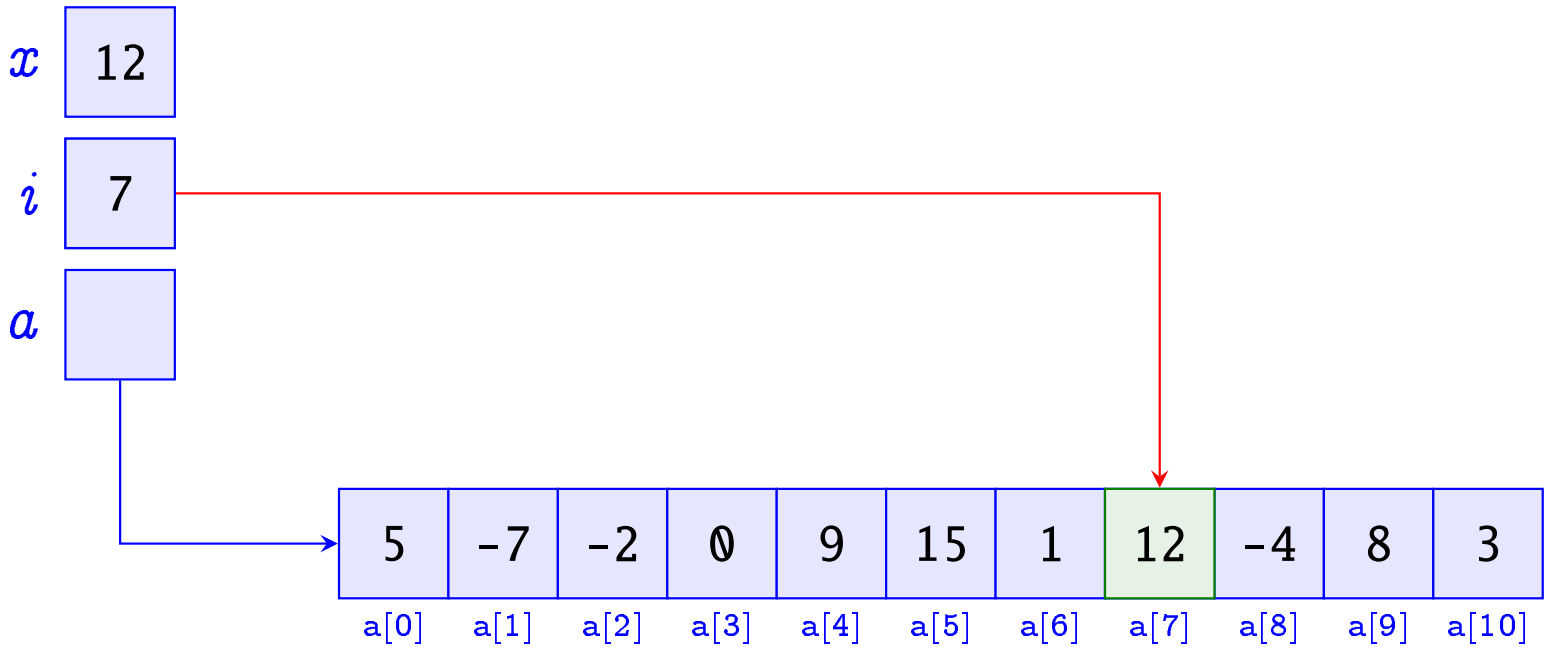


Massiivist elemendi otsimine



Massiivist elemendi otsimine

- Antud meetod on linaarse keerukusega $\mathcal{O}(n)$, kuna halvimal juhul läbitakse kogu massiiv.
- Kui sisendmassiiv on *järjestatud*, siis on võimalik otsida tunduvalt efektiivsemalt kasutades kahendotsimist.
- Kahendotsimise idee:
  - Kontrollime massiivi keskmist elementi.
  - Kui see on otsitav, siis olemegi elemendi leidnud.
  - Kui see on otsitavast väiksem, siis otsitav saab olla ainult järgmiste elementide hulgas.
  - Vastasel korral saab otsitav olla ainult eelnevate elementide hulgas.
  - Kordame kogu protsessi vastaval alammassiivil.

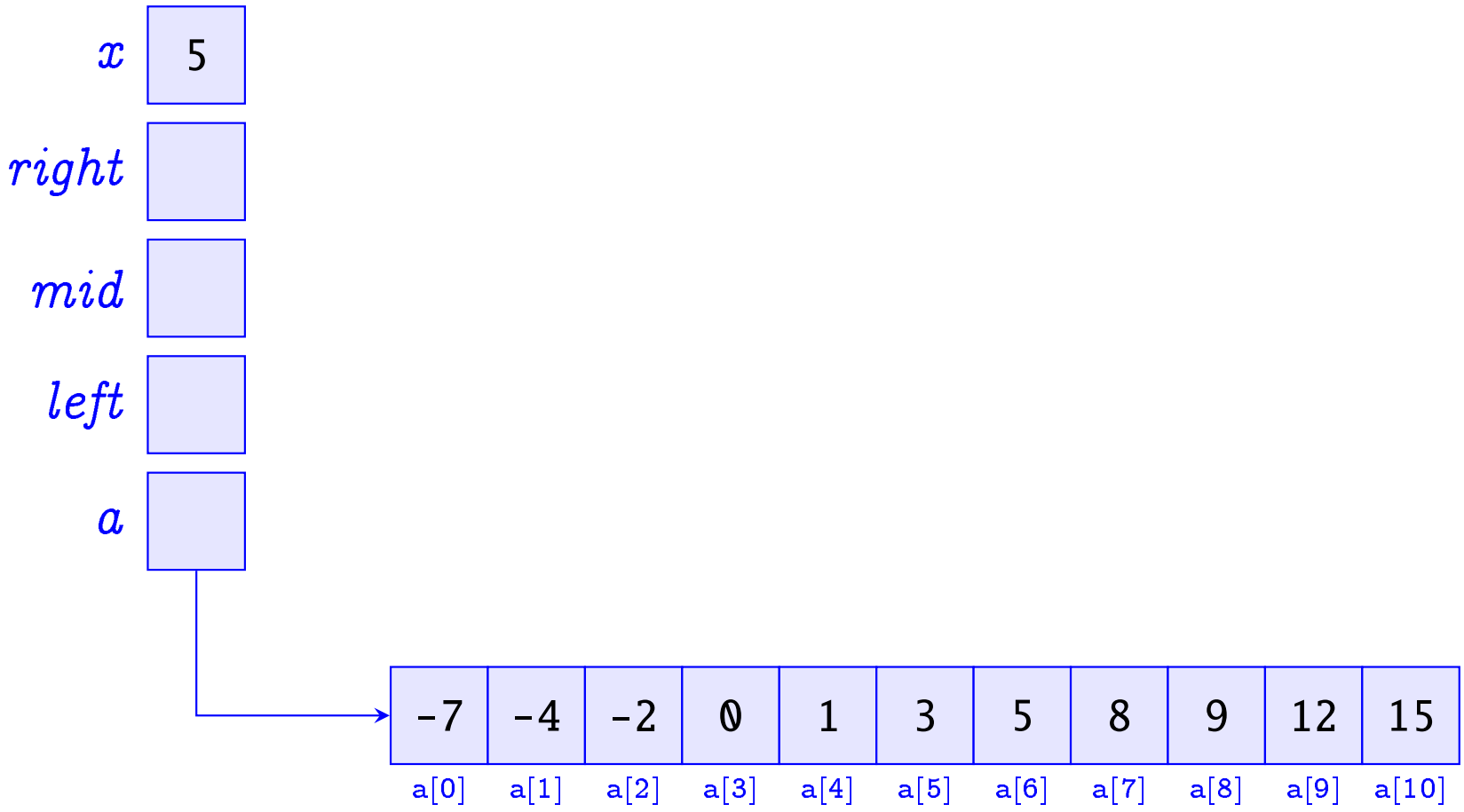
Kahendotsimine

Kahendotsimine

```
def recBinSearch(x, a, left, right):
    if left > right:
        return -1
    mid = (left+right) // 2
    if a[mid] == x:
        return mid
    elif a[mid] > x:
        return recBinSearch(x, a, left, mid-1)
    else:
        return recBinSearch(x, a, mid+1, right)

def binarySearch(x, a):
    return recBinSearch(x, a, 0, len(a)-1)
```

Kahendotsimine



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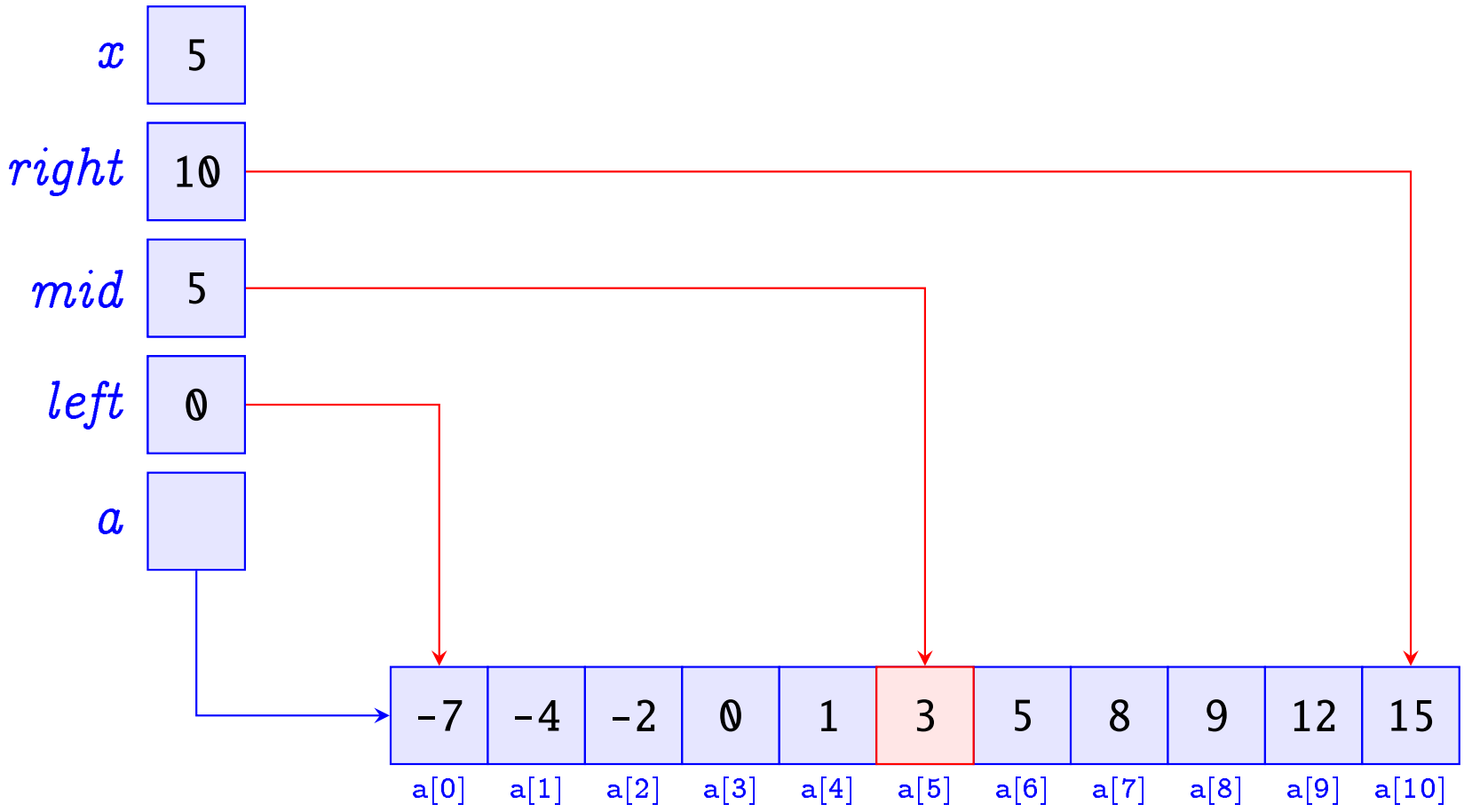

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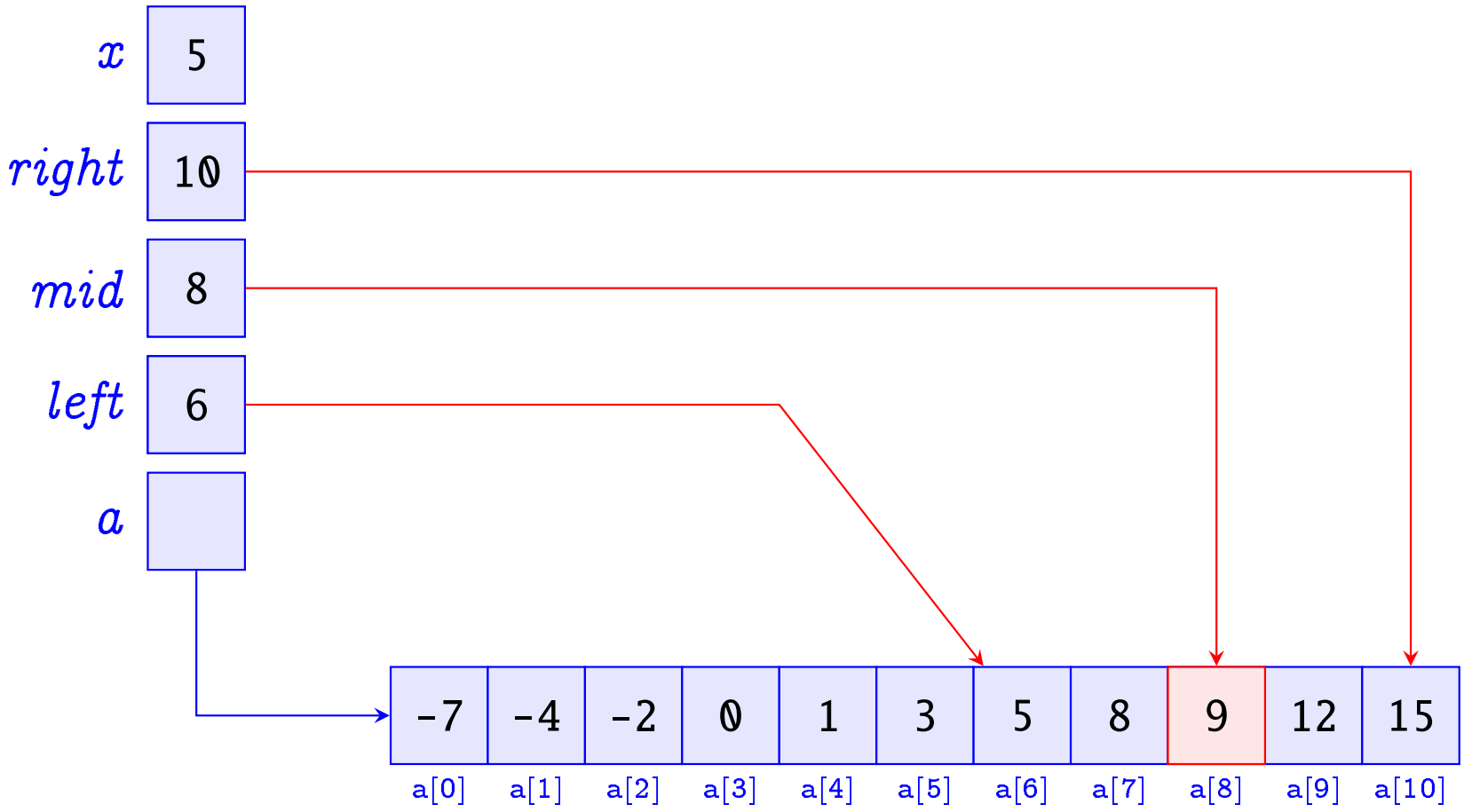
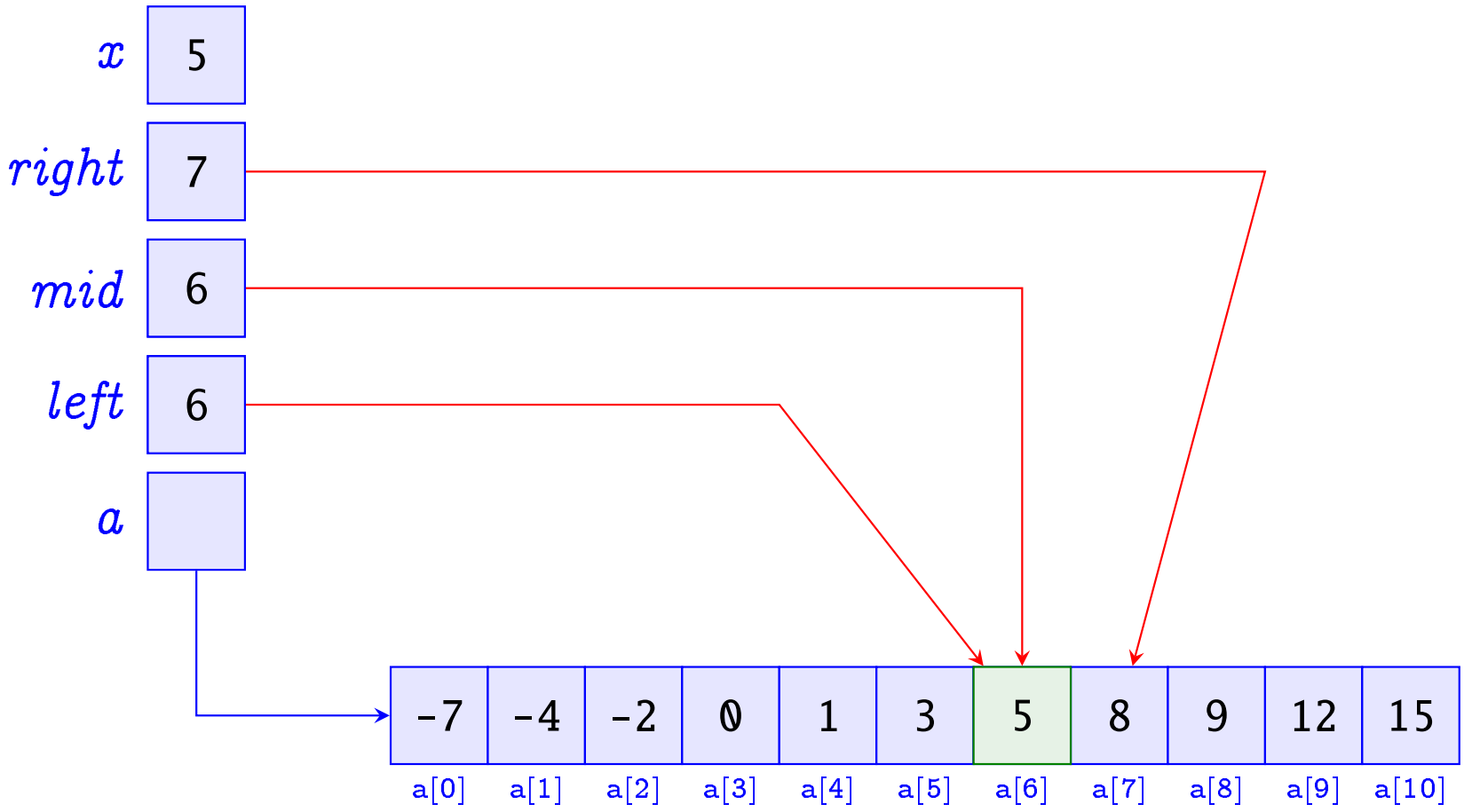


Kahendotsimine



Kahendotsimine

- Igal iteratsioonisammul välistatakse pool massiivist.
- Seega on kahendotsimine logaritmilise keerukusega $\mathcal{O}(\log_2(n))$.
- Suurte massiivide korral on erinevus efektiivsuses vägagi märgatav.
  - Näiteks, kui massiivis on $2^{20} = 1048576$ elementi, siis lineaarse otsimisega tuleb halvimal juhul kogu massiiv läbida.
  - Kahendotsimise korral tuleb halvimal juhul teha vaid 20 võrdlust.
- NB! Kahendotsimine eeldab, et sisendmassiiv on järjestatud; lineaarne otsimine seda ei eelda.

Sorteerimine

- Sorteerimine on andmete järjestamine nende väärtuste järgi.
- Kõigis meie näidetes vaatleme massiivielementide järjestamist kasvavasse järjekorda.
- Sorteerimiseks on mitmeid erinevaid meetode, milledest alljärgnevalt vaatleme kahte:
  - valikumeetod,
  - ühildusmeetod.

Valikumeetodil sorteerimine

Valikumeetodi idee

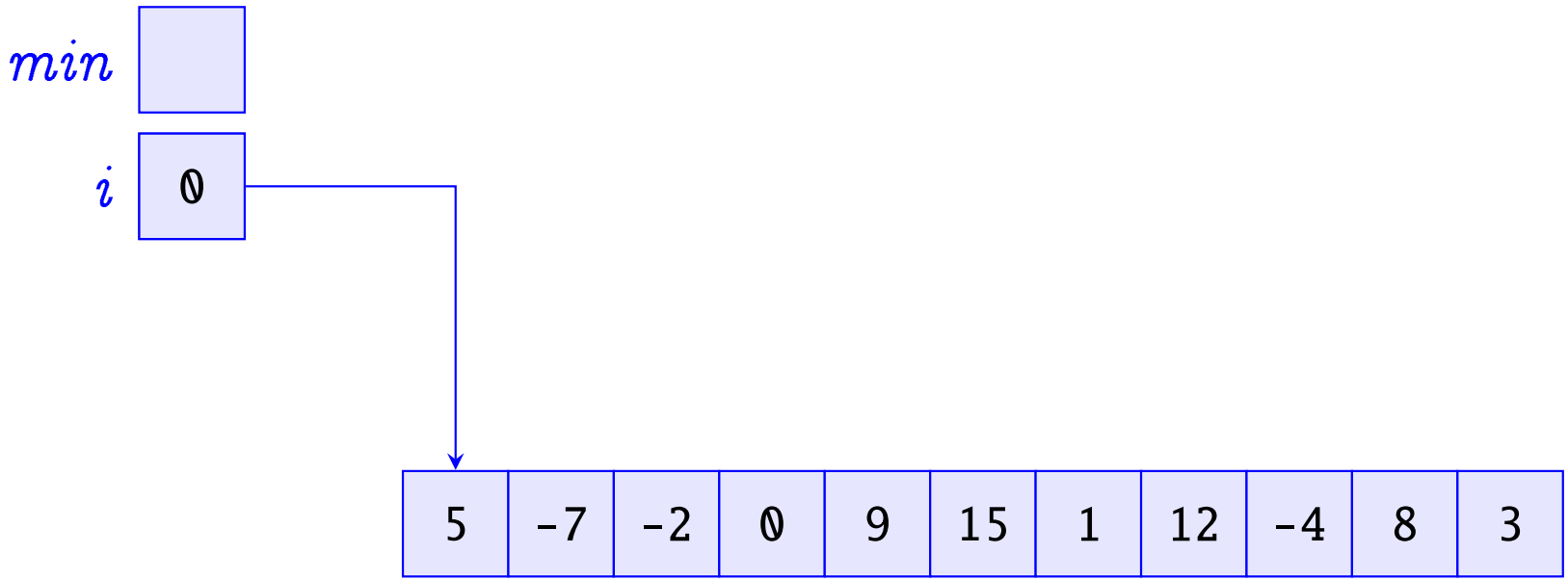
- Leiame elemendi, mis peaks olema kõige esimene.
  - Selleks otsime massiivist minimaalse elemendi.
- Vahetame selle elemendi massiivi tegeliku esimese elemendiga.
- Massiivi algus (so. esimene element) on nüüd sorteeritud ja edaspidi seda enam ei muuda.
- Ülejäänud massiiv on aga veel sorteerimata.
- Kordame kogu protsessi sorteerimata massiiviosal, kuni kogu massiiv saab sorteeritud.

Valikumeetodil sorteerimine

Valikumeetodil sorteerimine

```
def selectionSort(a):
    for i in range(len(a)):
        min = i
        for j in range(i+1,len(a)):      # otsi minimaalne
            if a[j] < a[min]:  min = j
        if i != min:                     # vaheta elemendid
            tmp    = a[i]
            a[i]   = a[min]
            a[min] = tmp
```

Valikumeetodil sorteerimine



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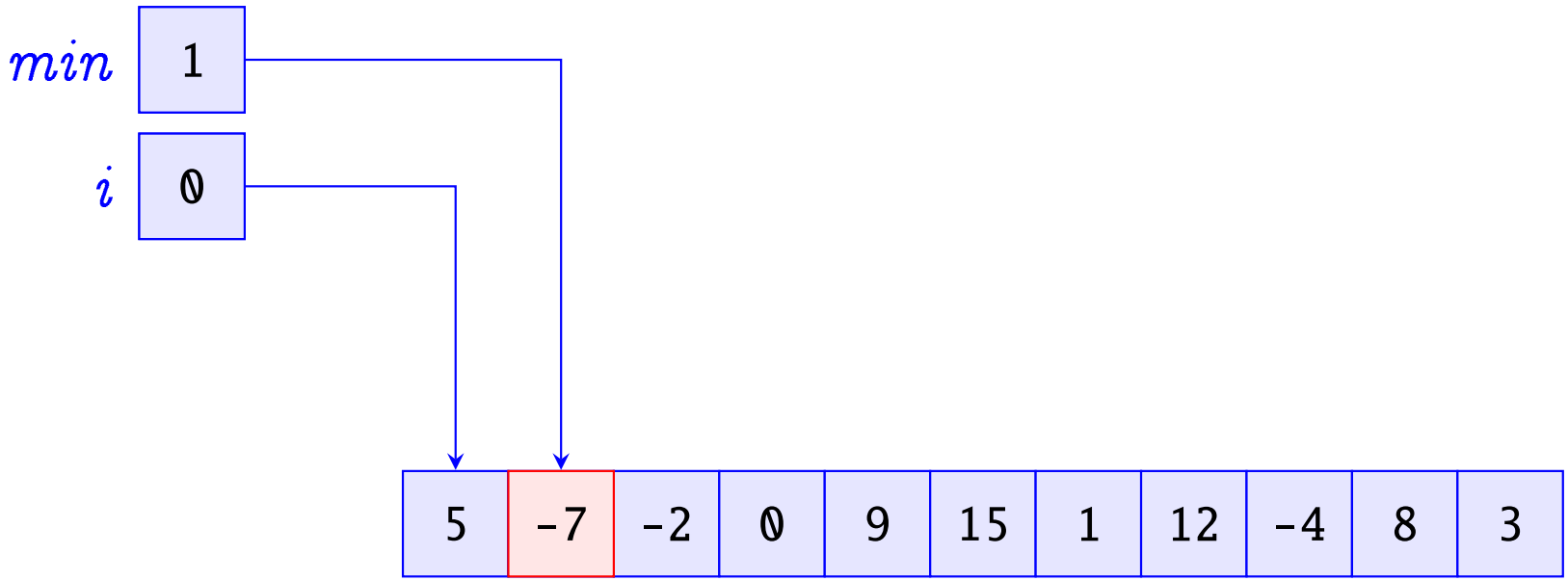

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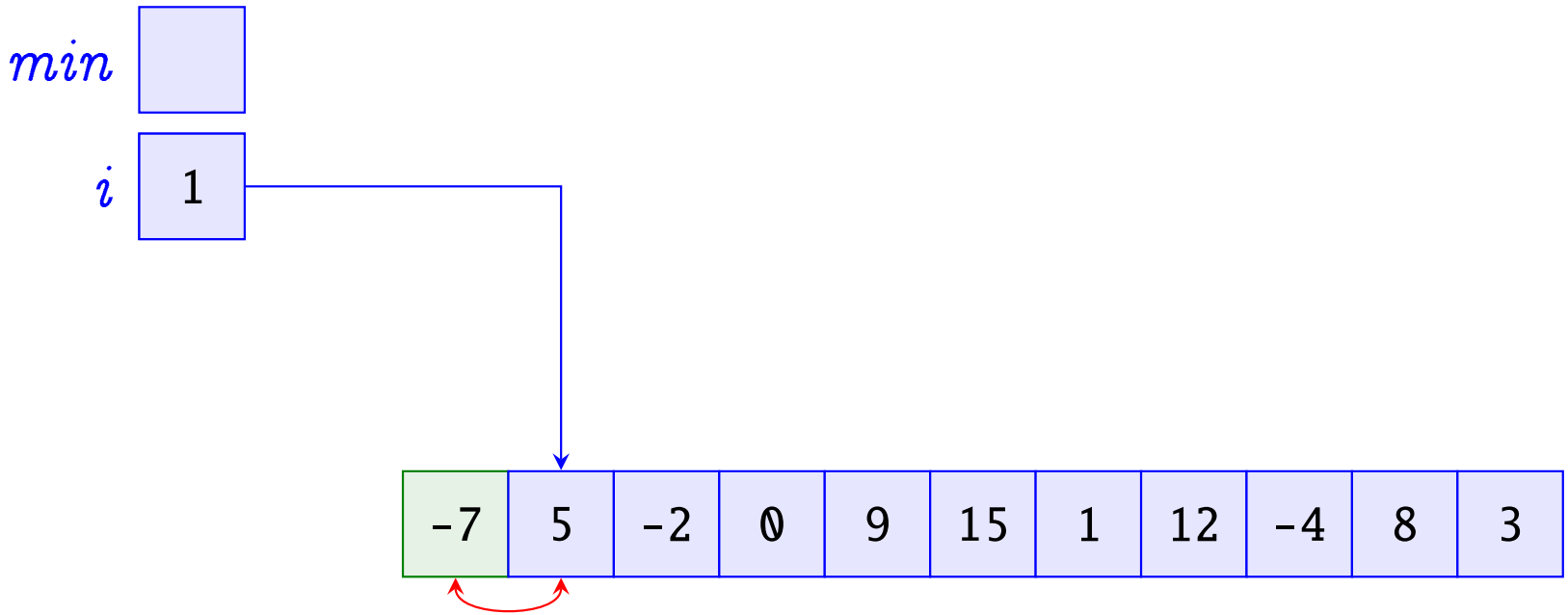

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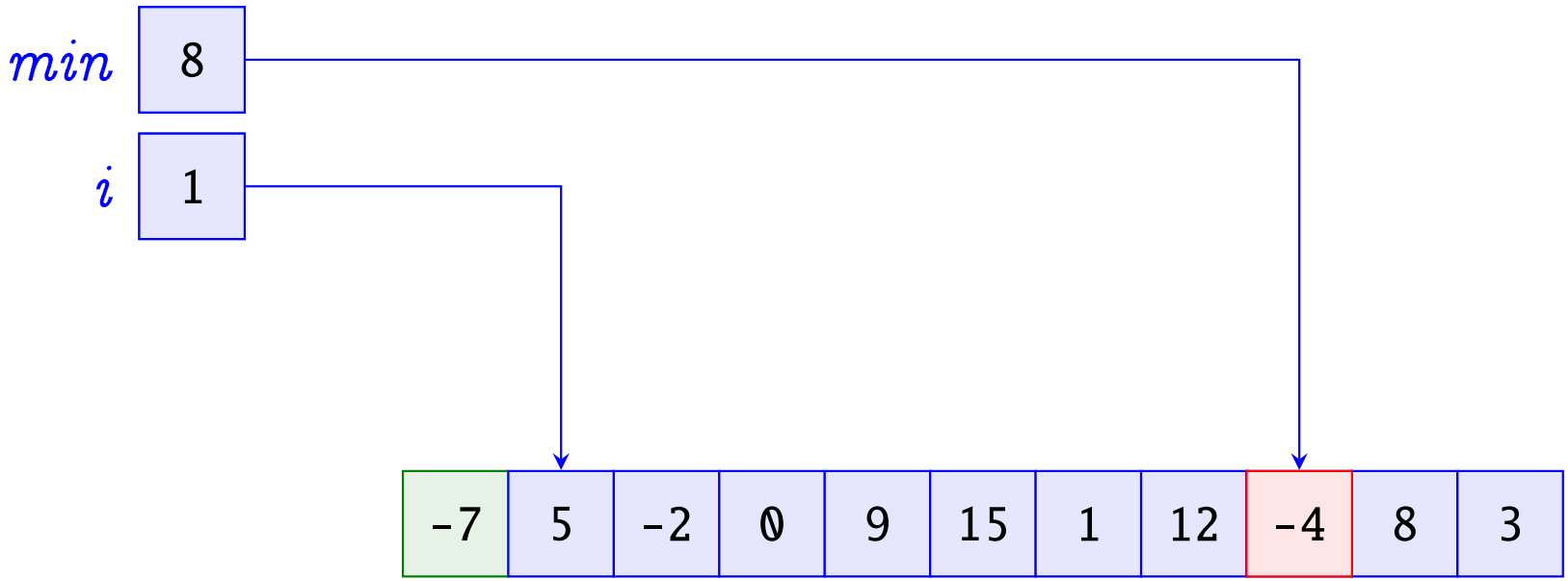

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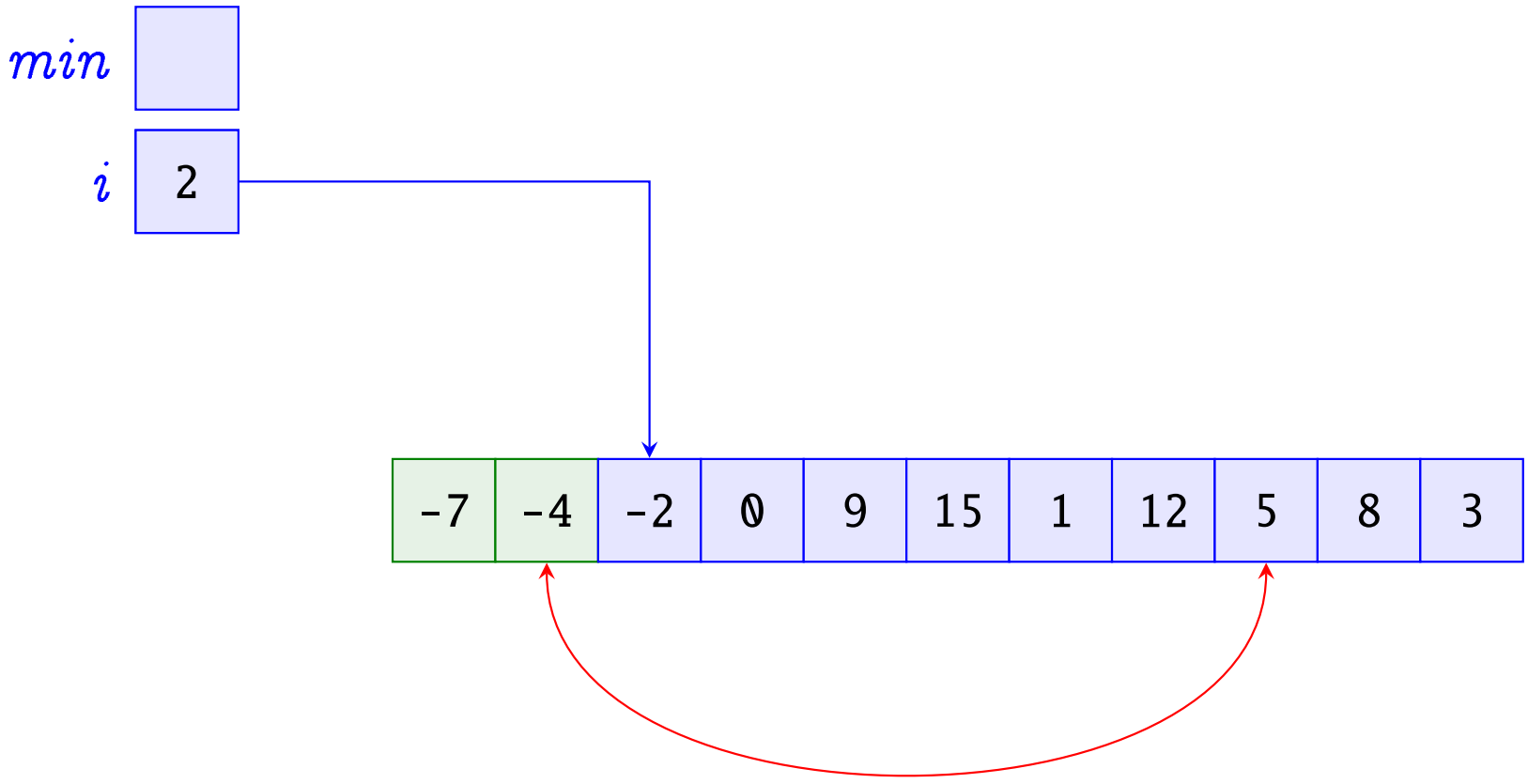

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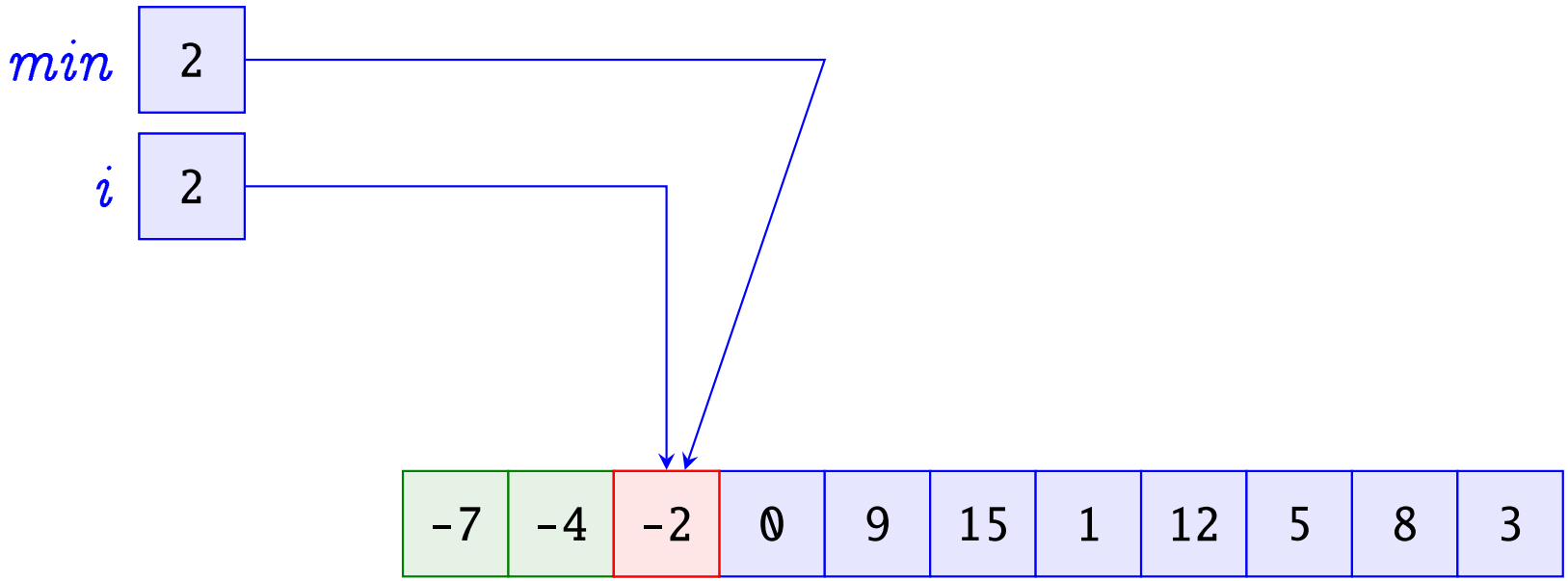

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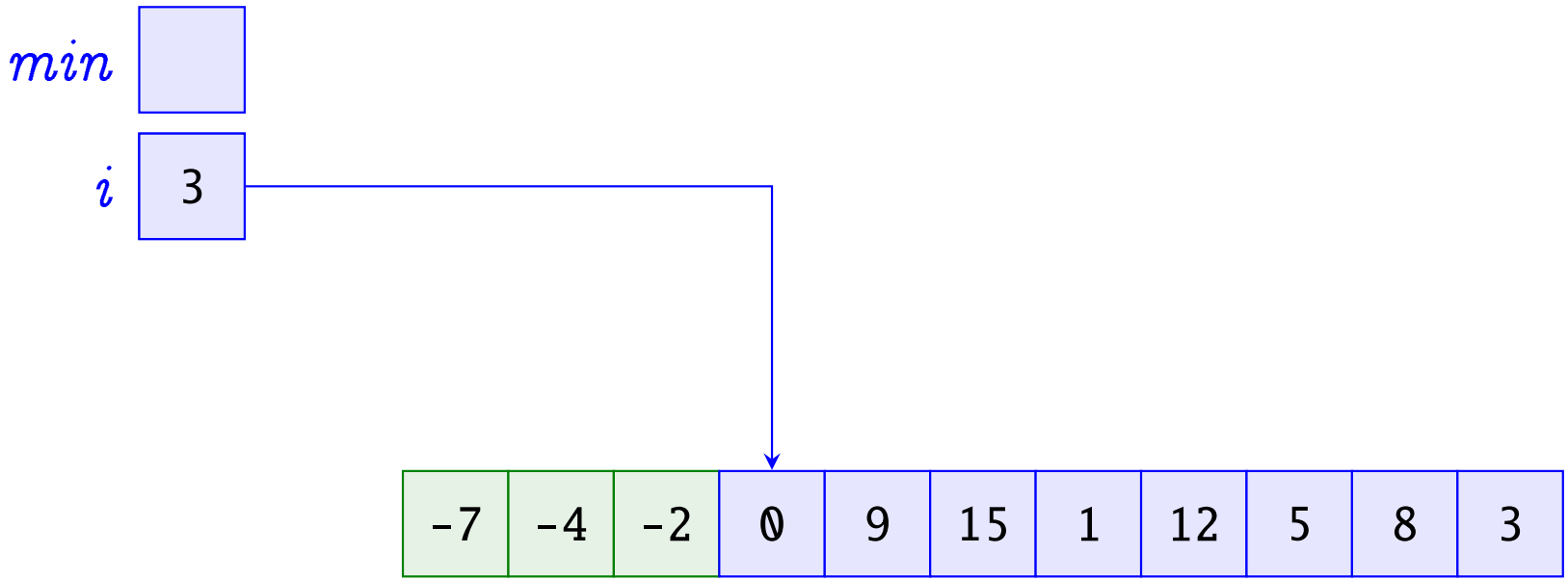

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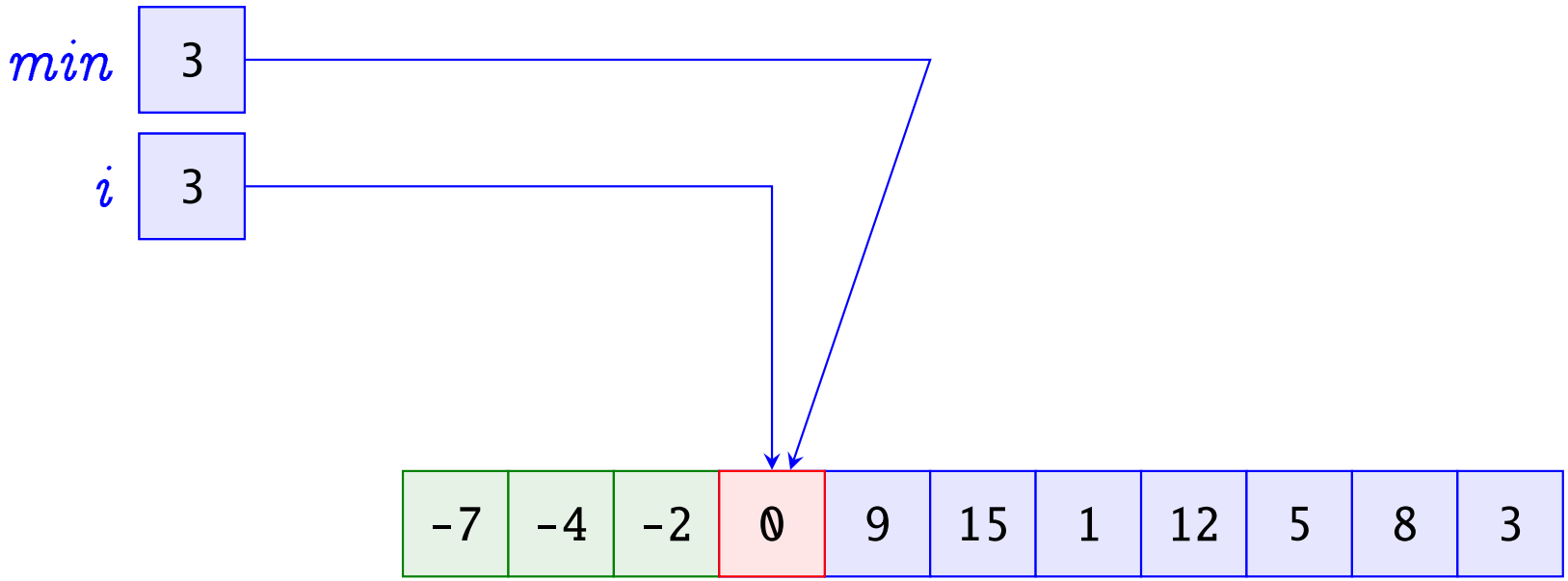

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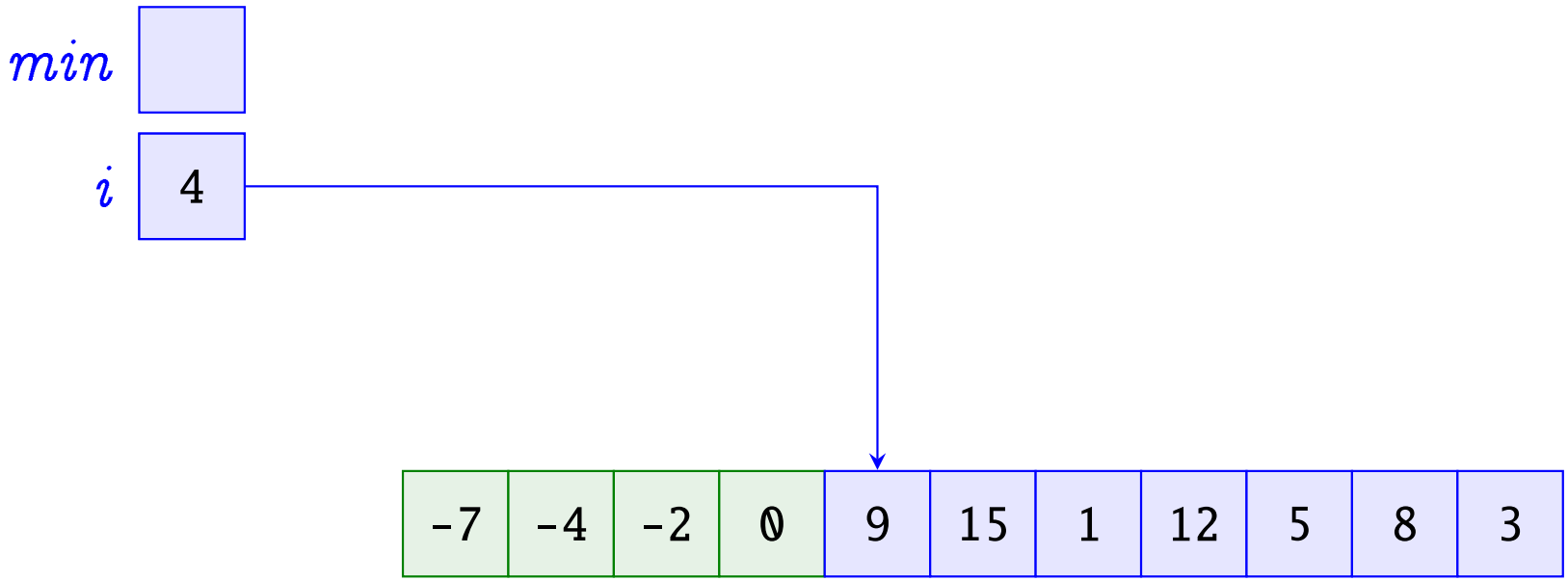

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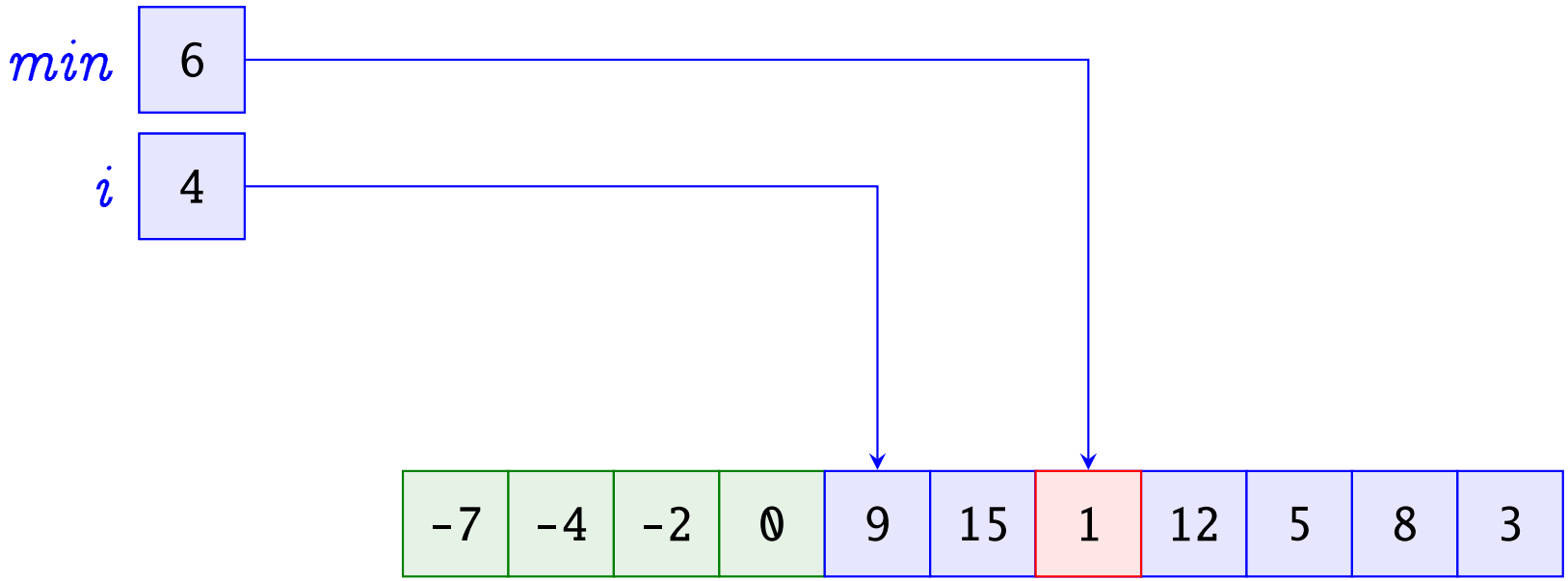

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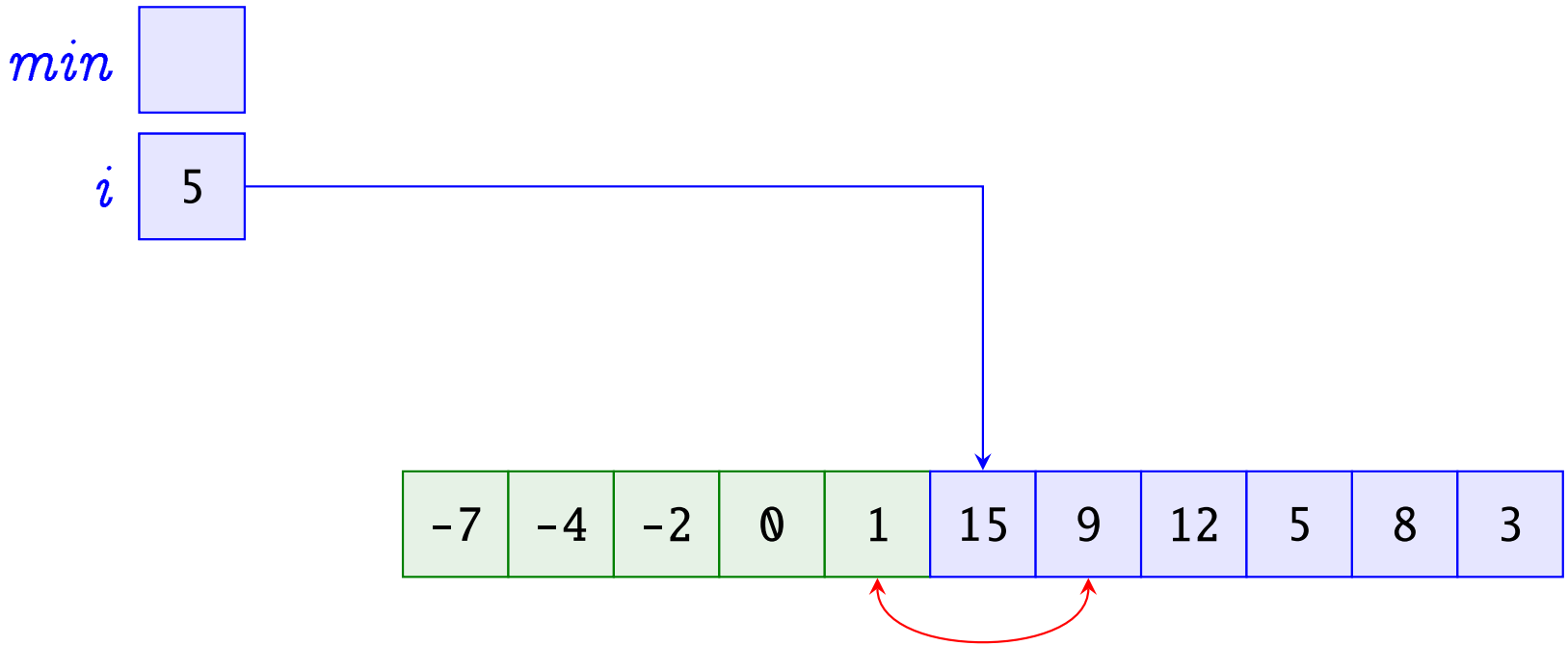

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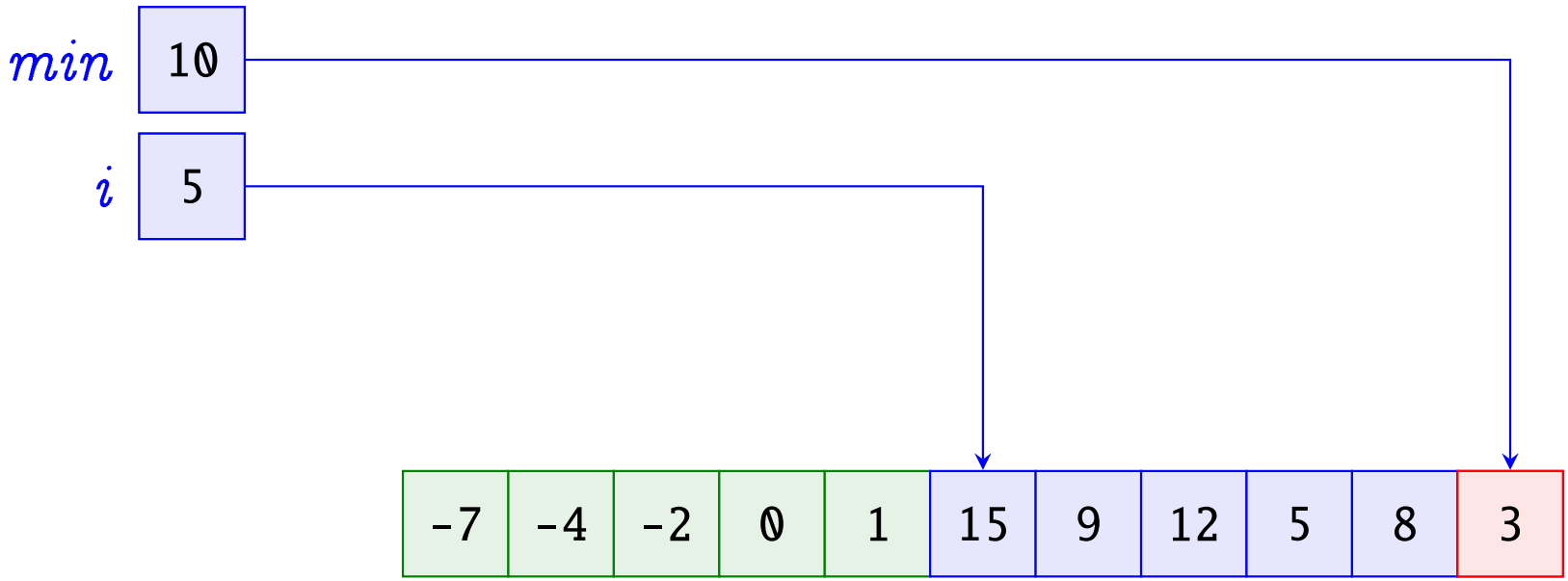

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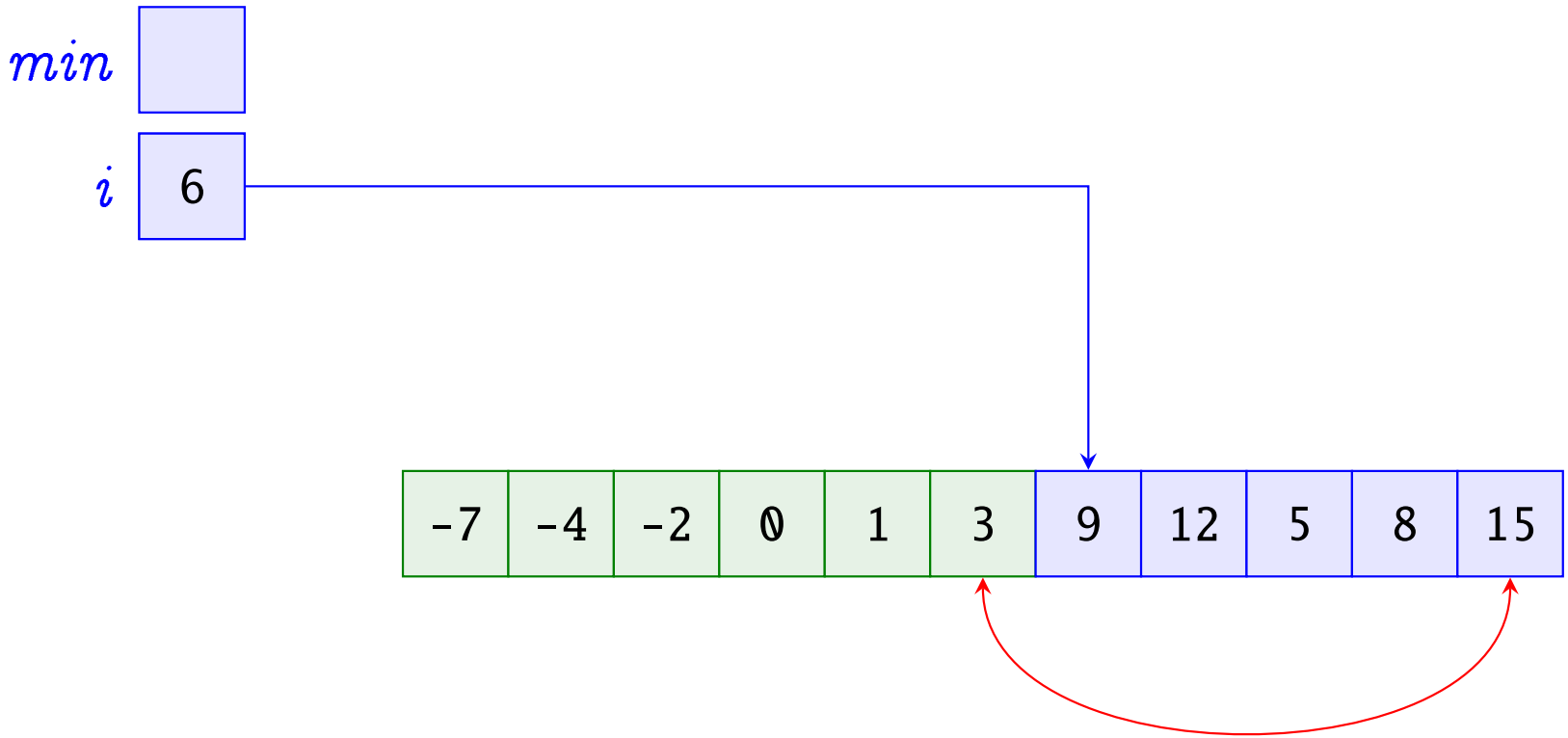

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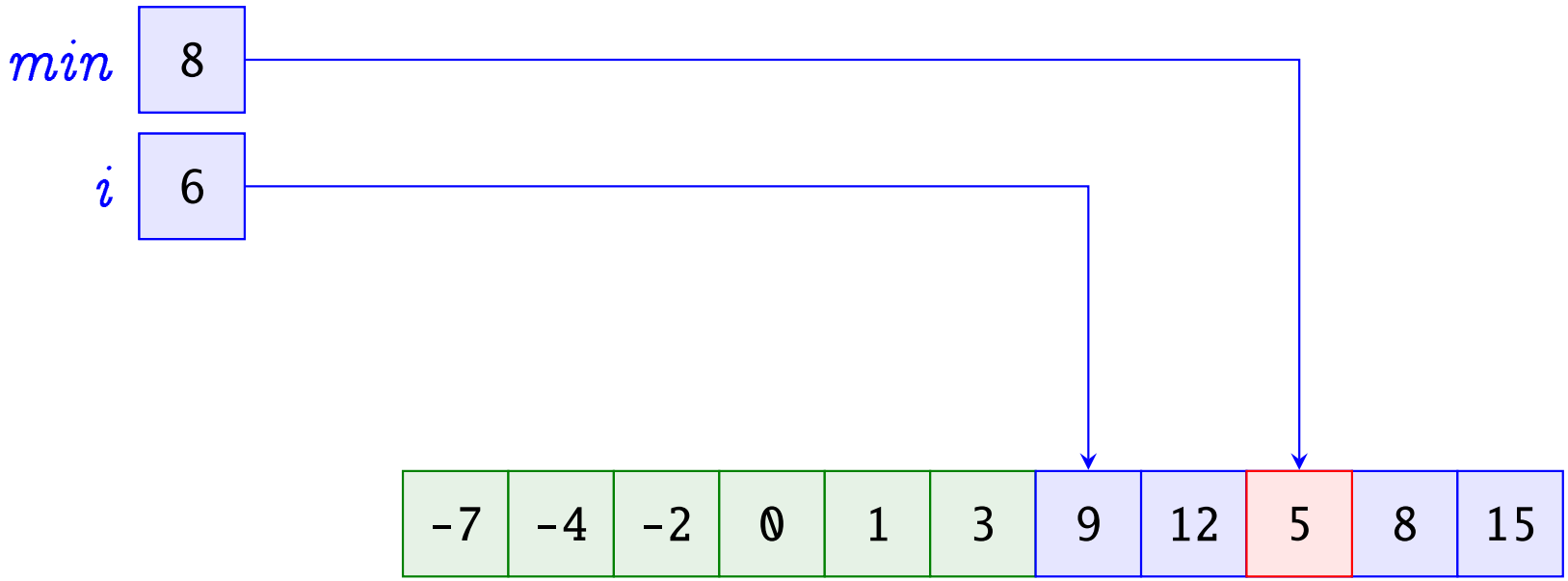

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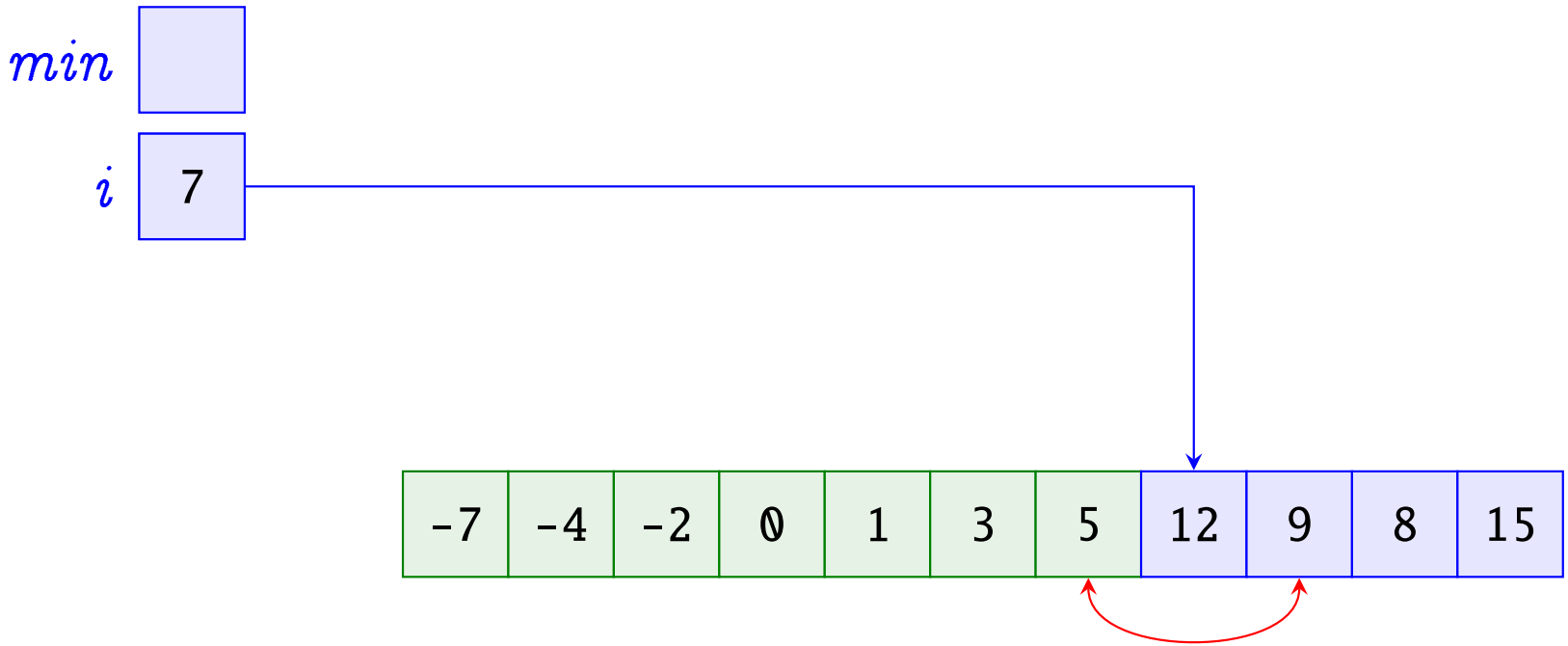

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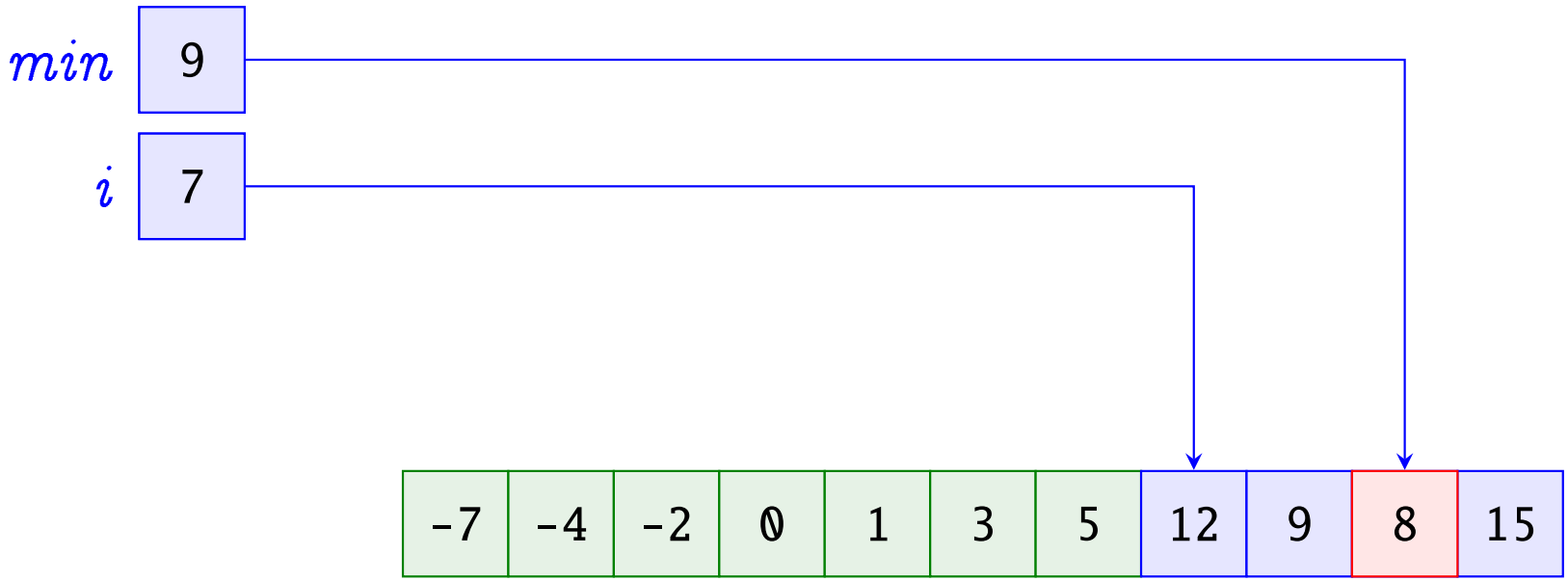

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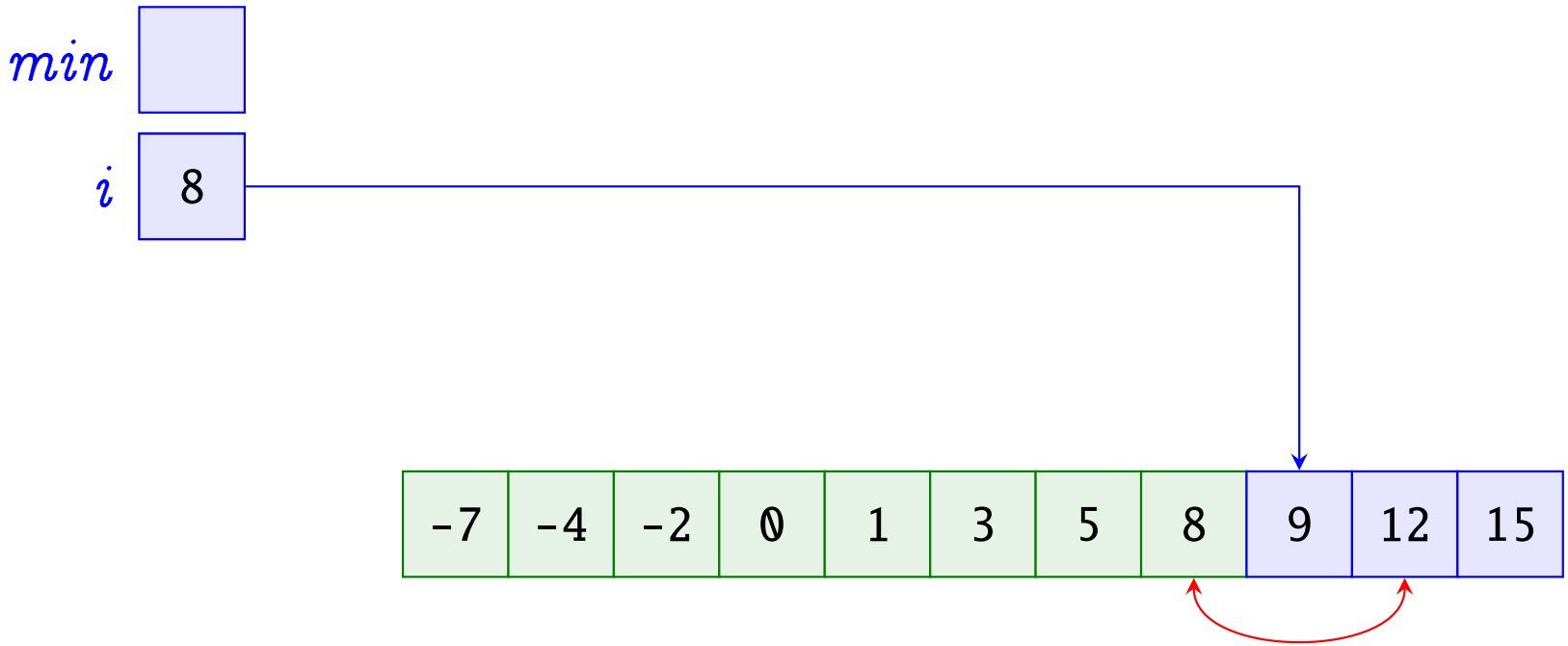

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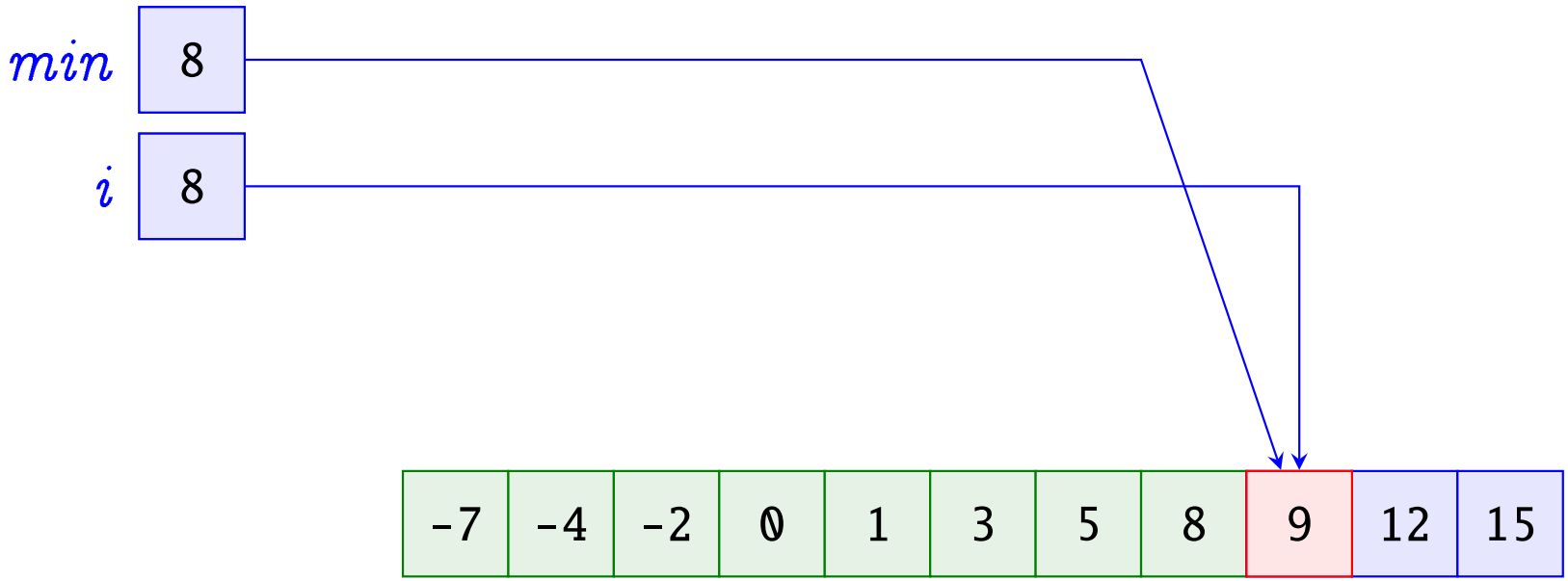

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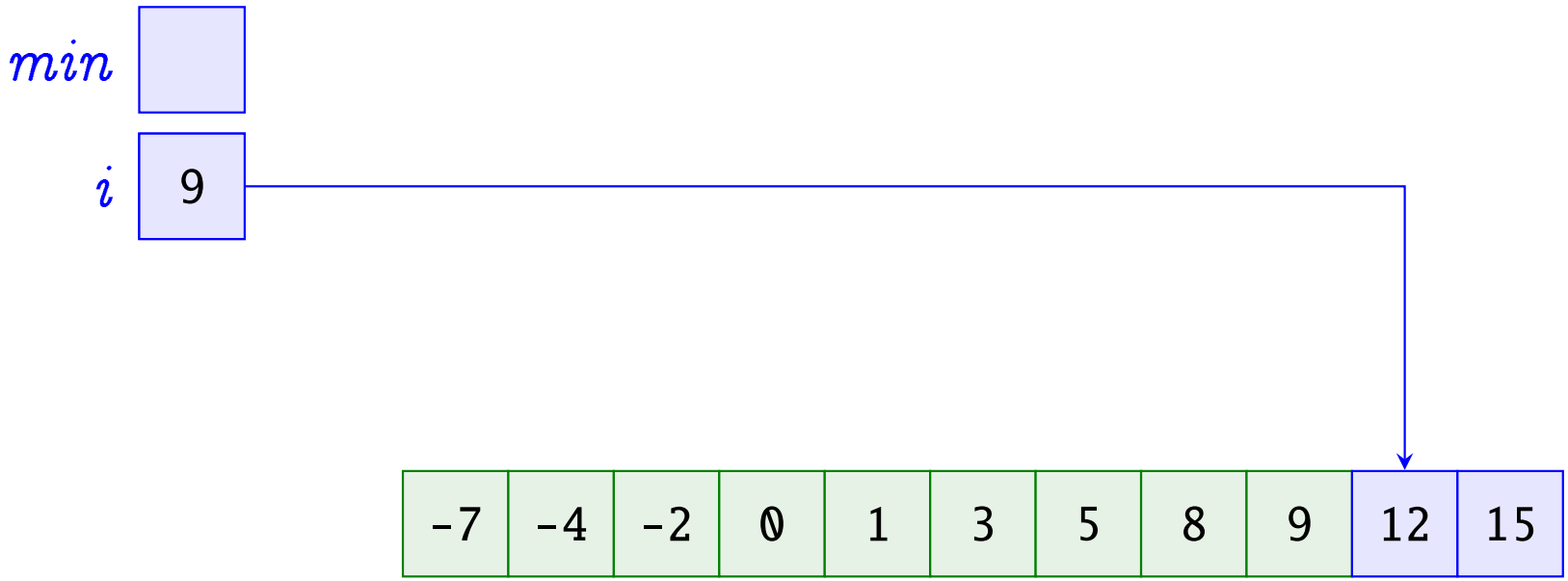

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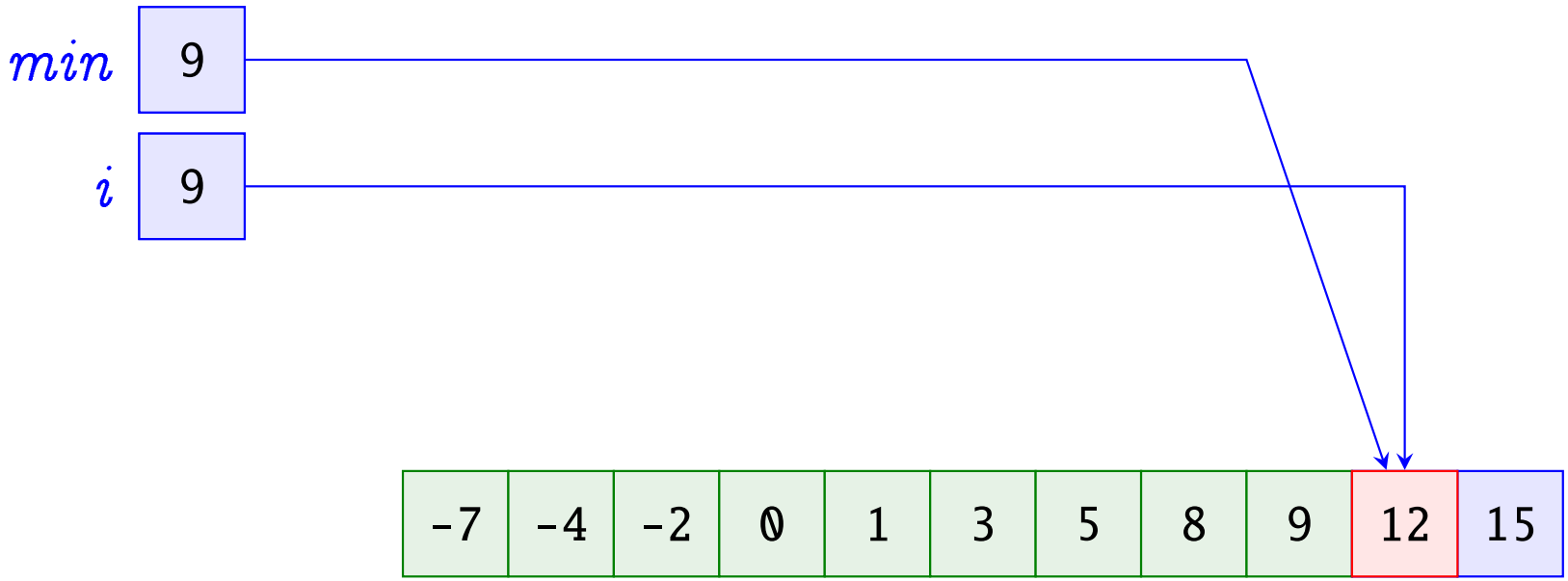

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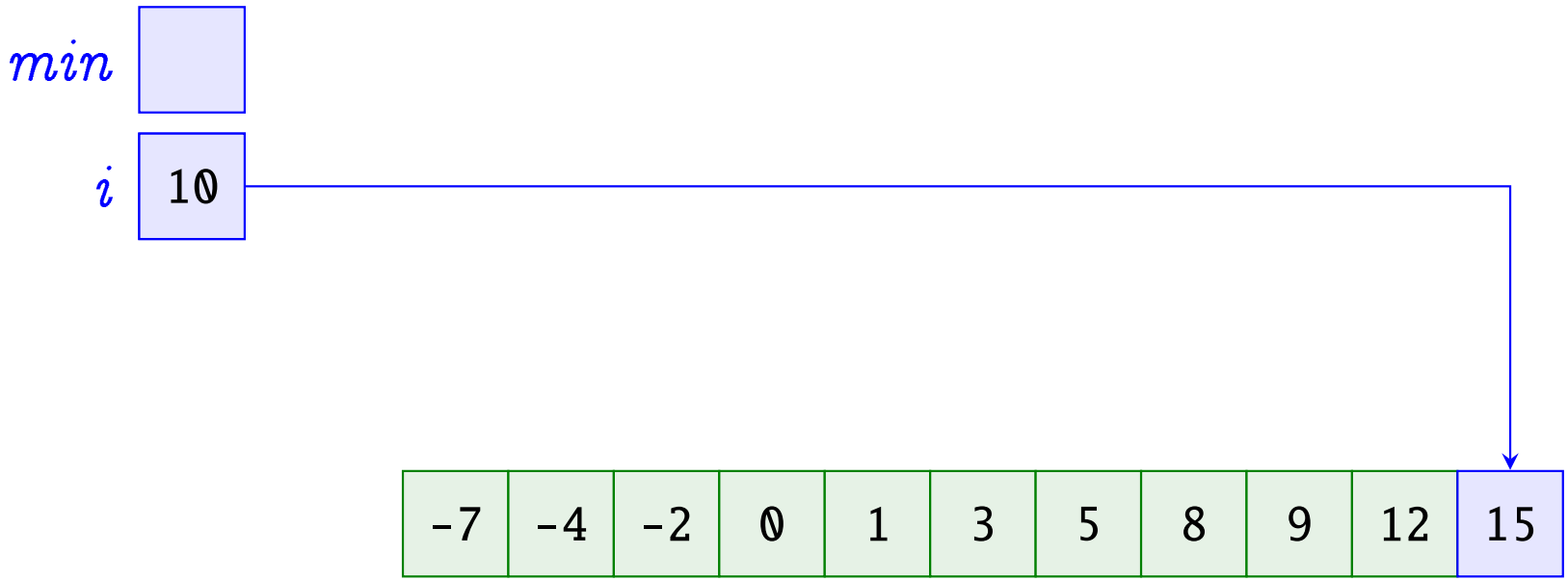

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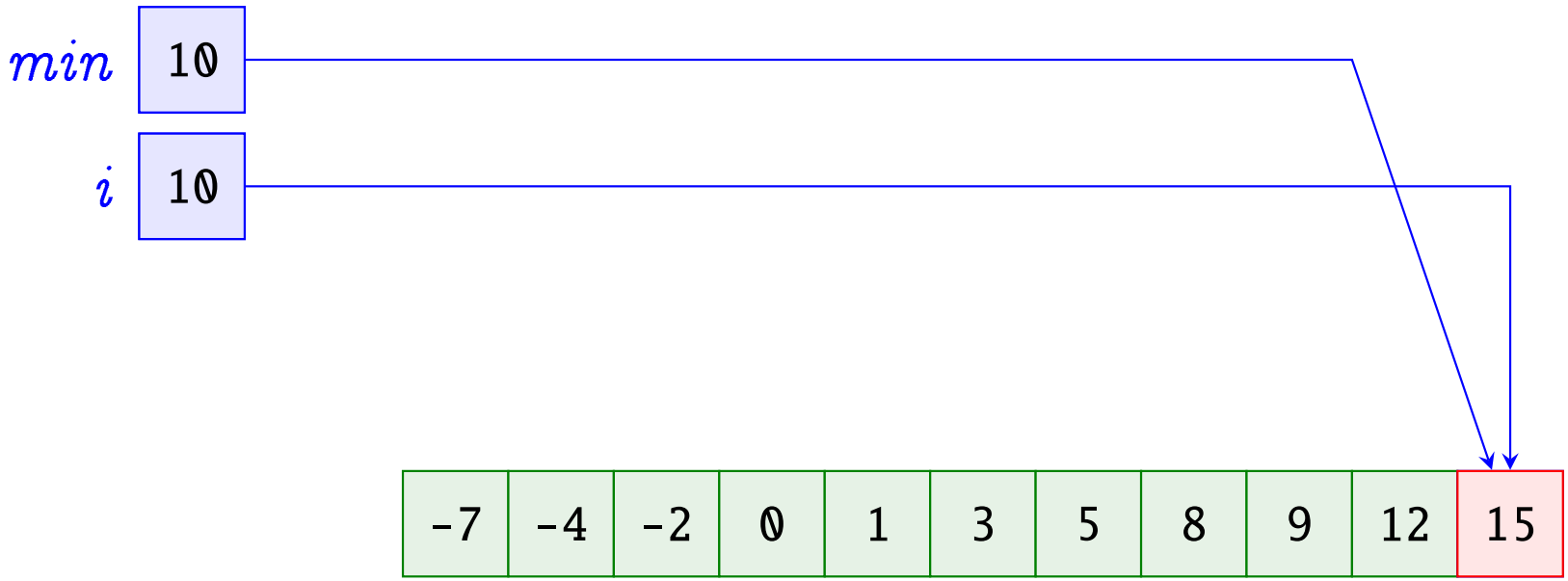

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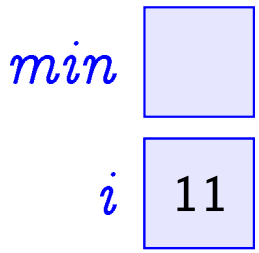


-7	-4	-2	0	1	3	5	8	9	12	15

Valikumeetodil sorteerimine

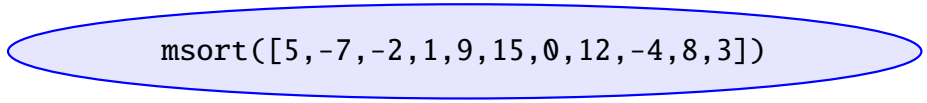
- Valikumeetodil otsitakse igal välisel iteratsioonisammul ülejäänud massiivist vähimat elementi.
- See nõuab kogu ülejäänud massiivi läbimist.
- Seega on valikumeetod ruutkeerukusega $\mathcal{O}(n^2)$ massiivi pikkuse n suhtes.

Ühildusmeetodil sorteerimine

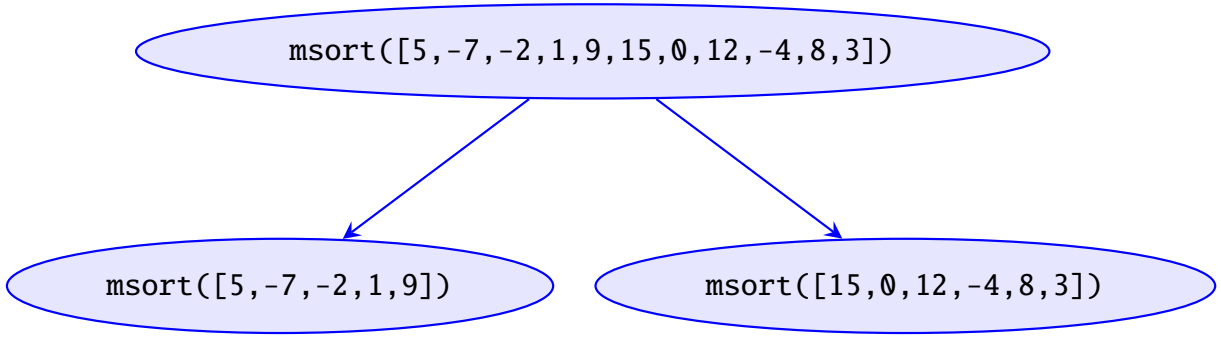
Ühildusmeetodil idee

- Jaotame massiivi kaheks võrdse pikkusega alammassiiviks.
  - Kui massiivis on paaritu arv elemente, siis esimene on ühe võrra lühem.
- Sorteerime alammassiivid rekursiivselt.
  - Rekursiooni baasiks on massiivid mille pikkus on väiksem kui 2.
- Ühildame alammassivid (mis nüüd on juba sorteeritud) üheks terveks sorteeritud massiiviks.

Ühildusmeetodil sorteerimine



Ühildusmeetodil sorteerimine



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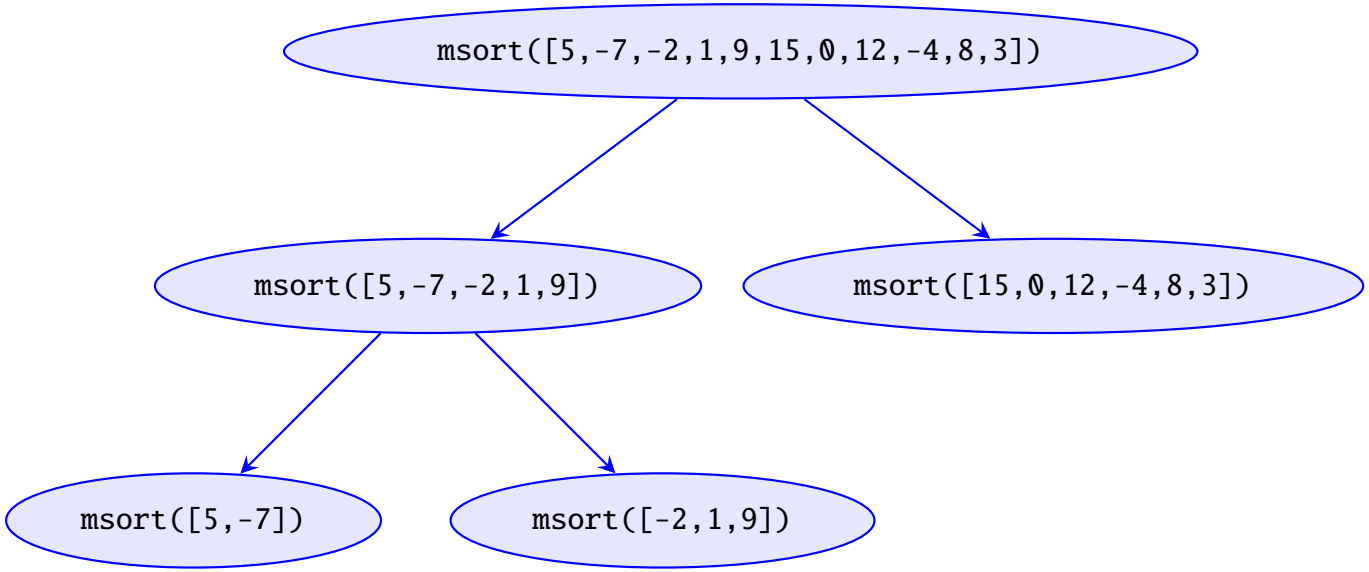
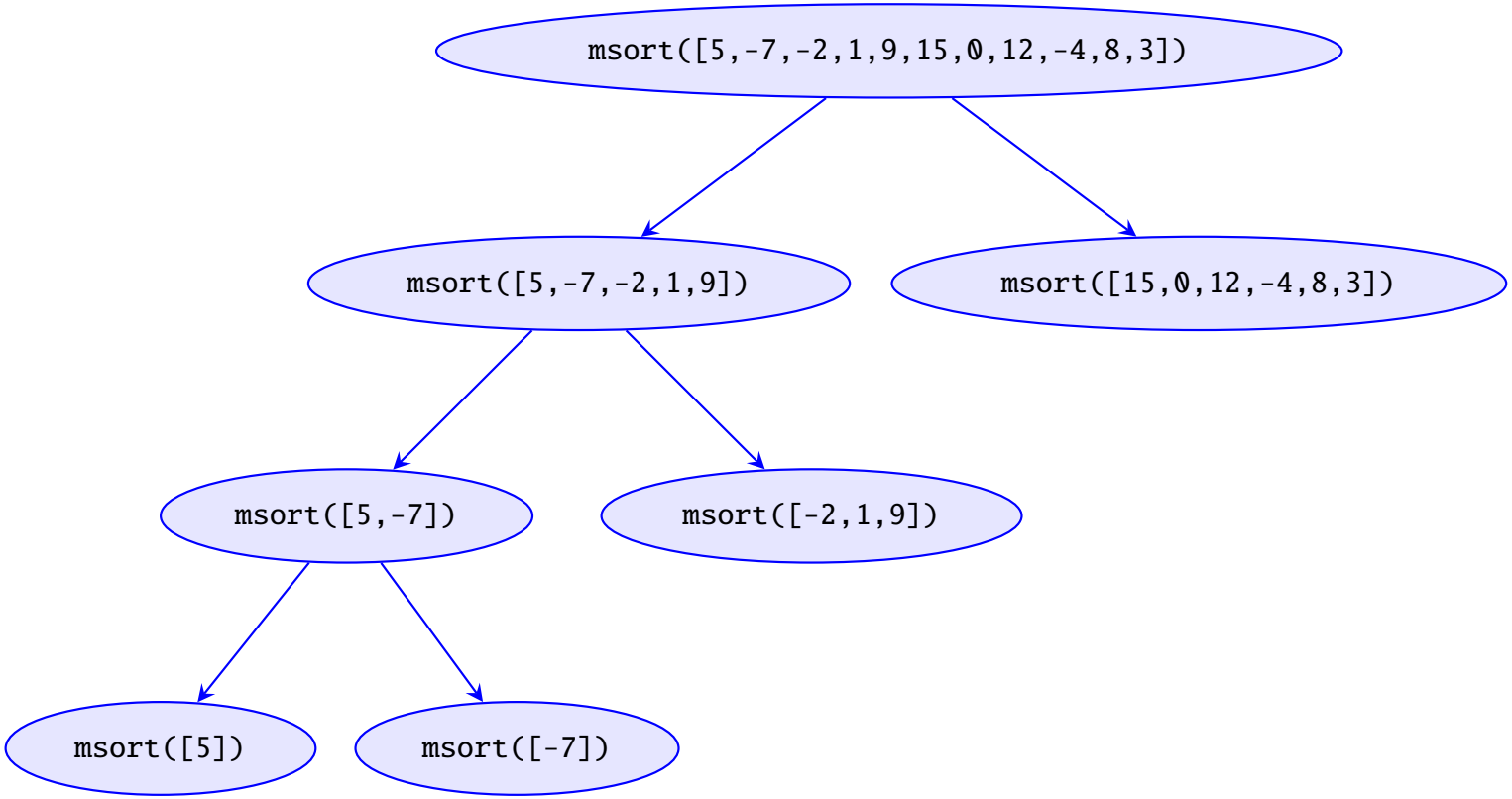


Ühildusmeetodil sorteerimine



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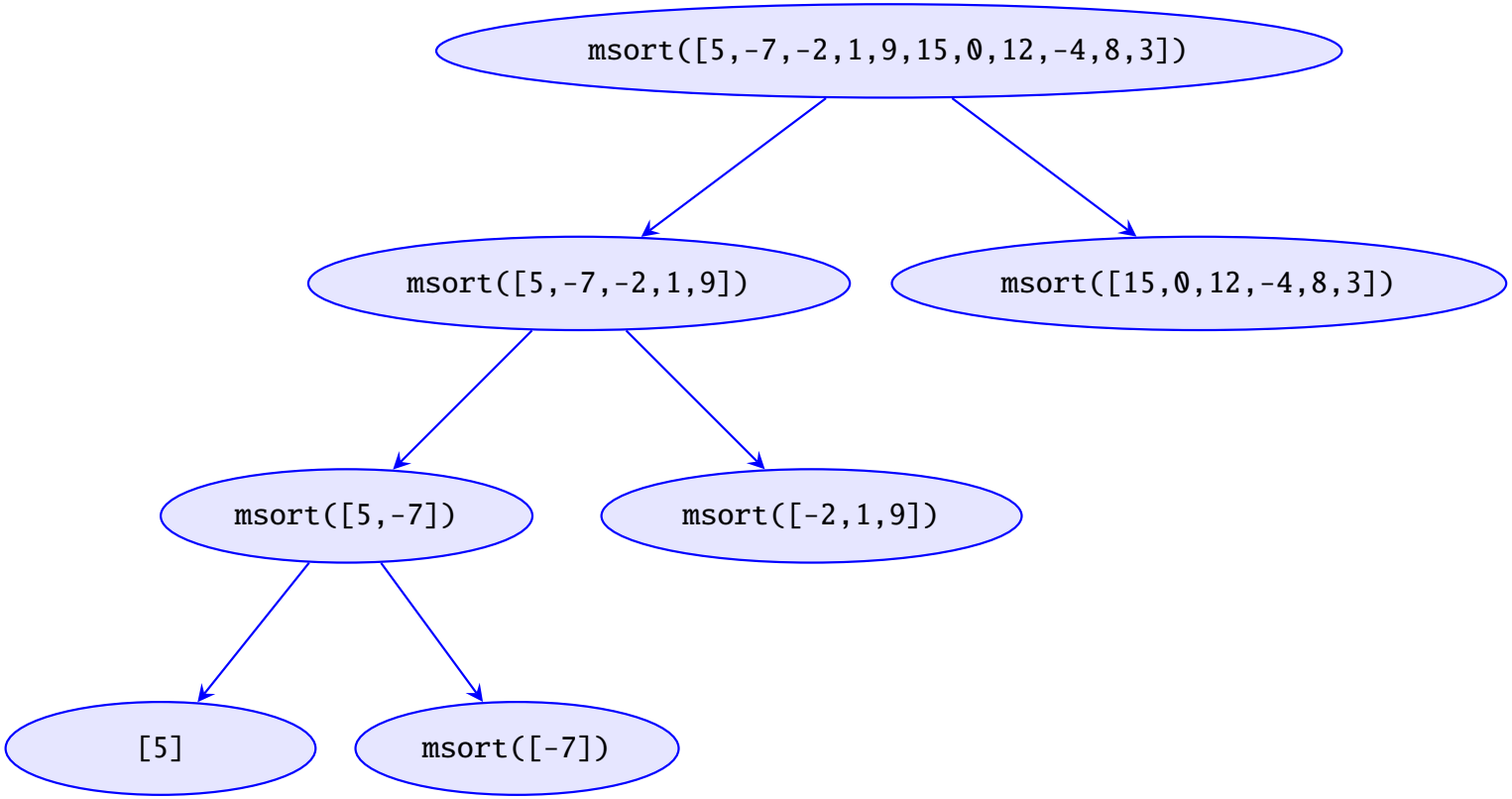

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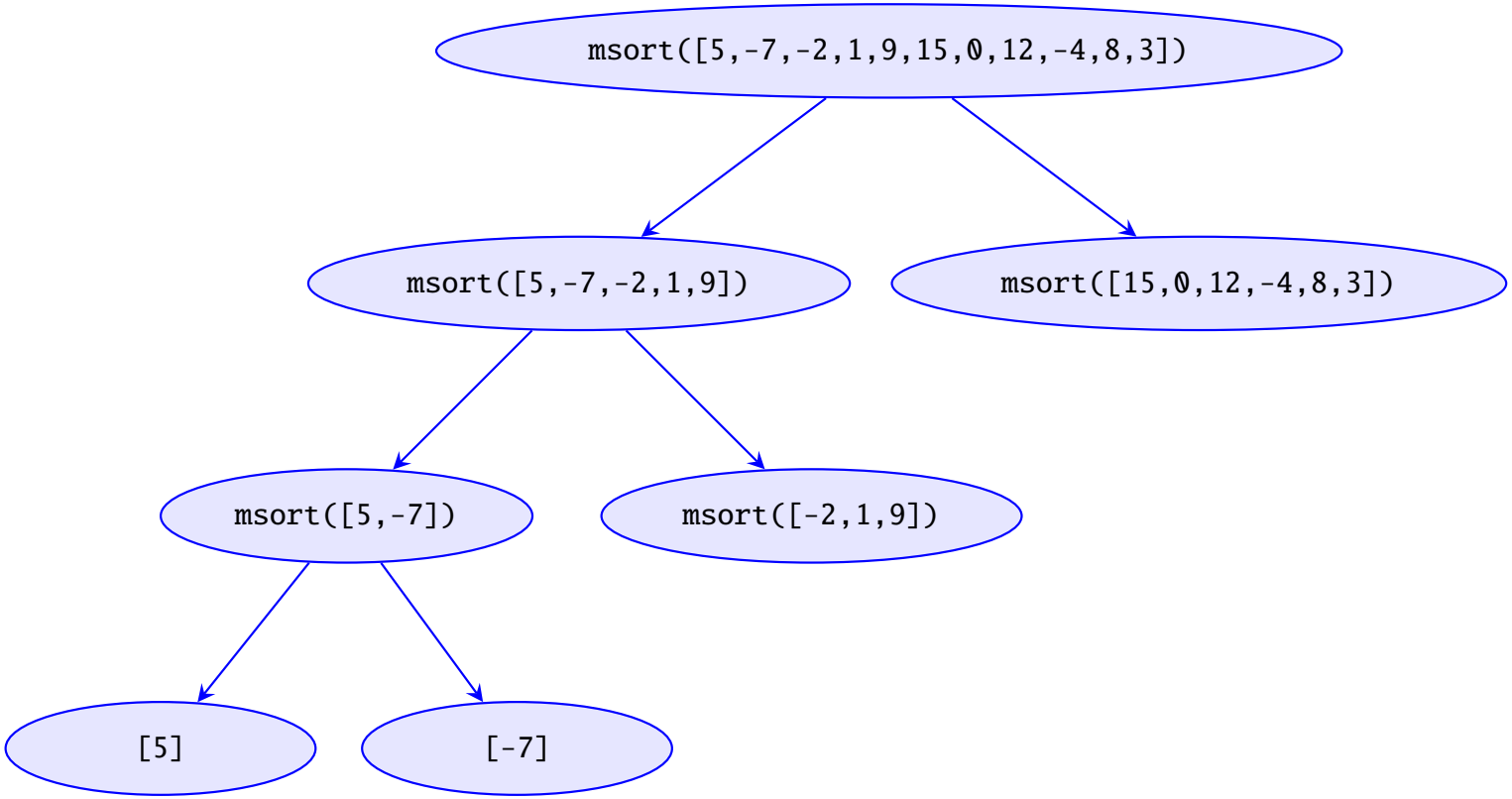

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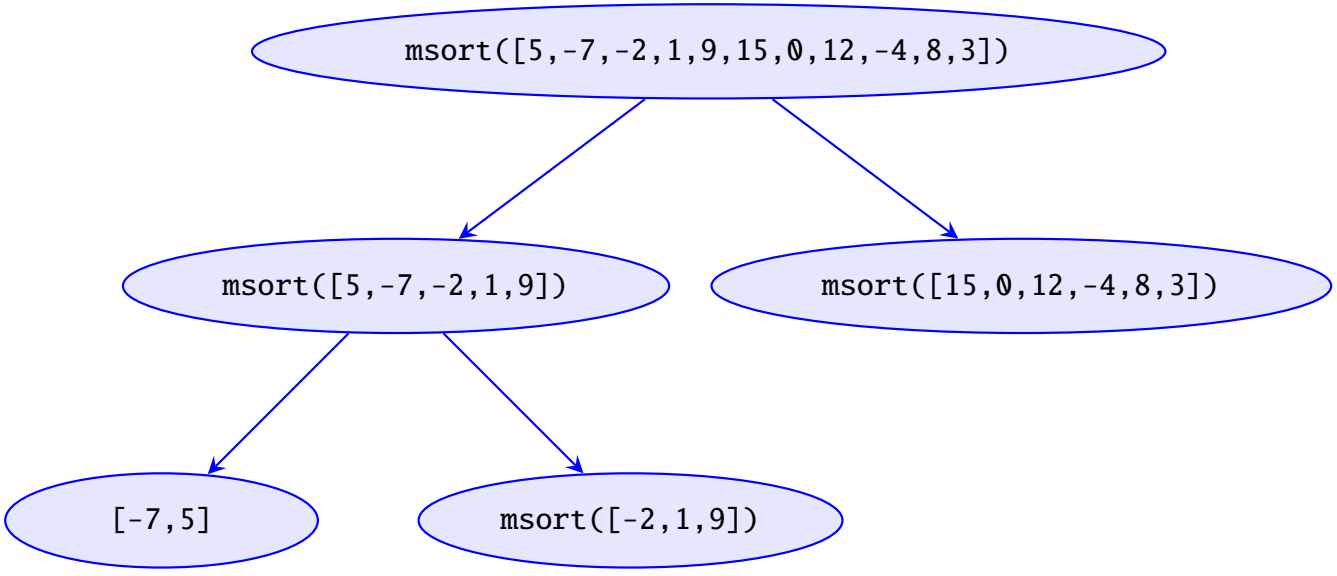

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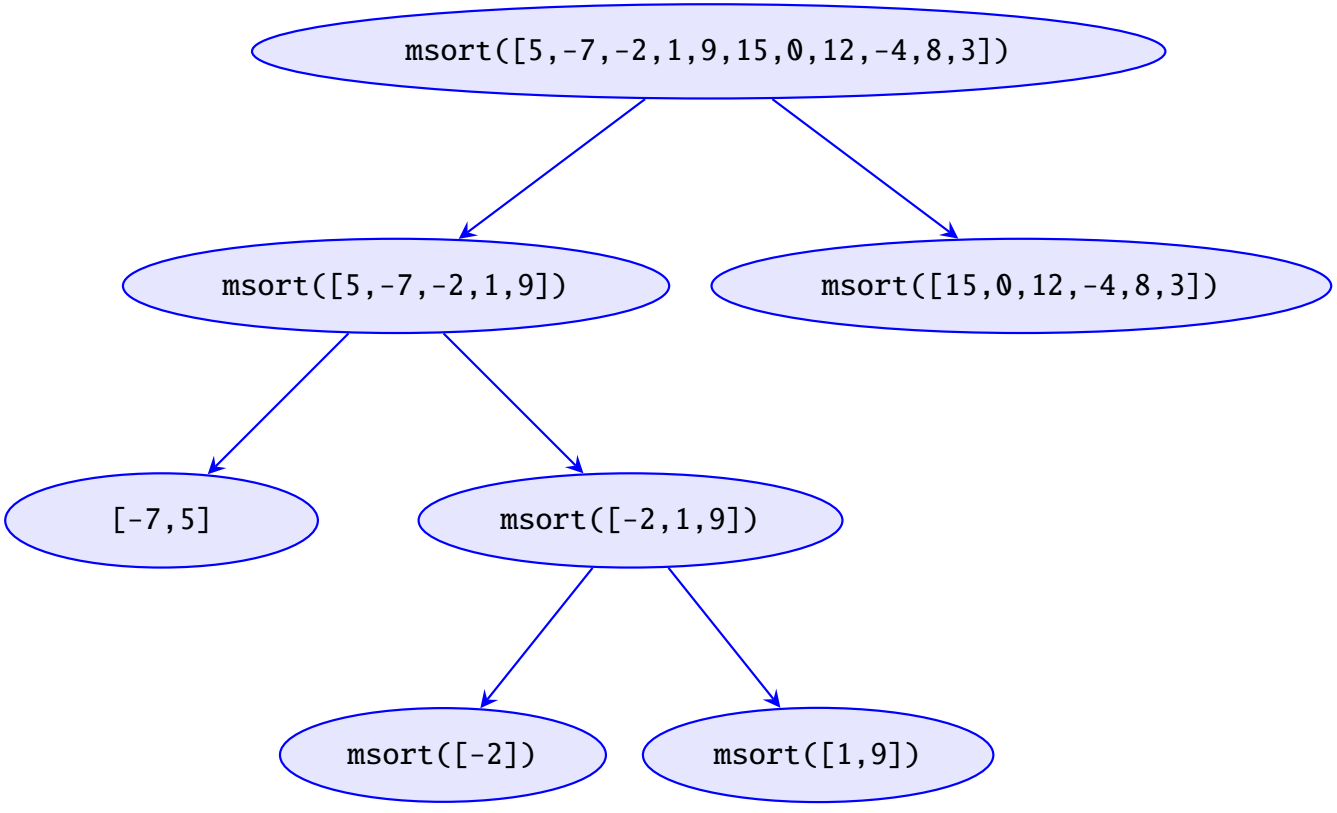

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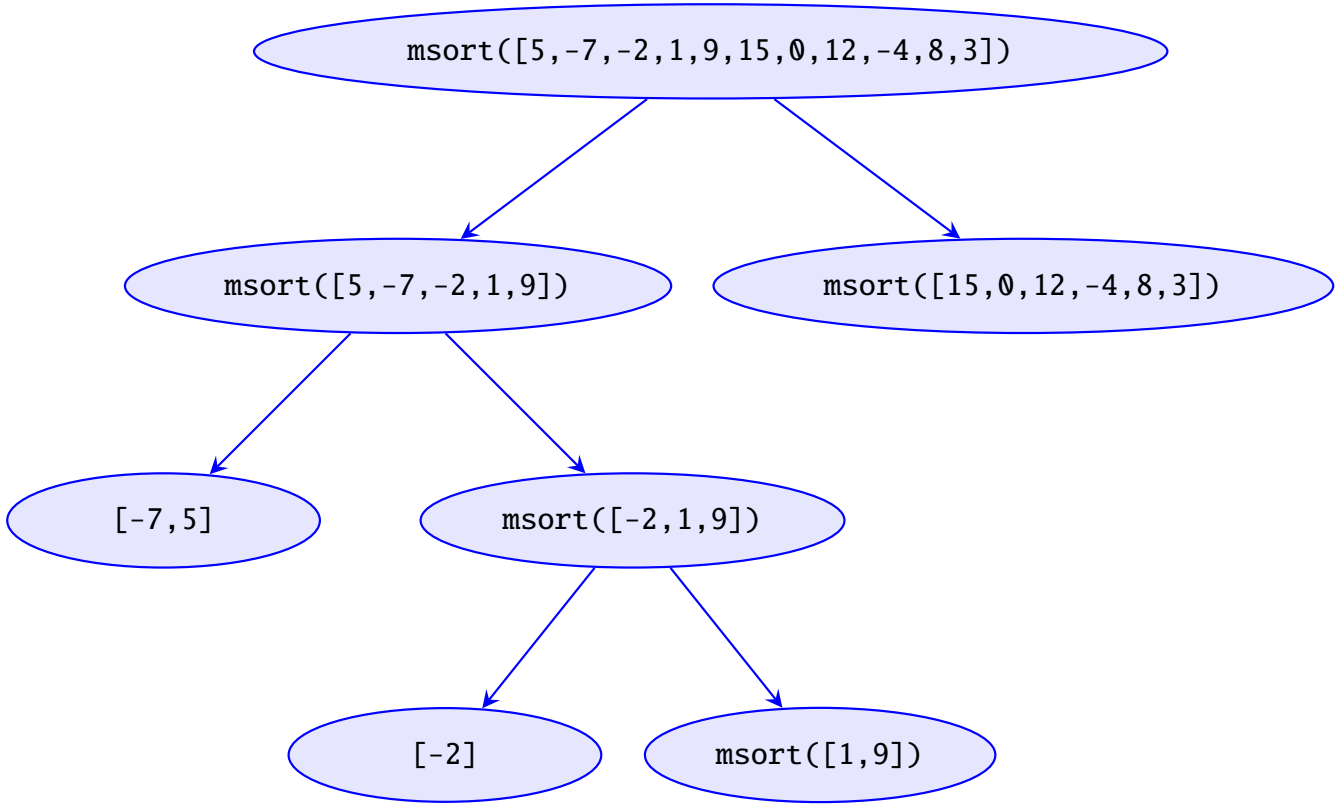

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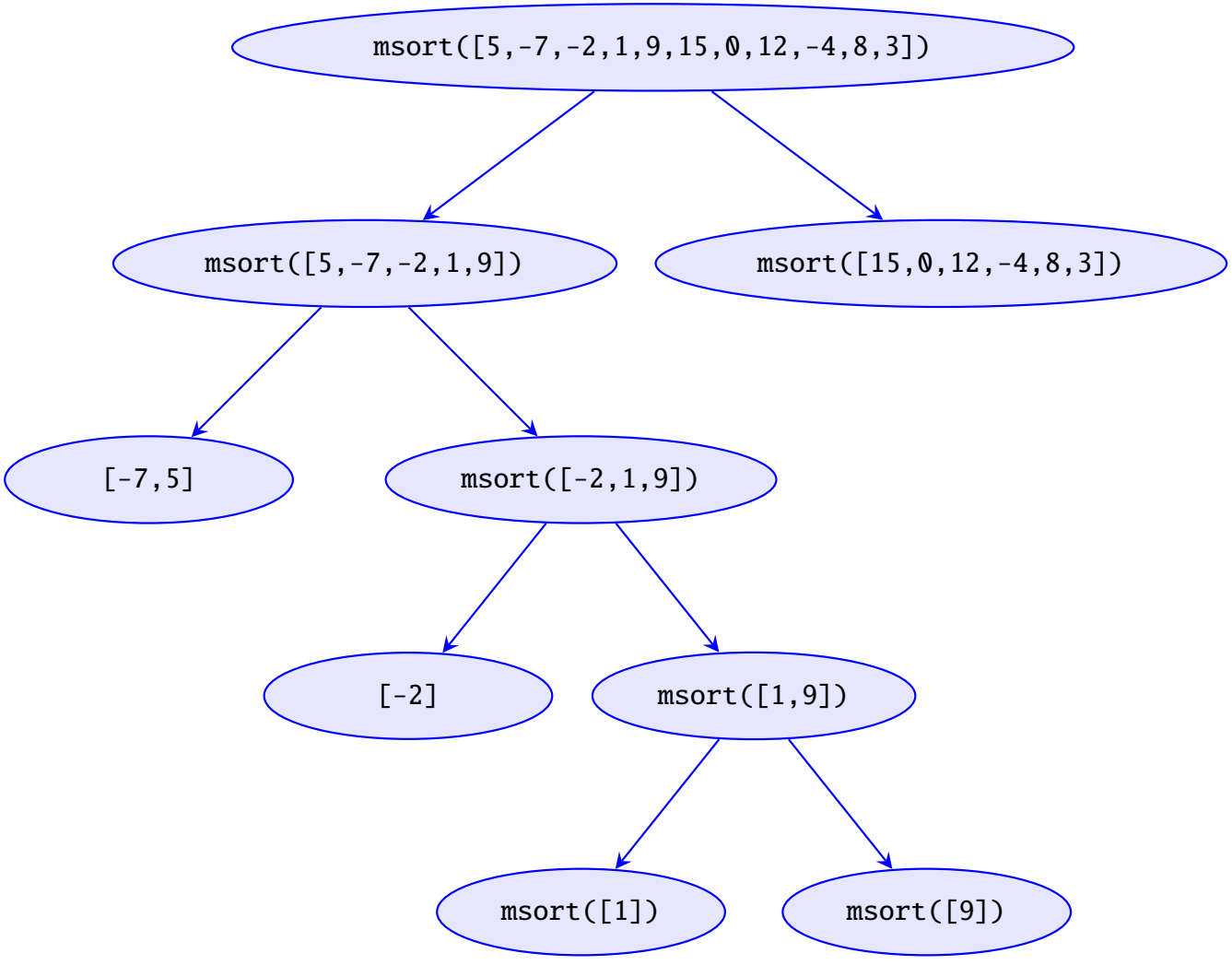

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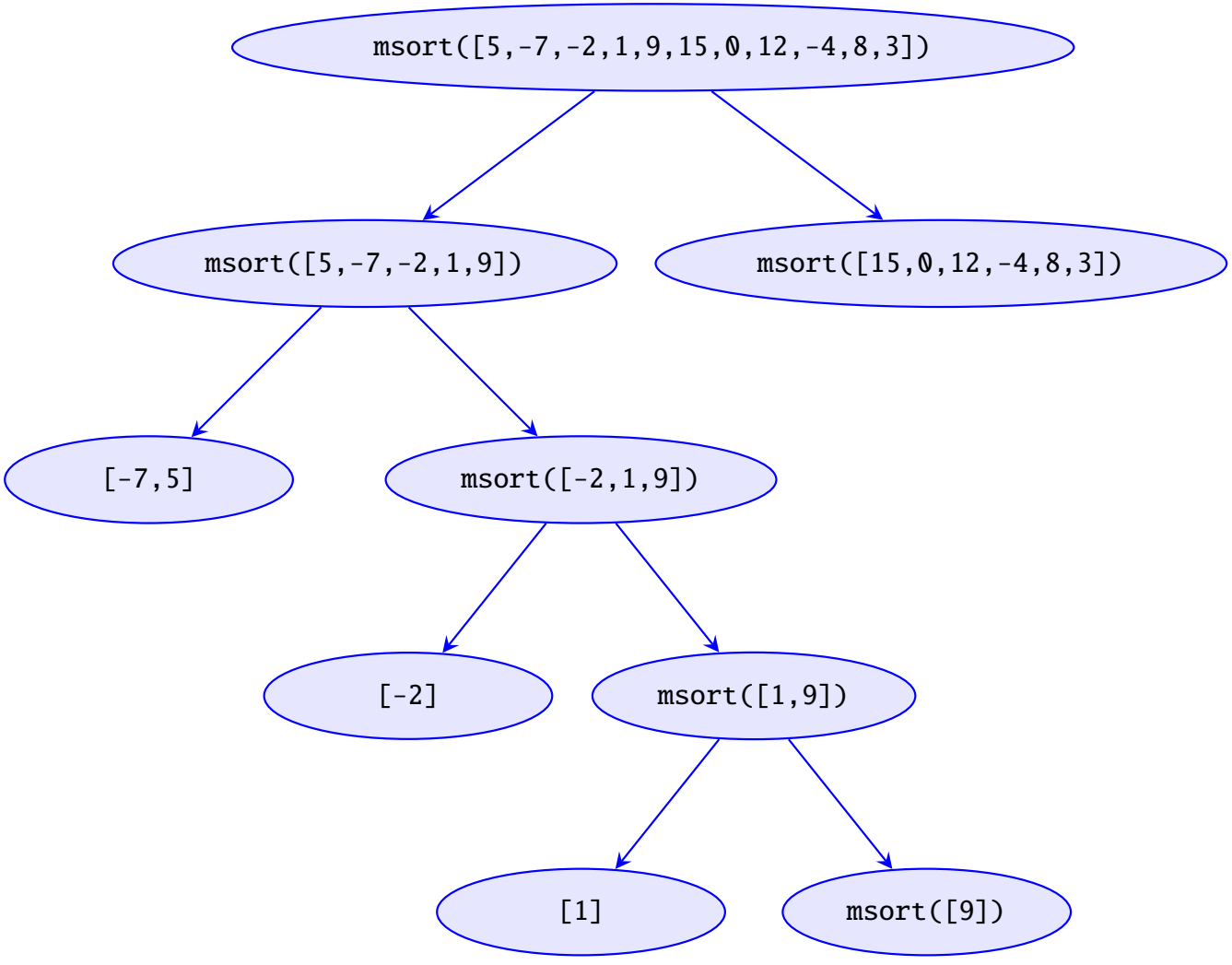

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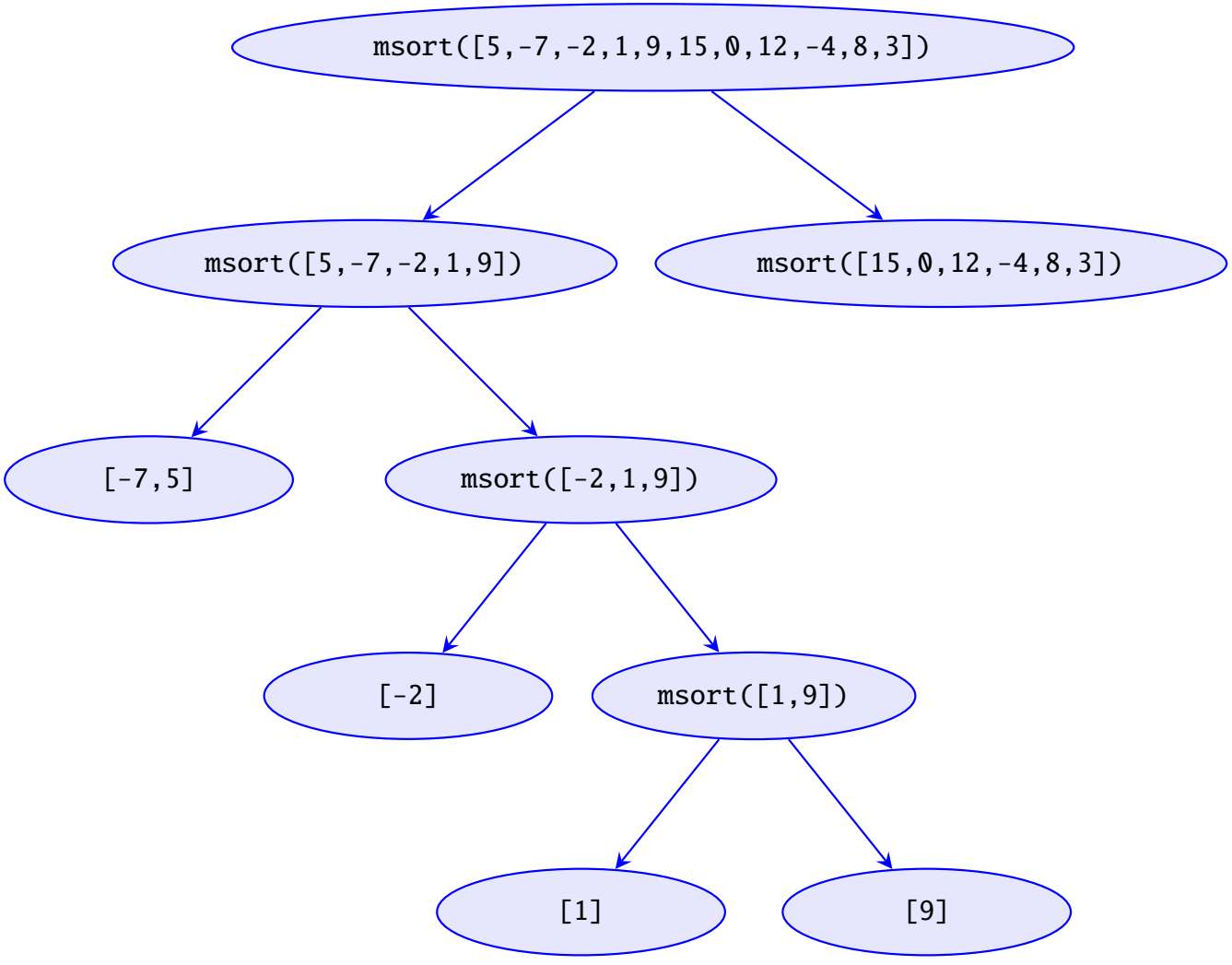

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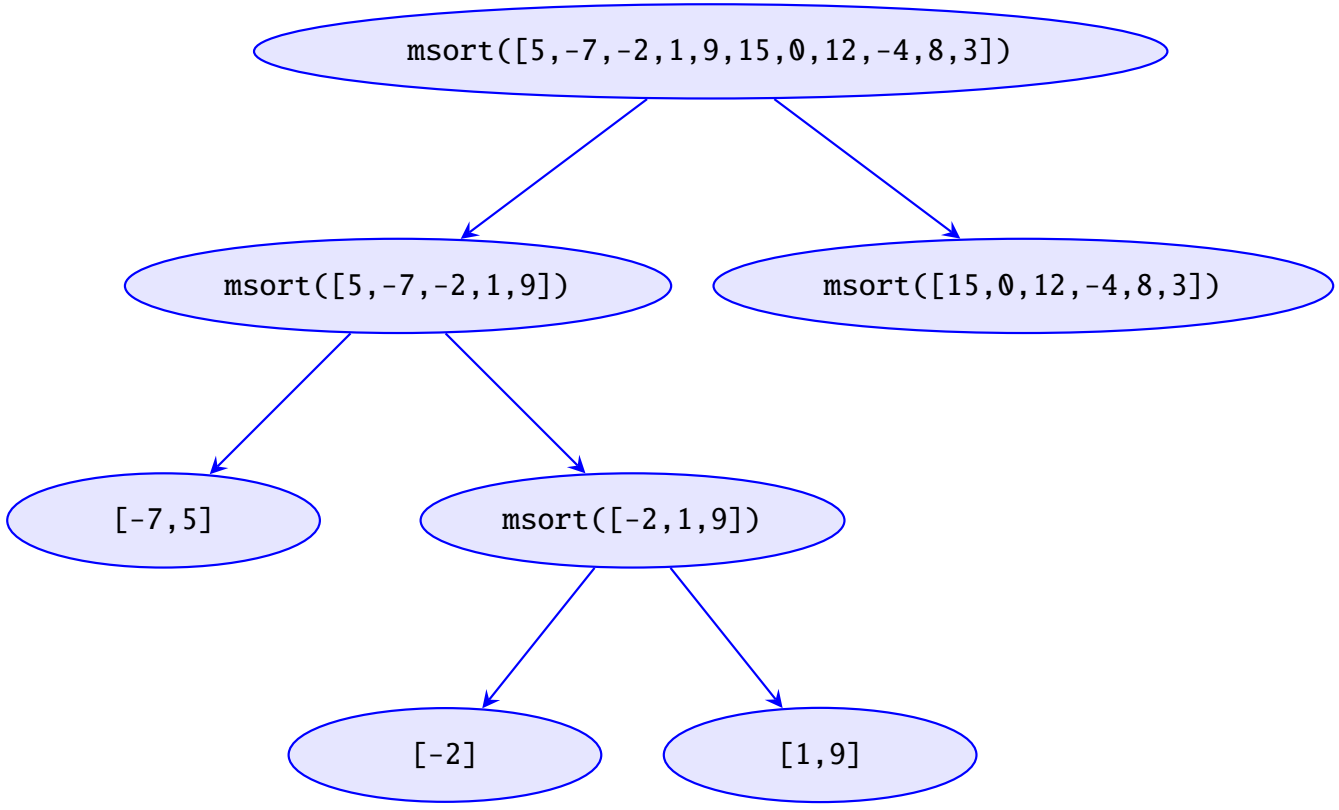

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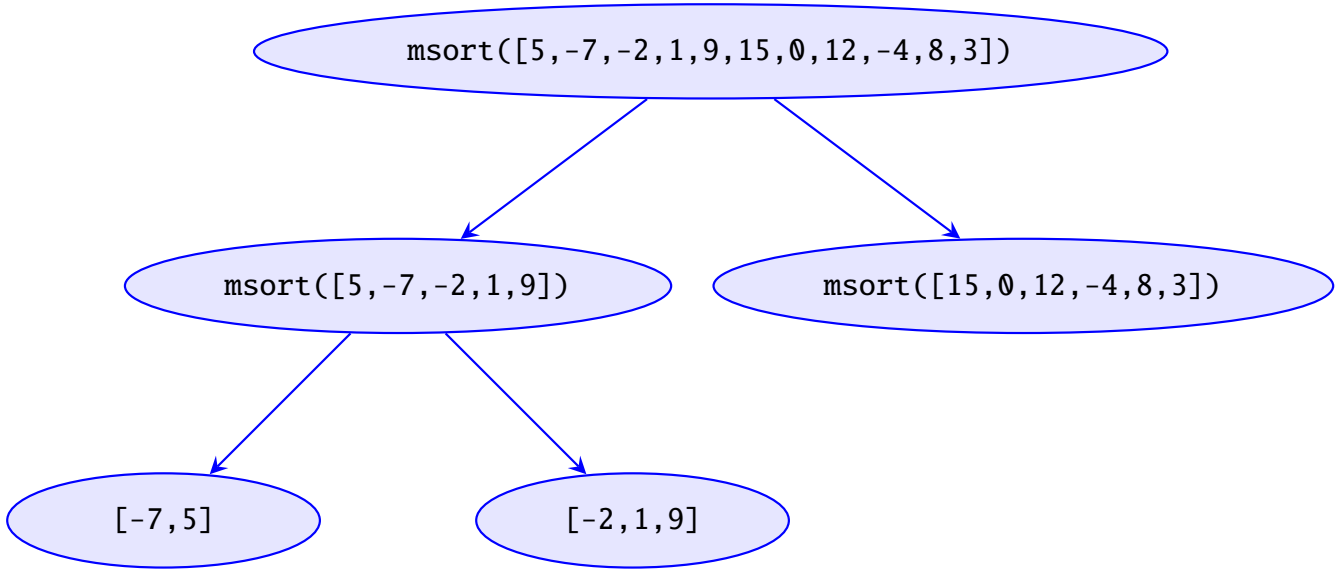

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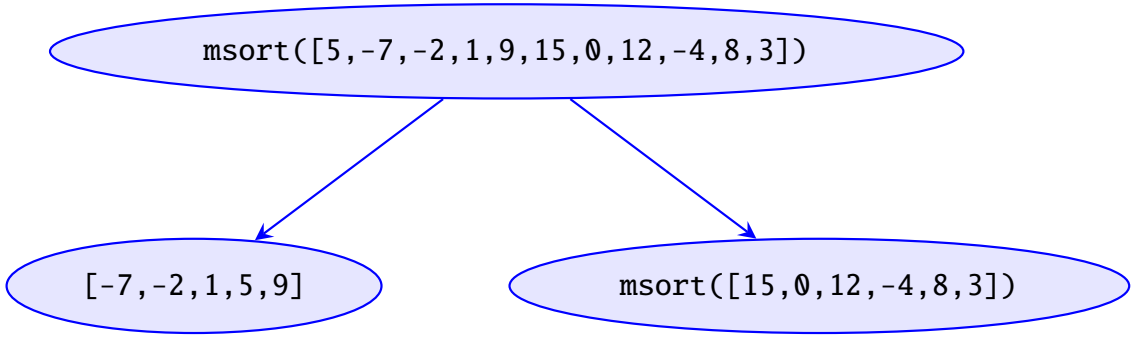

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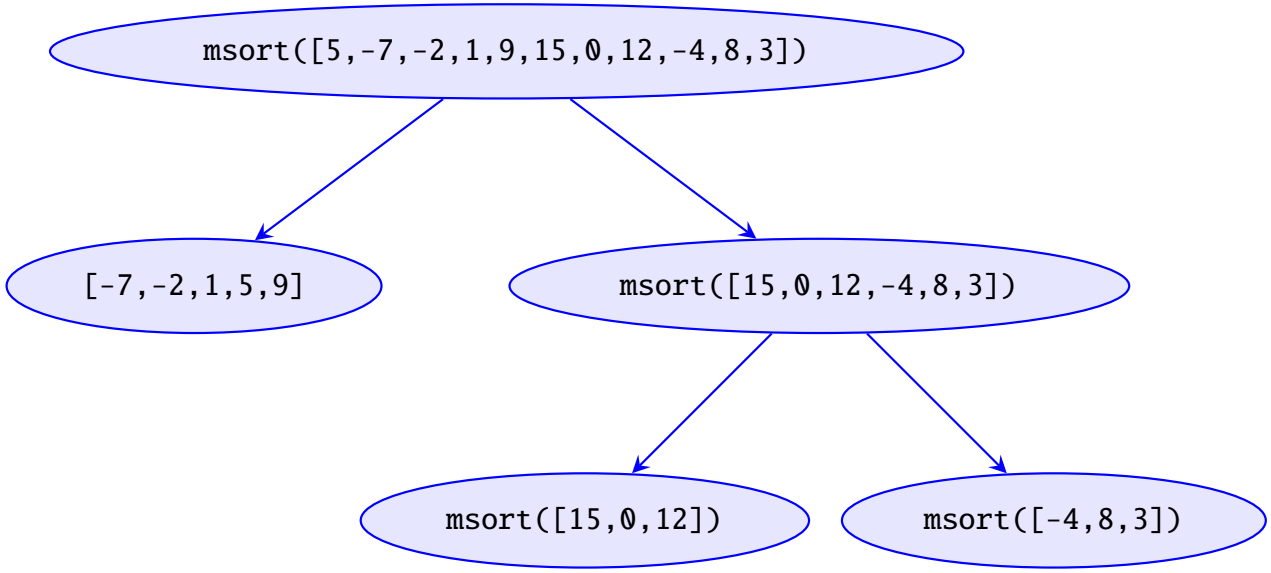

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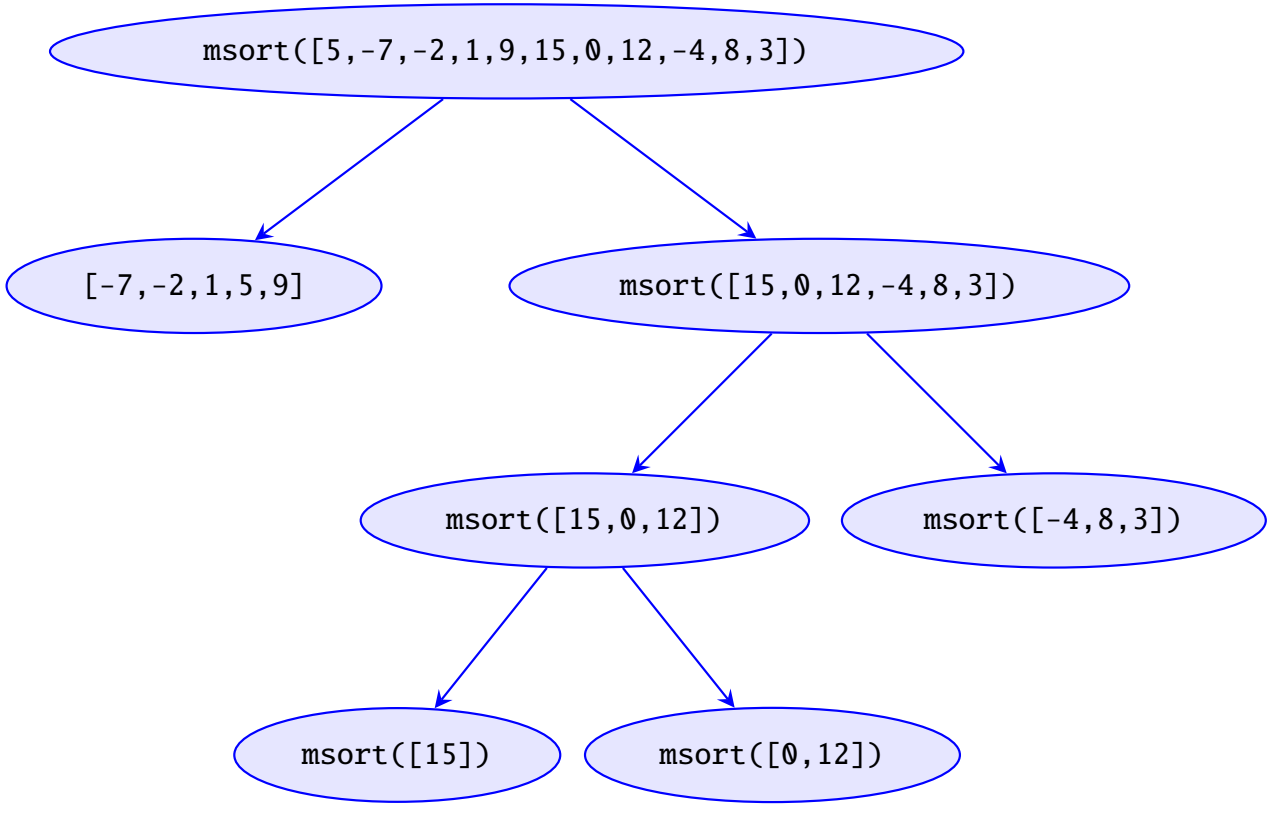

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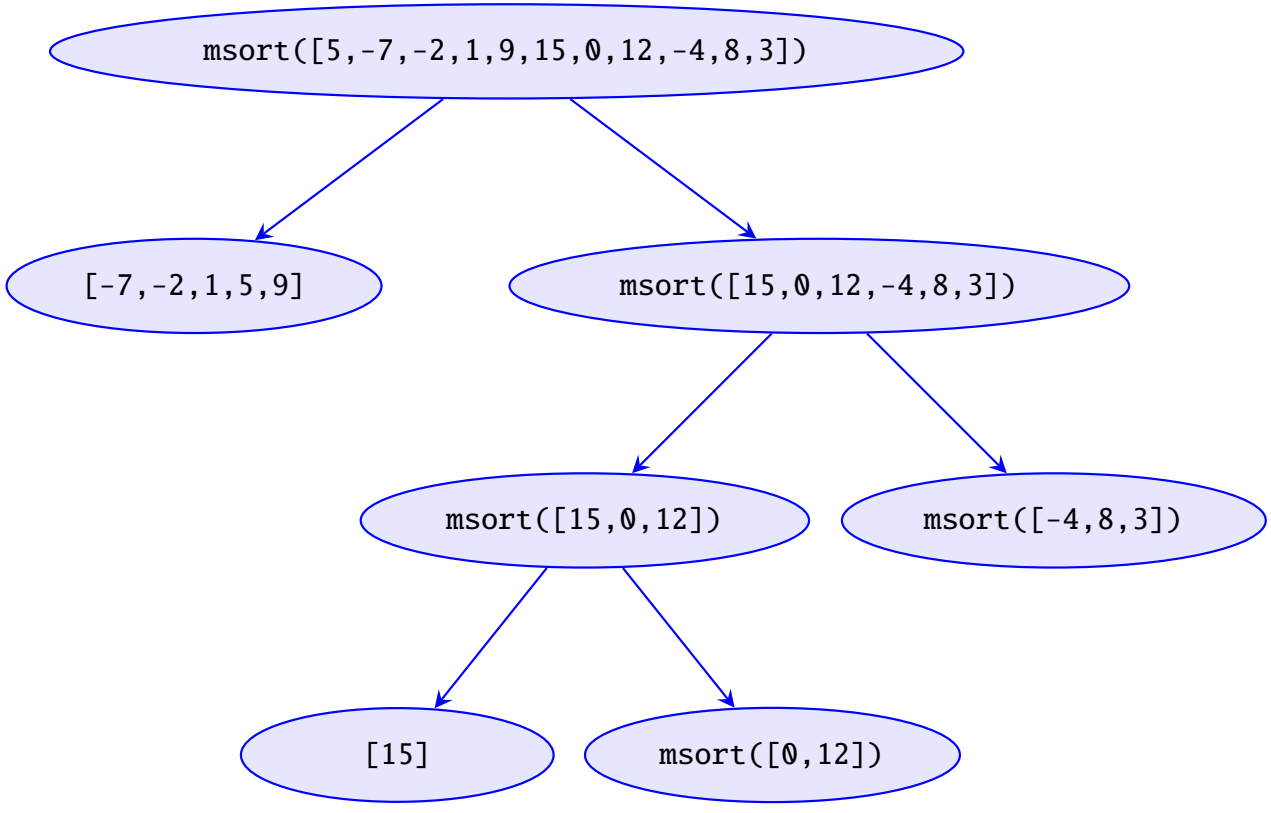

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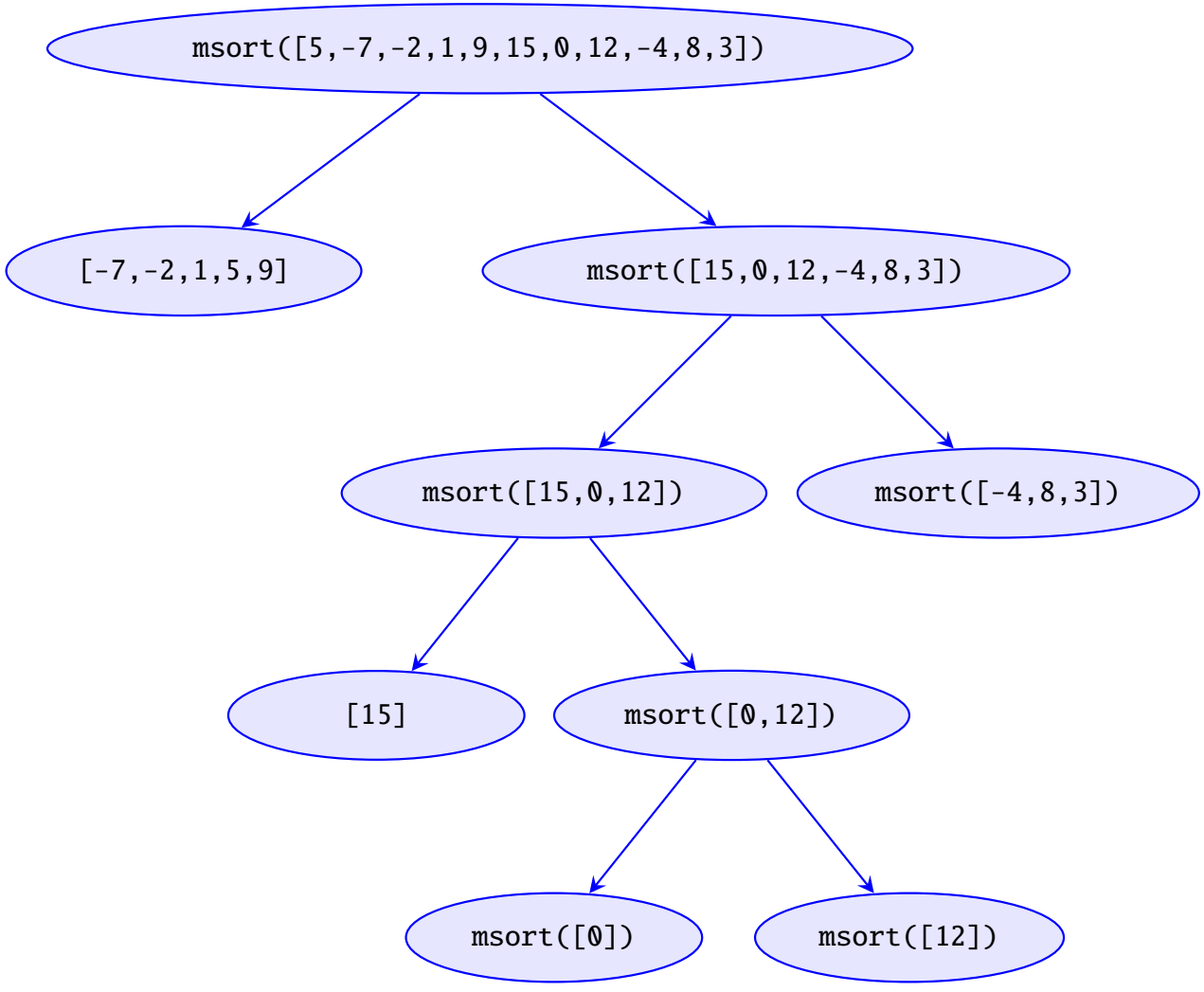

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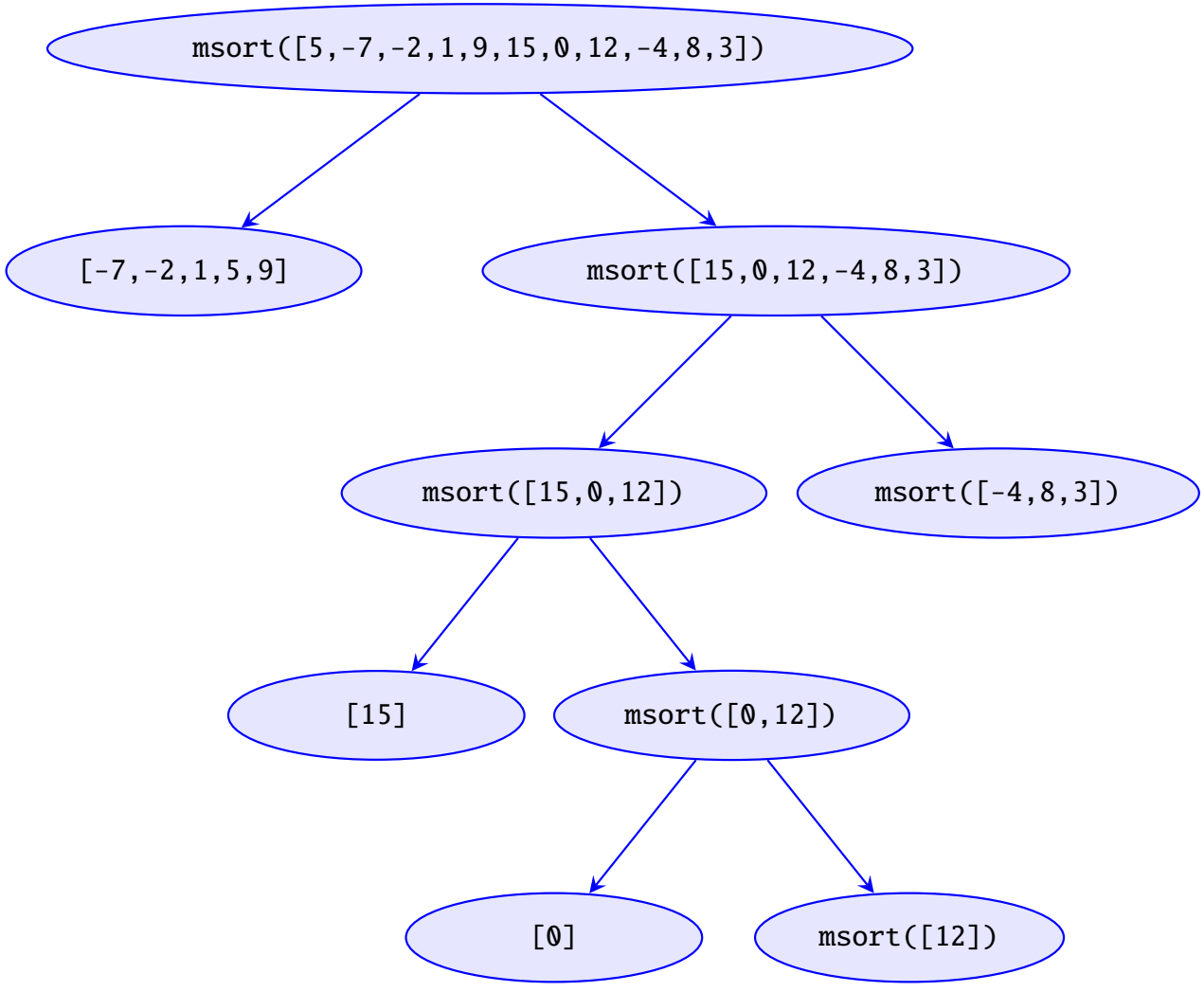

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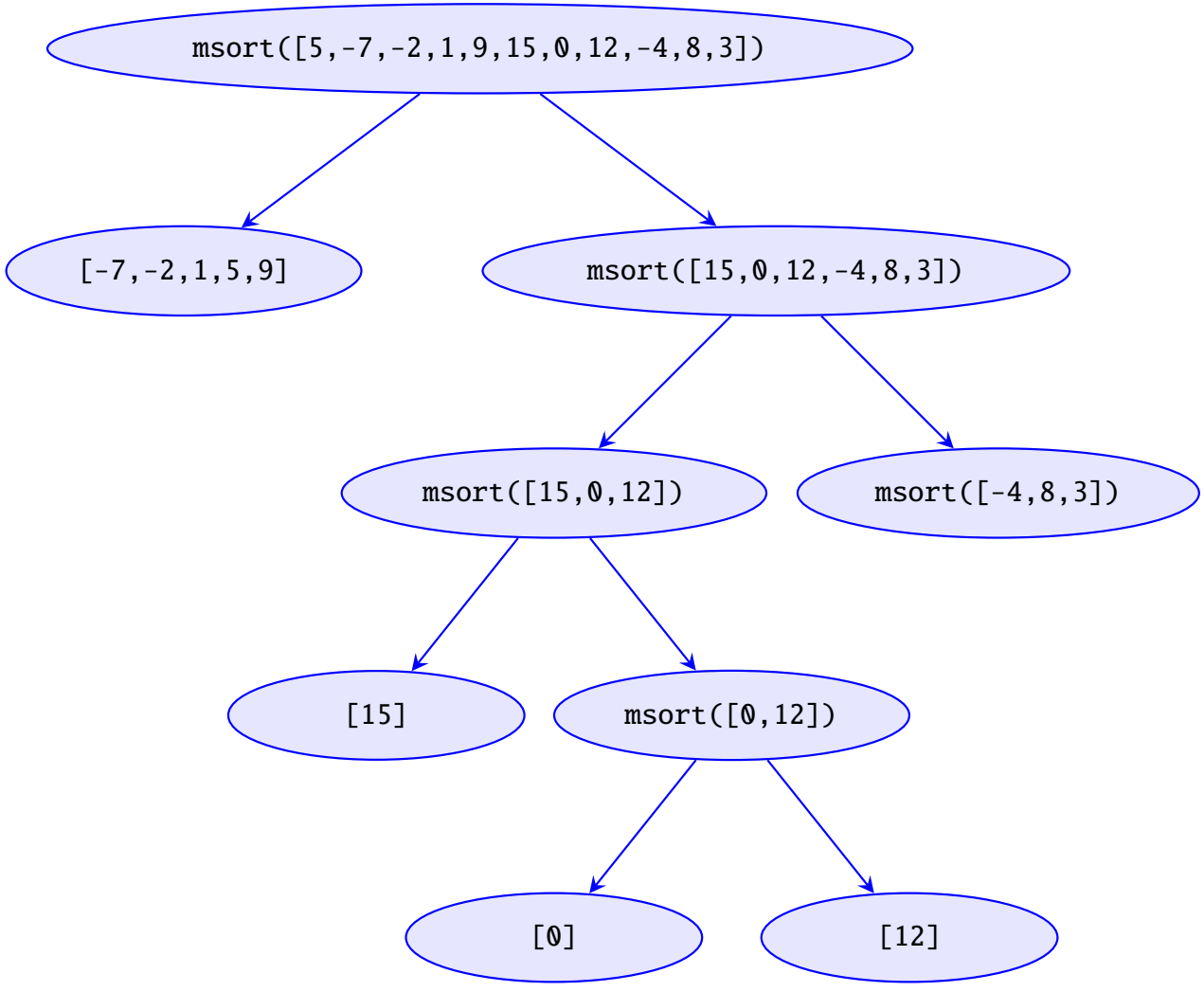

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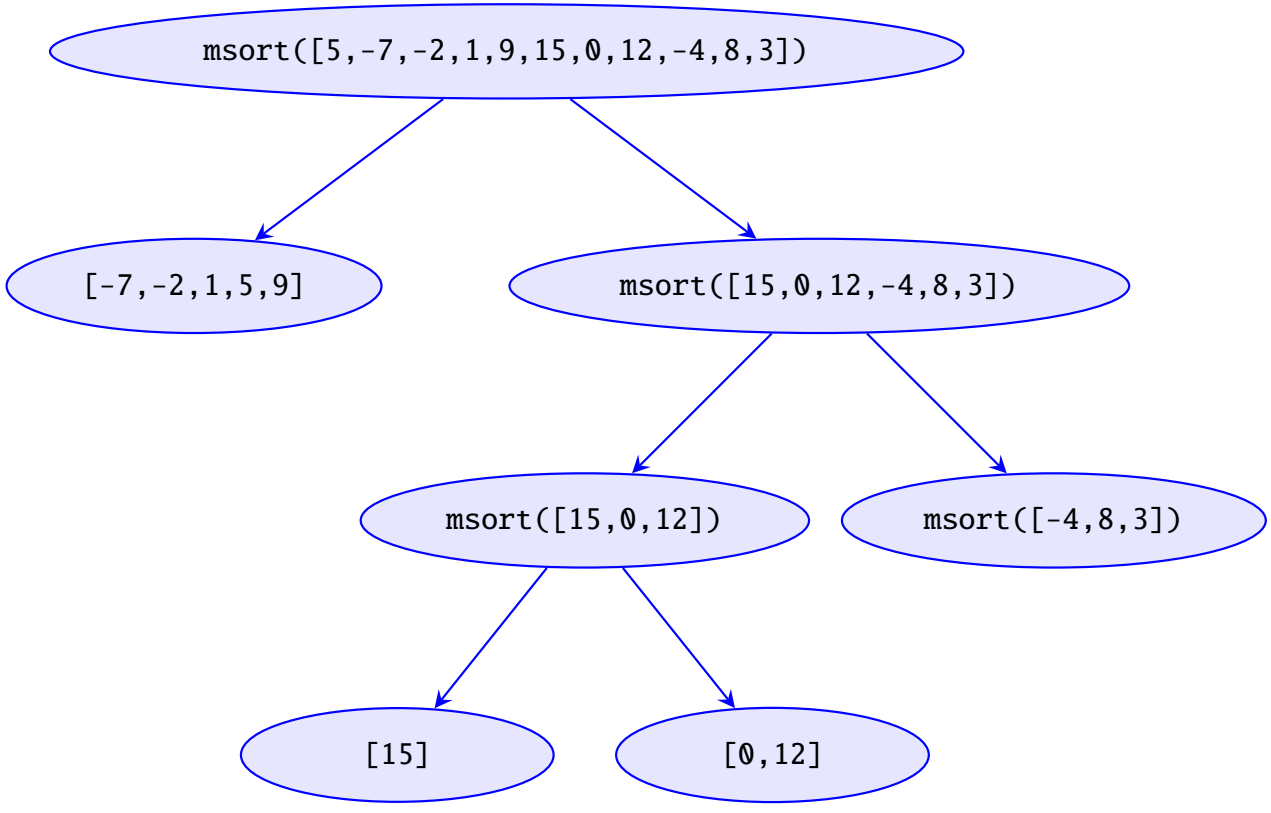

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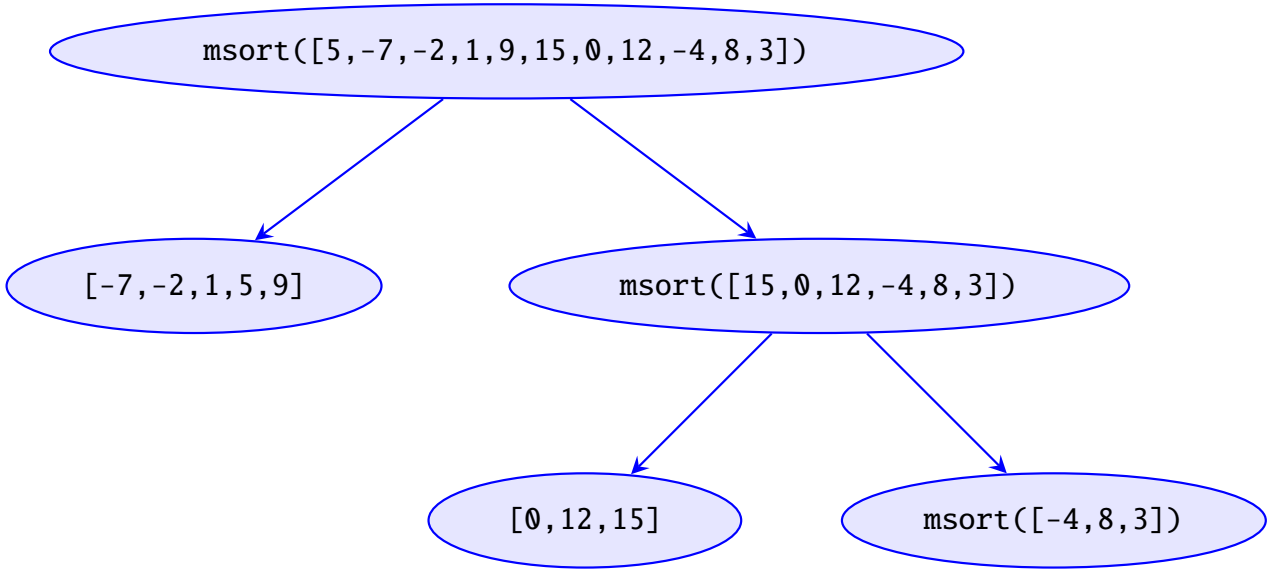

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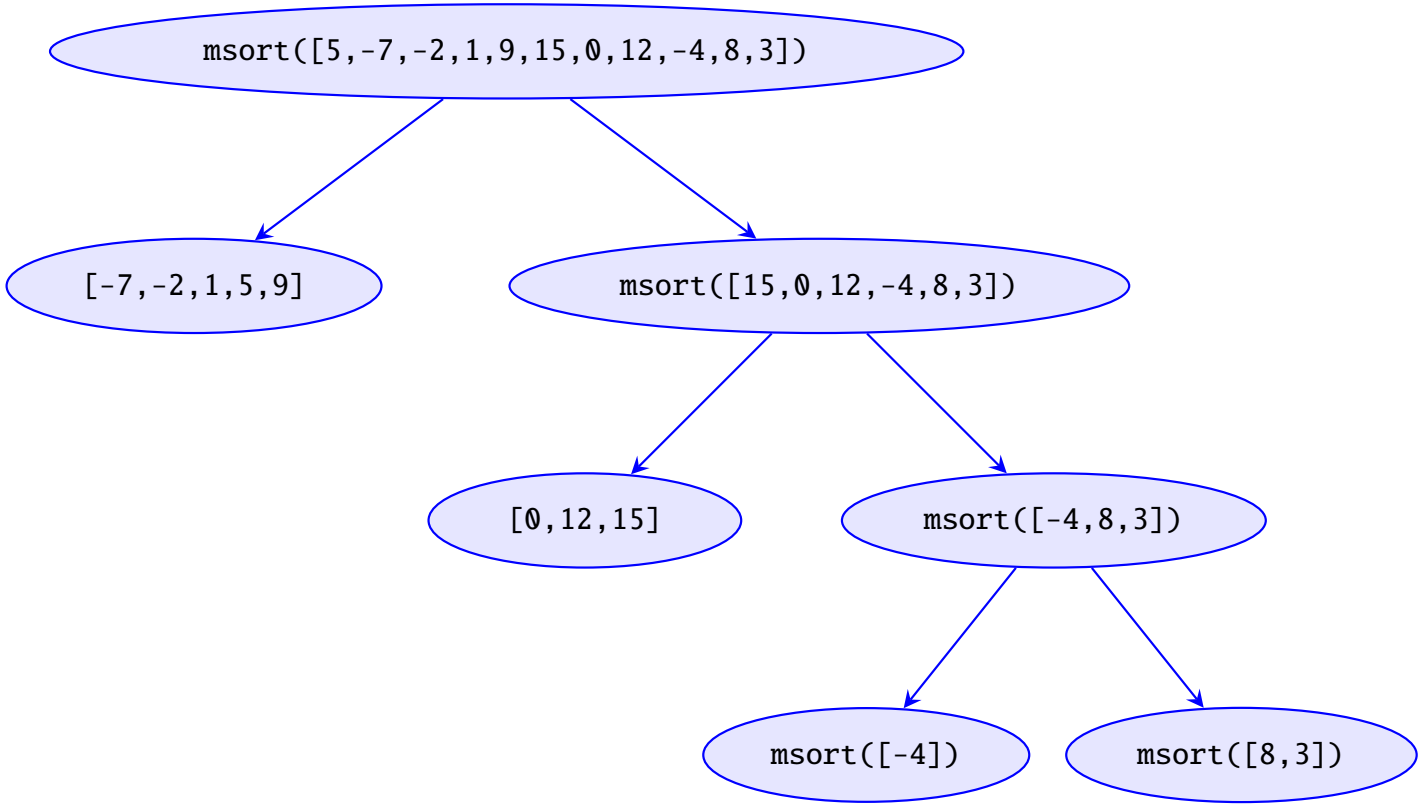

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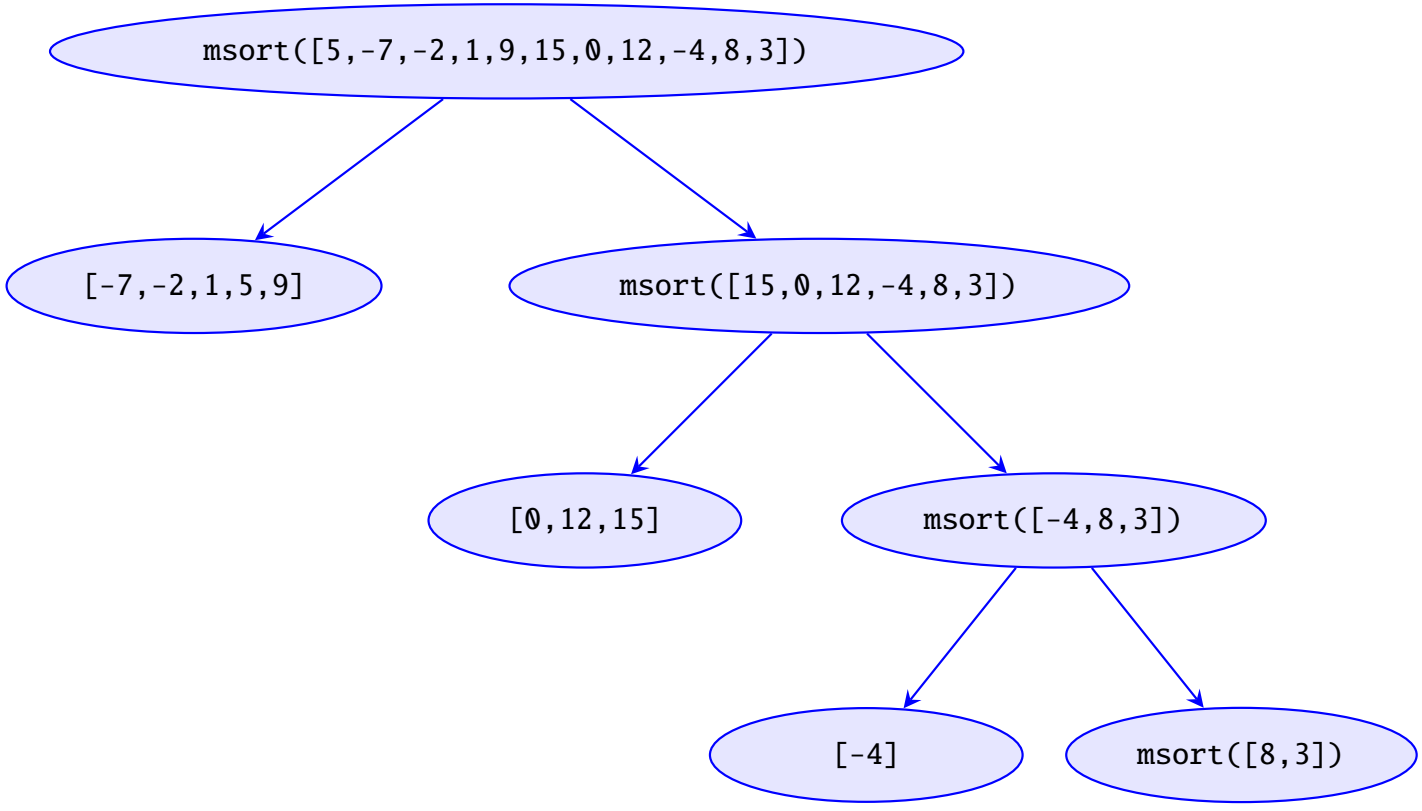

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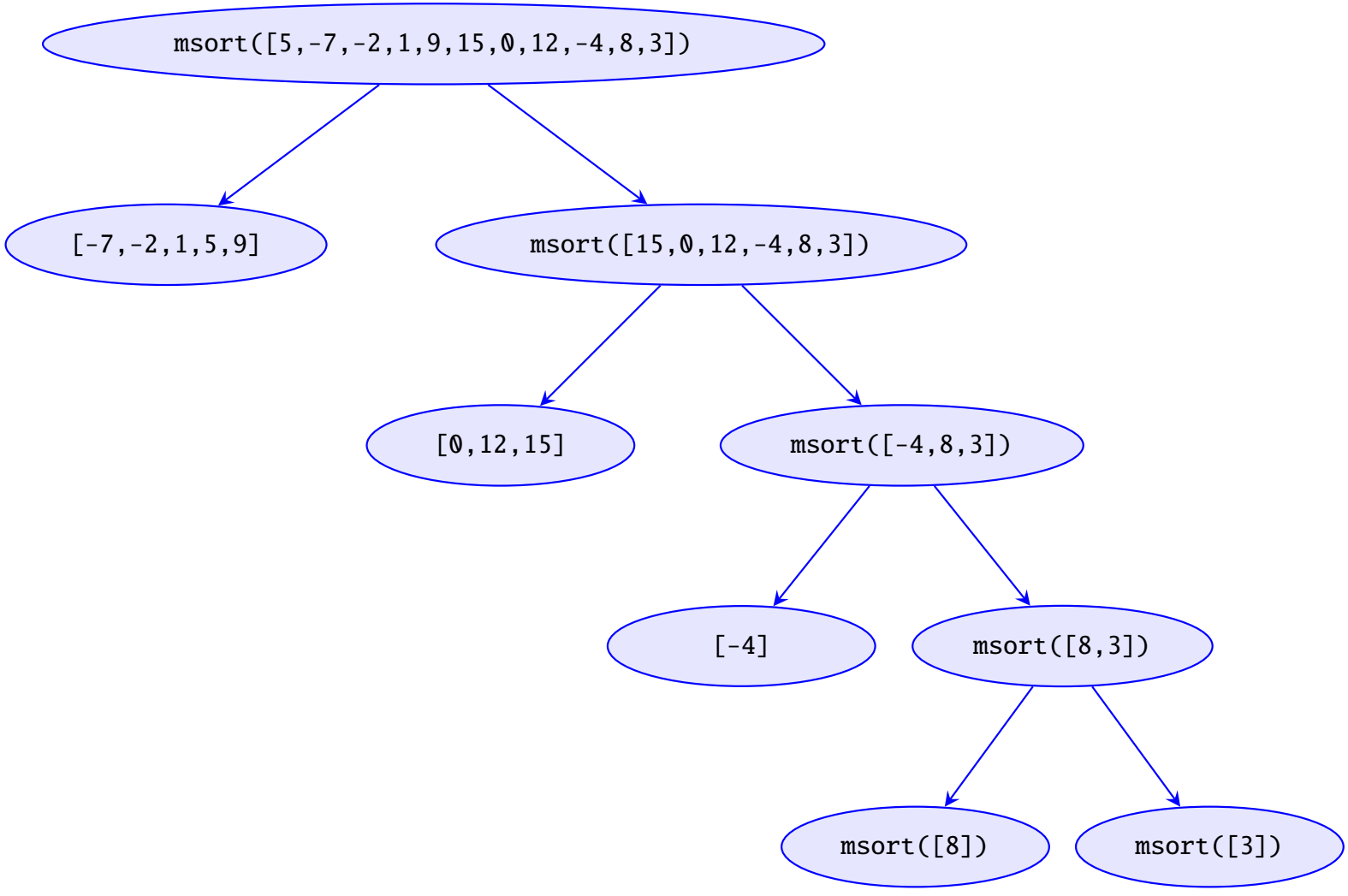

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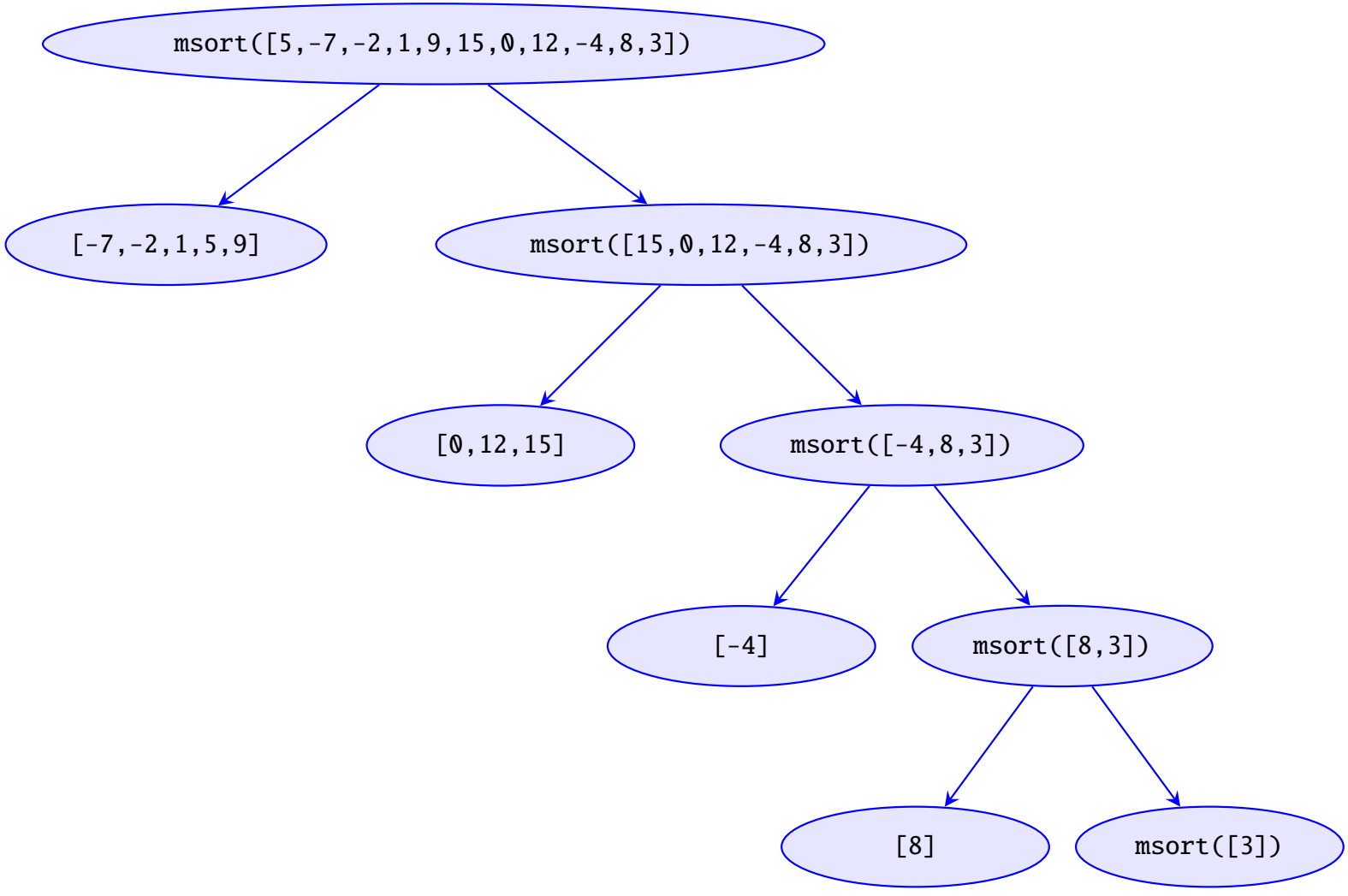

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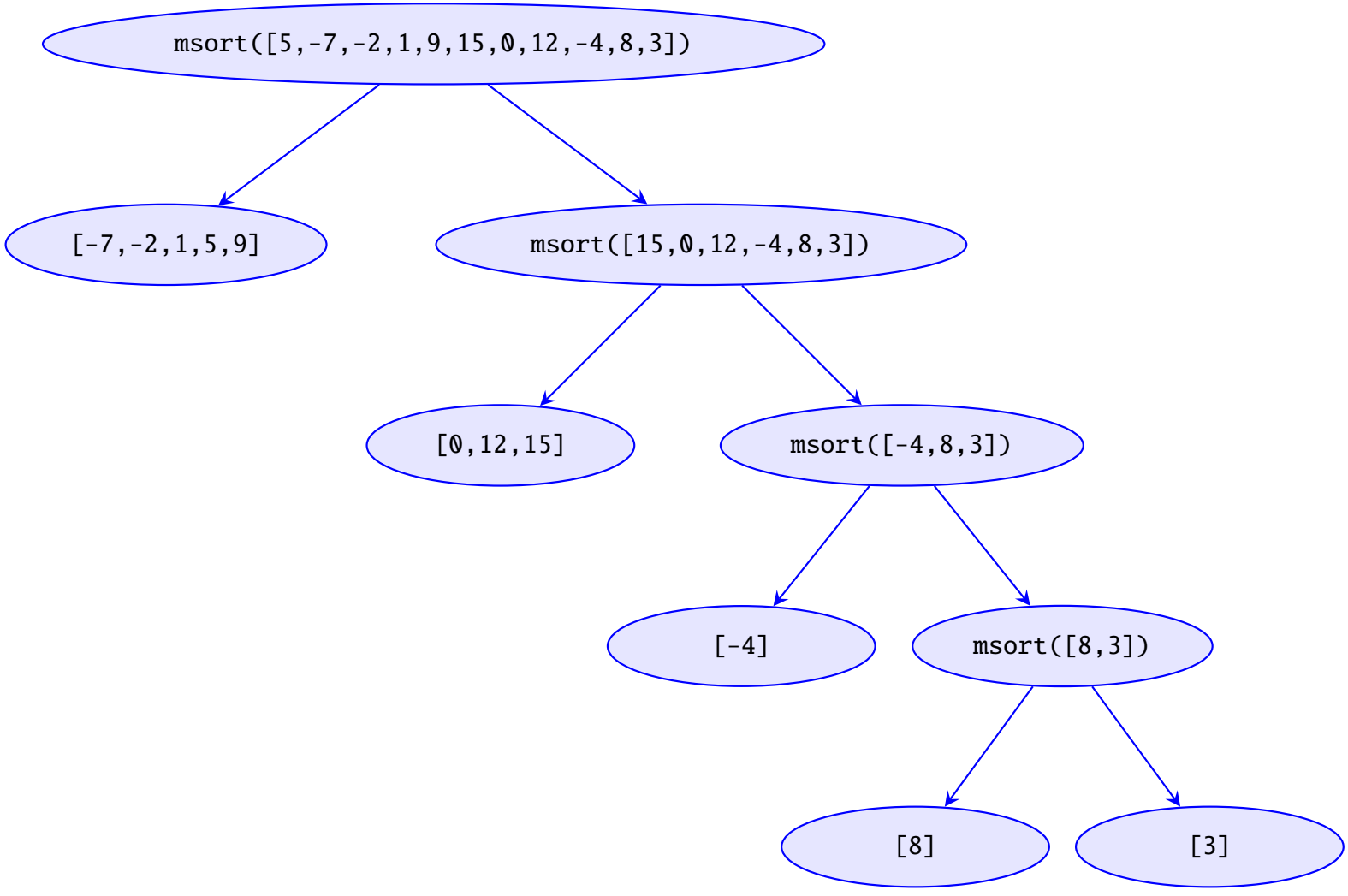

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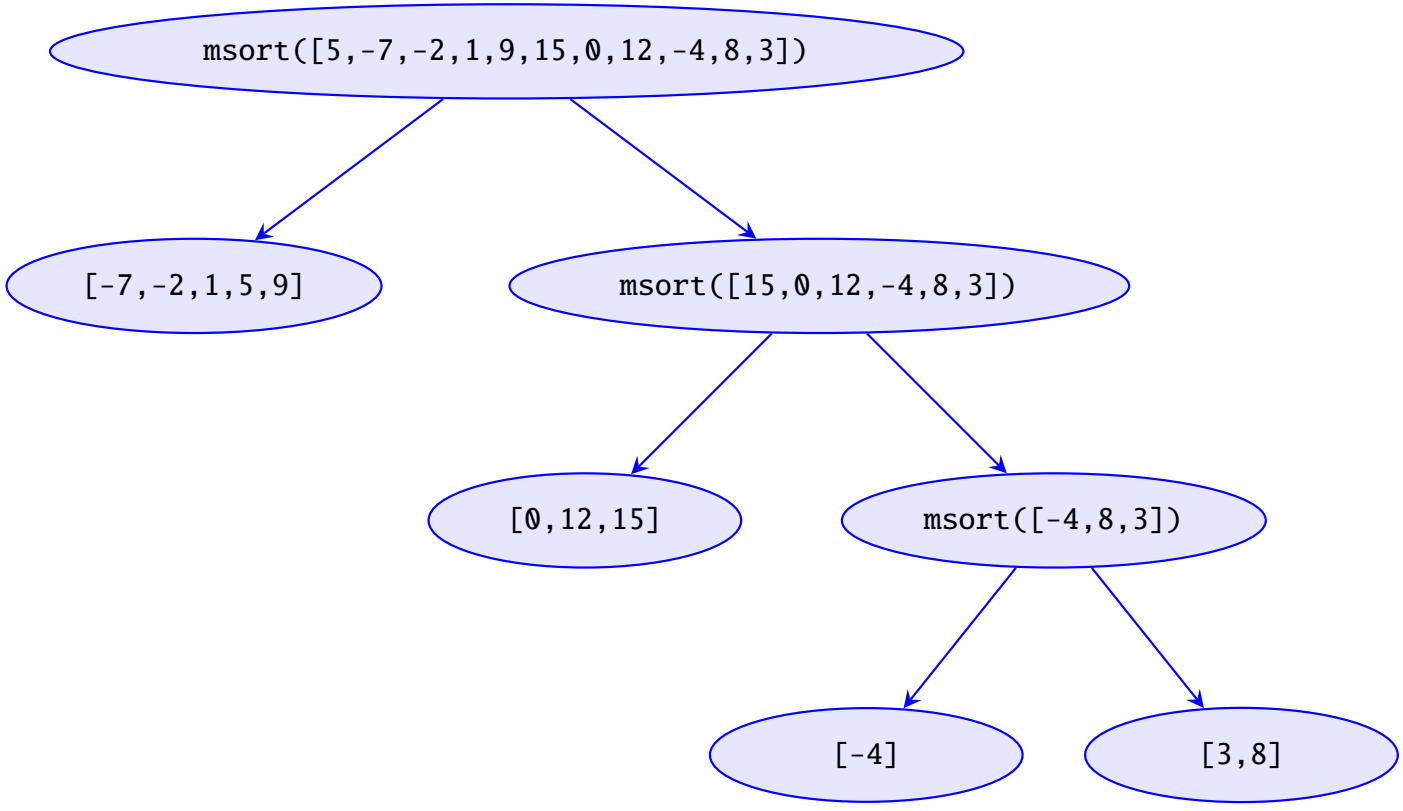

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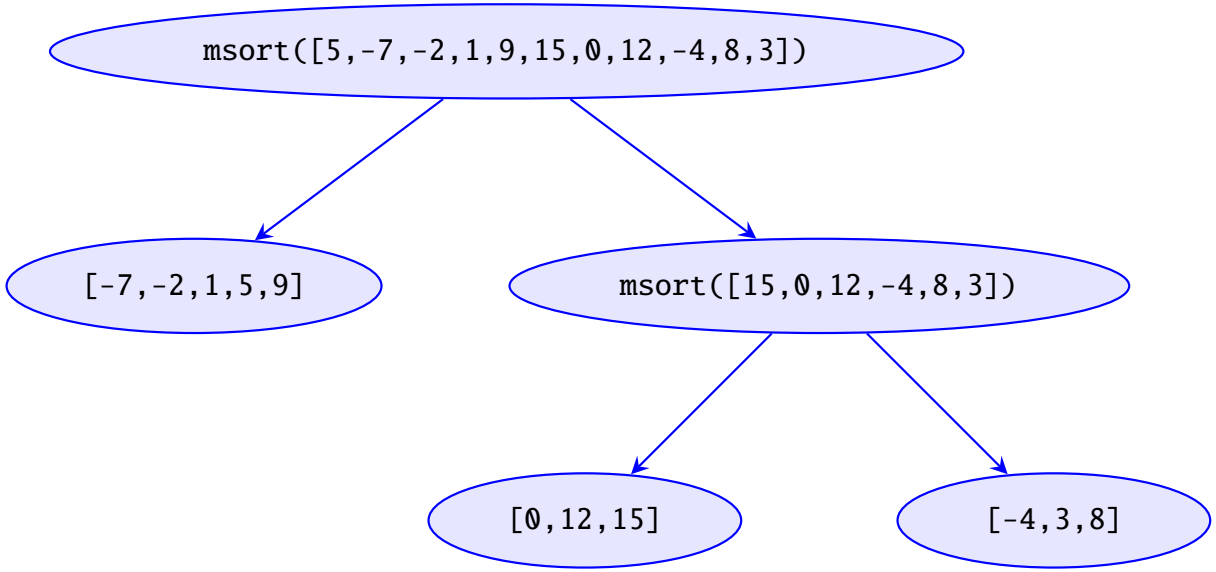

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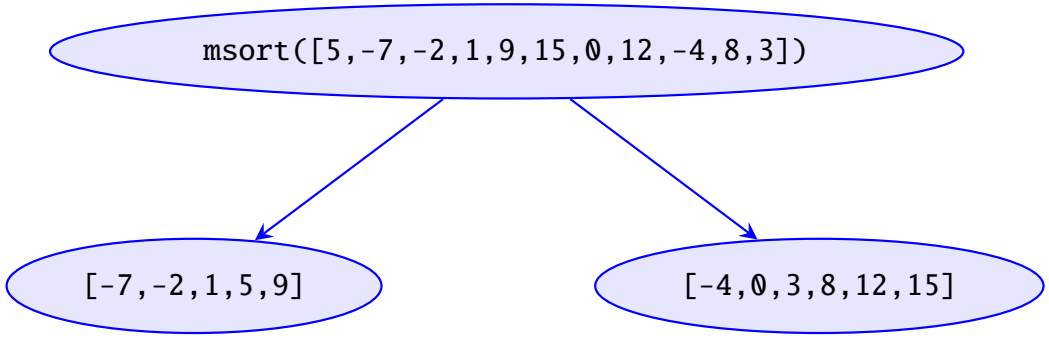

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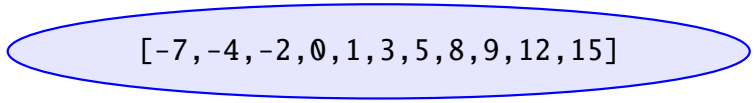

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```
def merge(left, right):
    result = []; i = 0; j = 0
    while (i < len(left) and j < len(right)):
        if (left[i] <= right[j]):
            result.append(left[i]);   i += 1
        else:
            result.append(right[j]);  j += 1
    result += left[i:]
    result += right[j:]
    return result

def mergeSort(a):
    if len(a) < 2:    return a
    mid   = len(a) // 2
    left  = mergeSort(a[:mid])
    right = mergeSort(a[mid:])
    return merge(left,right)
```

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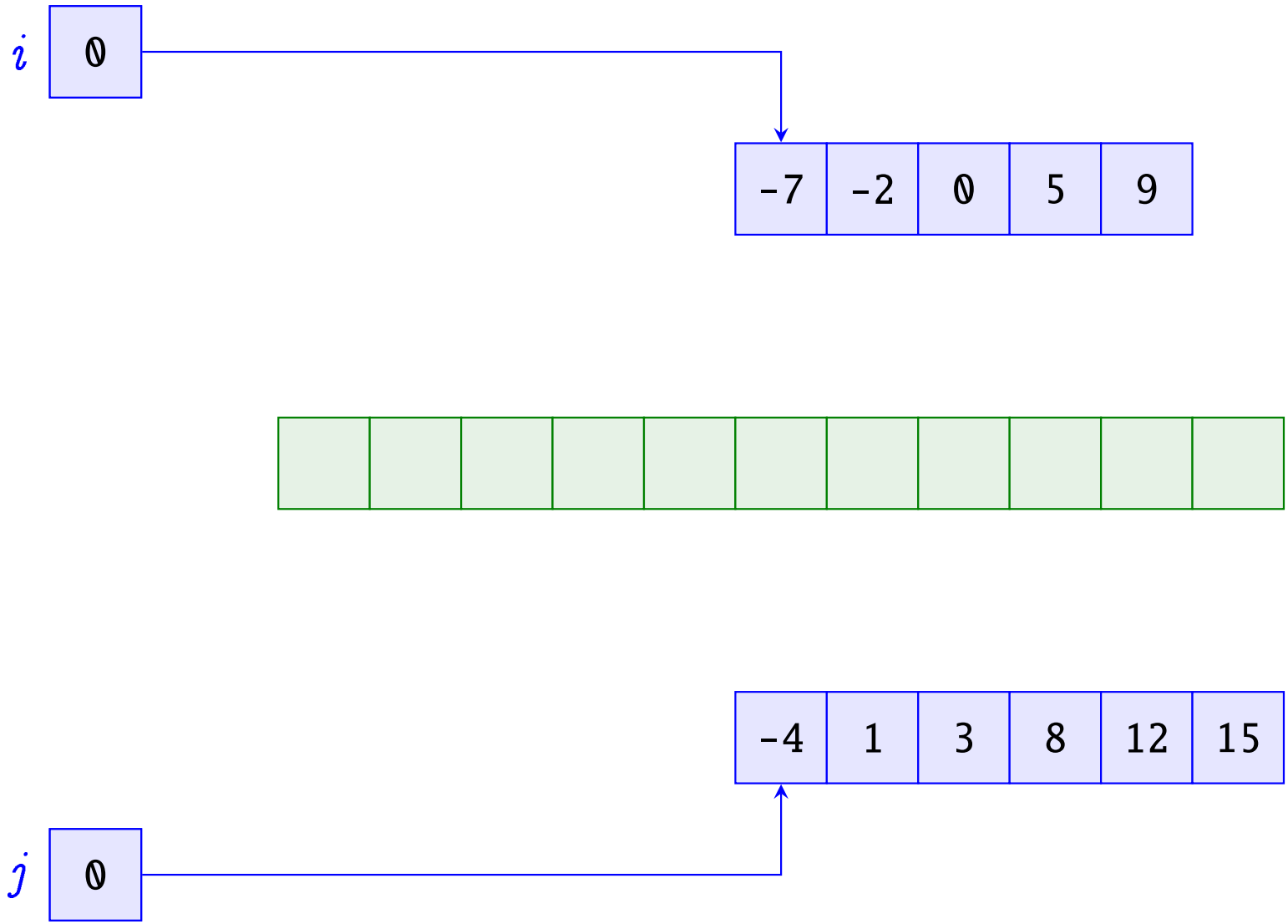

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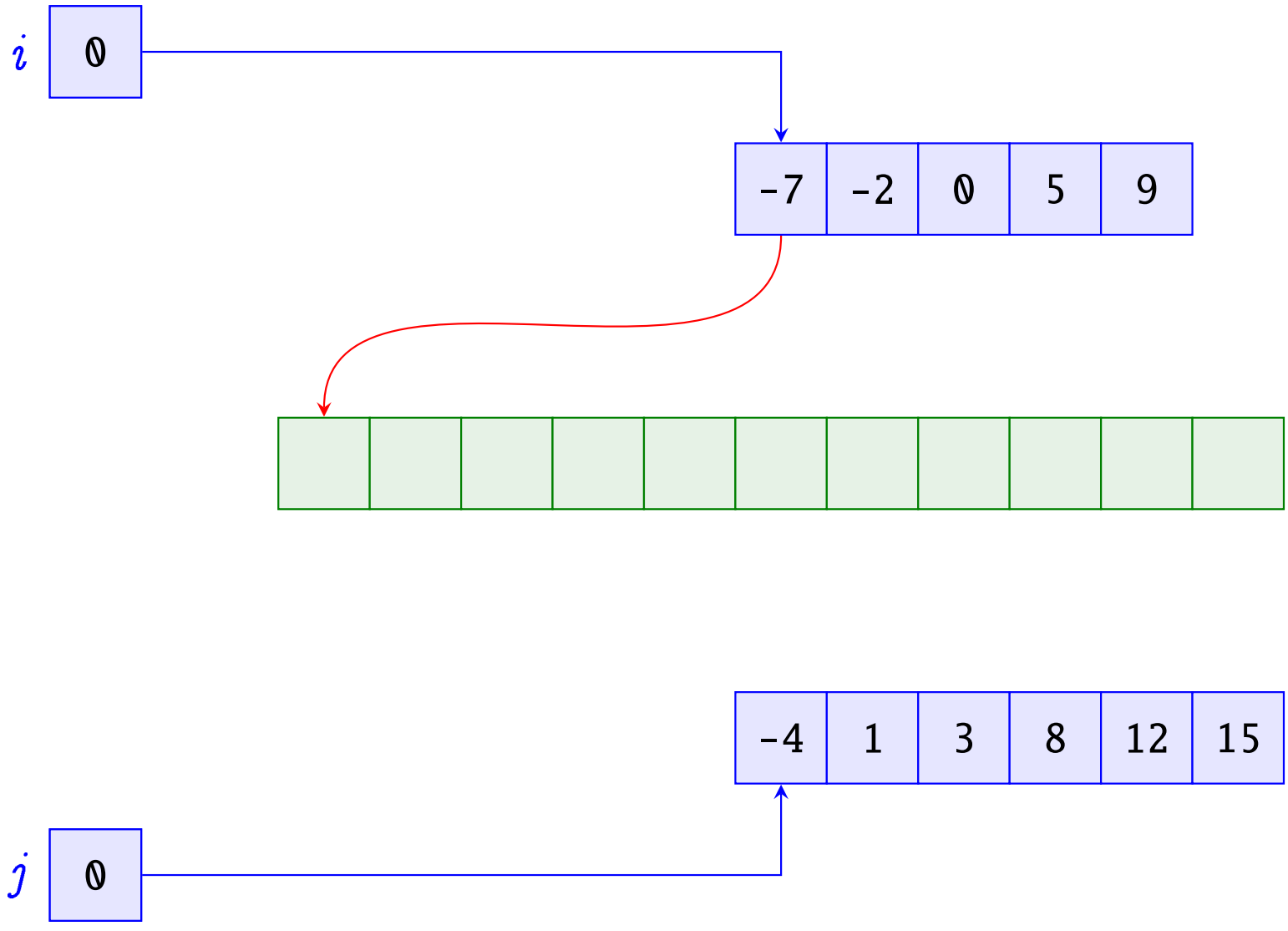

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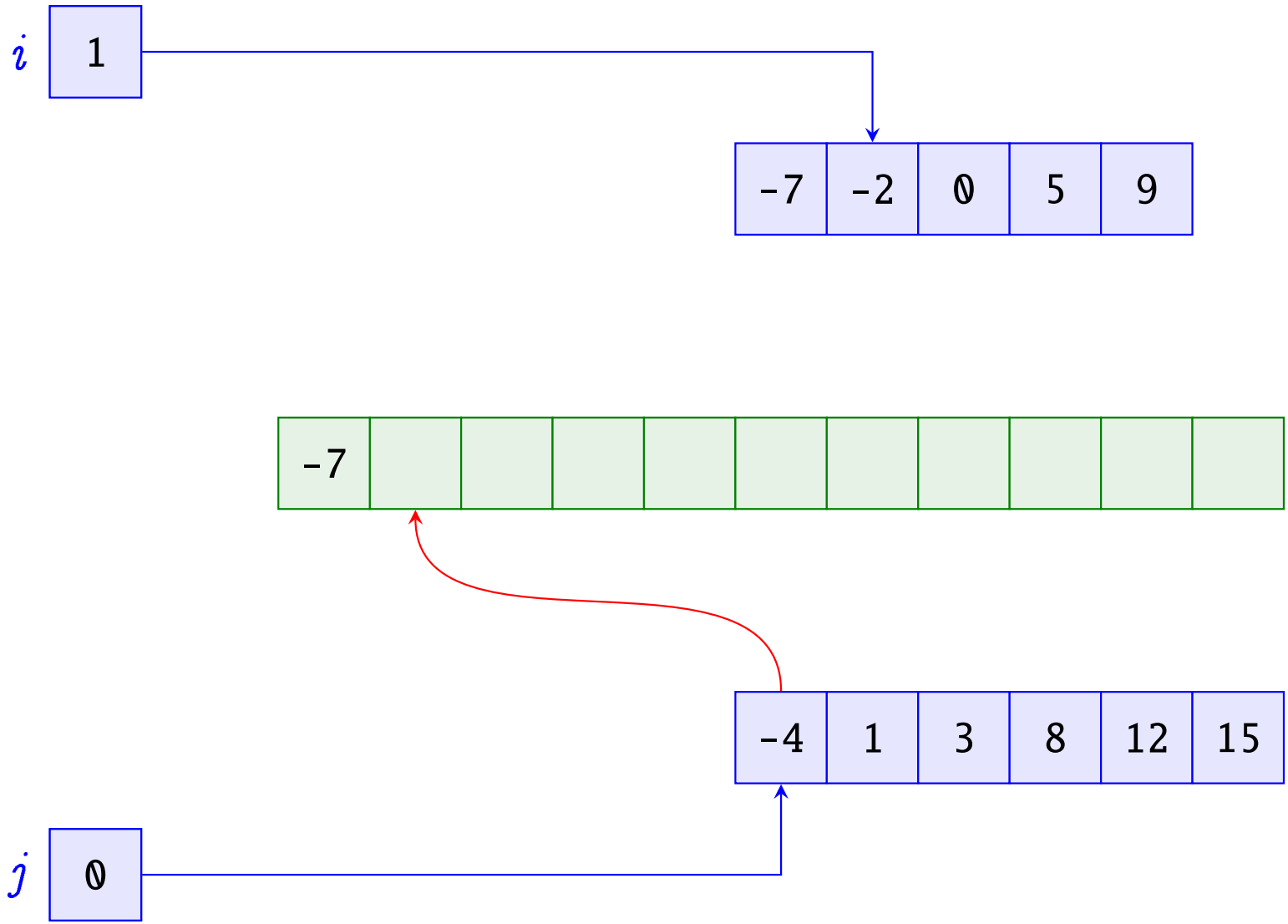

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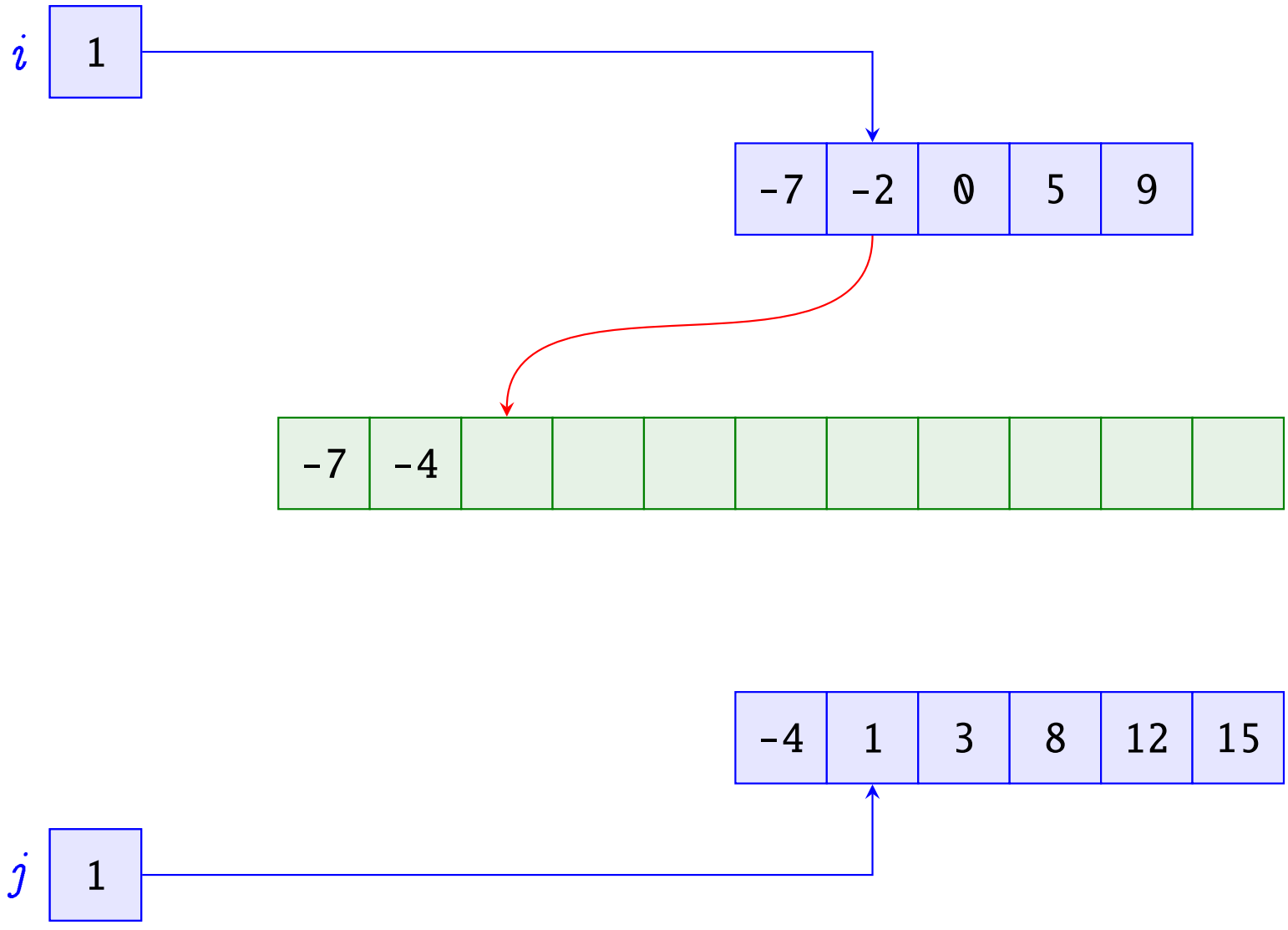

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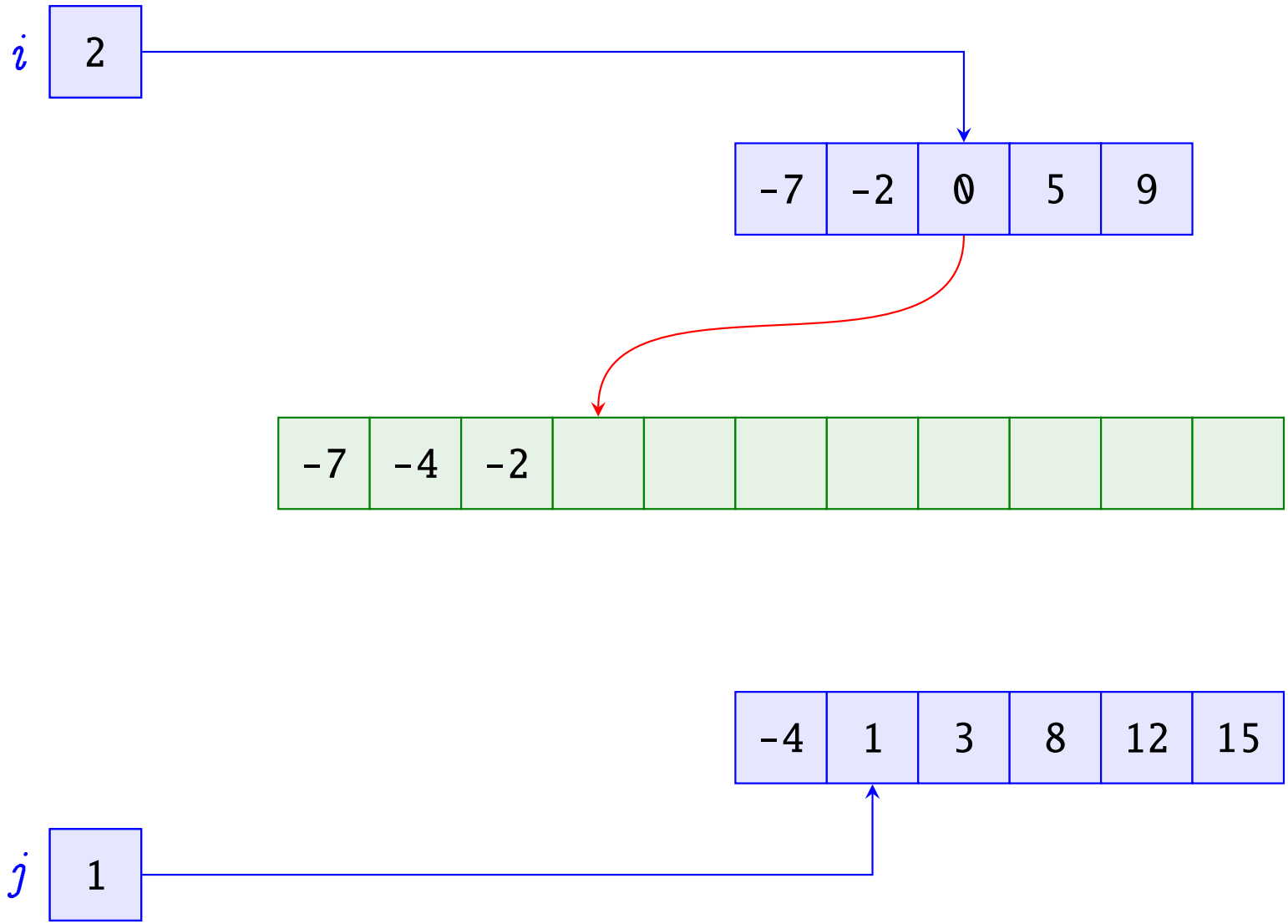

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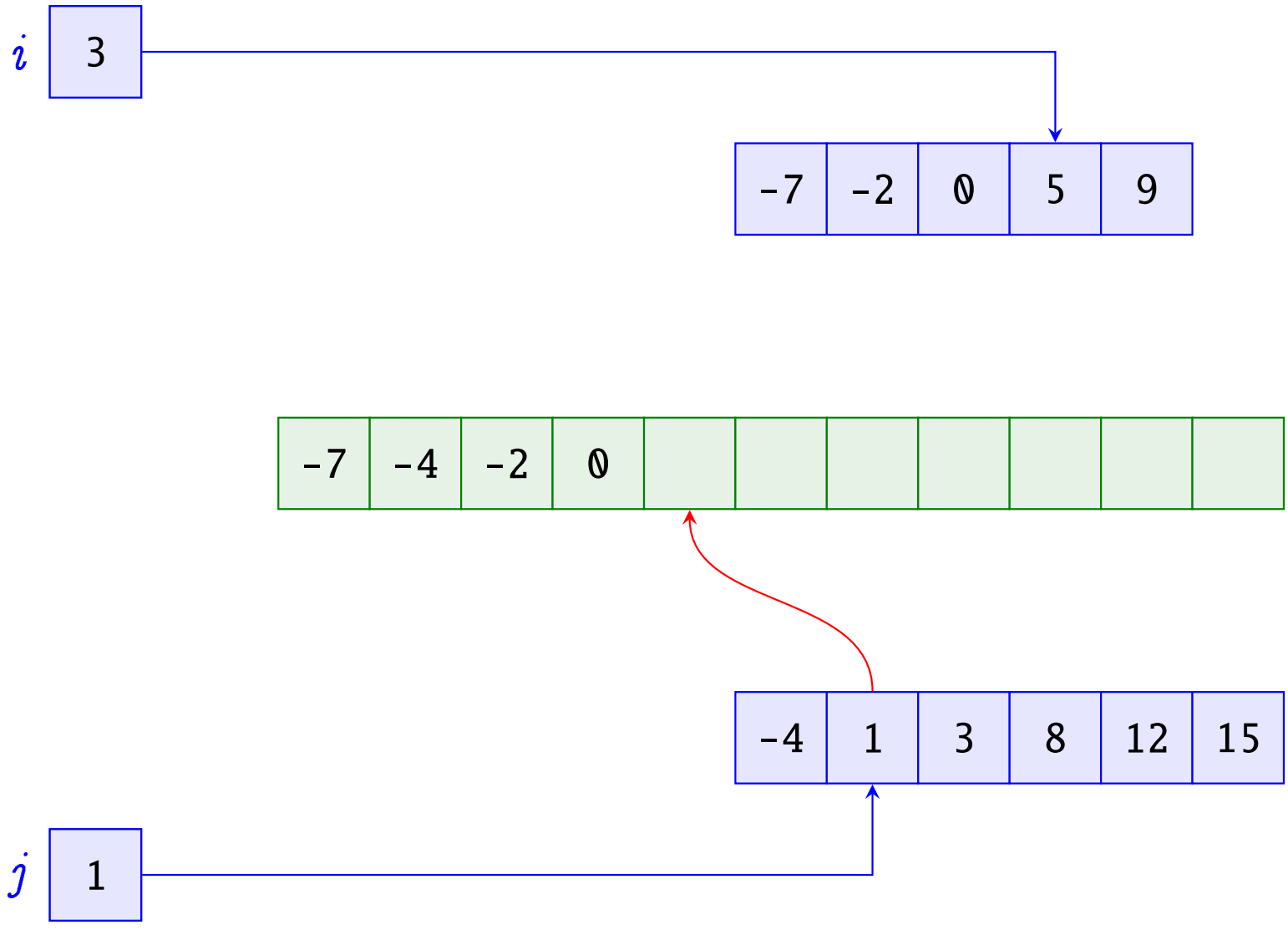

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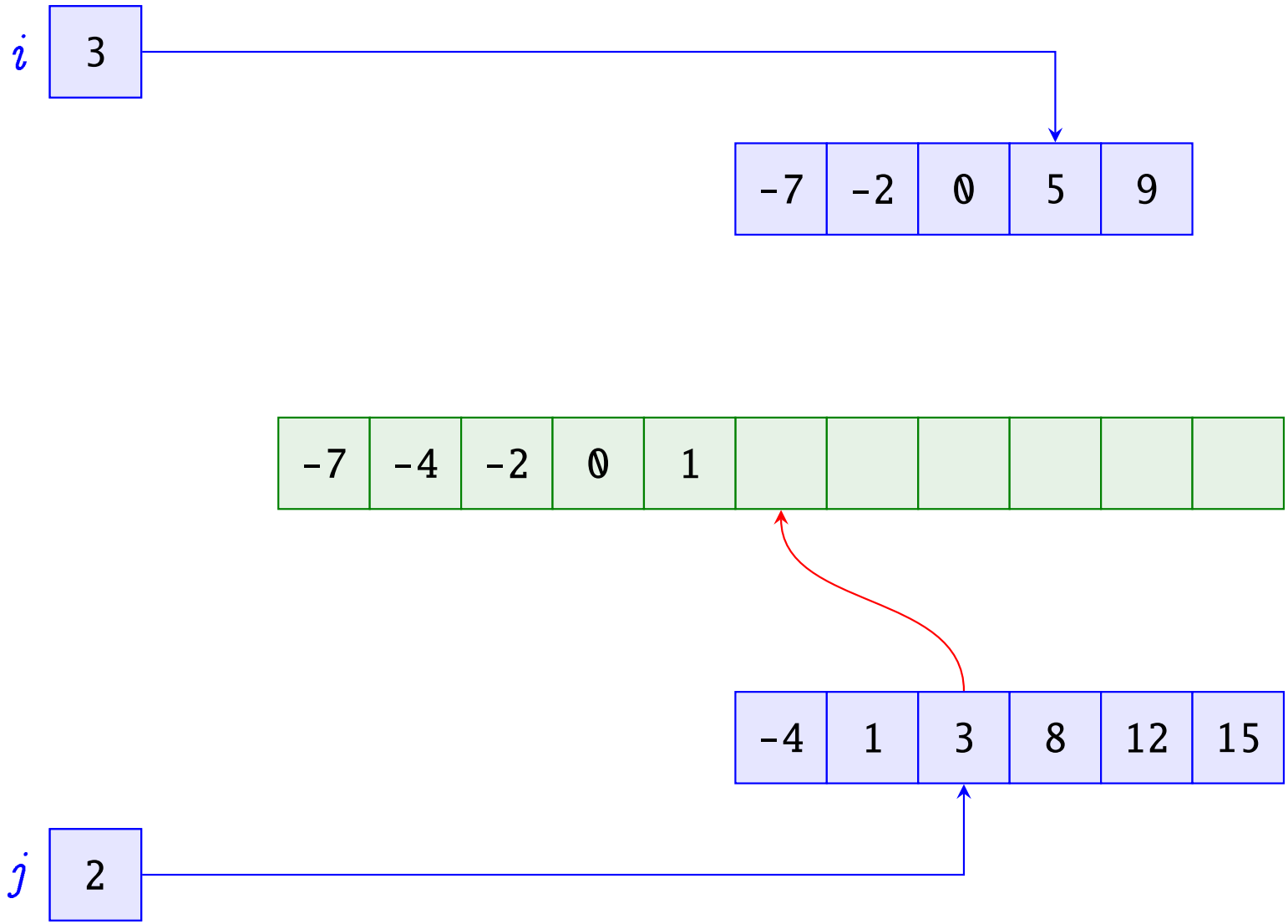

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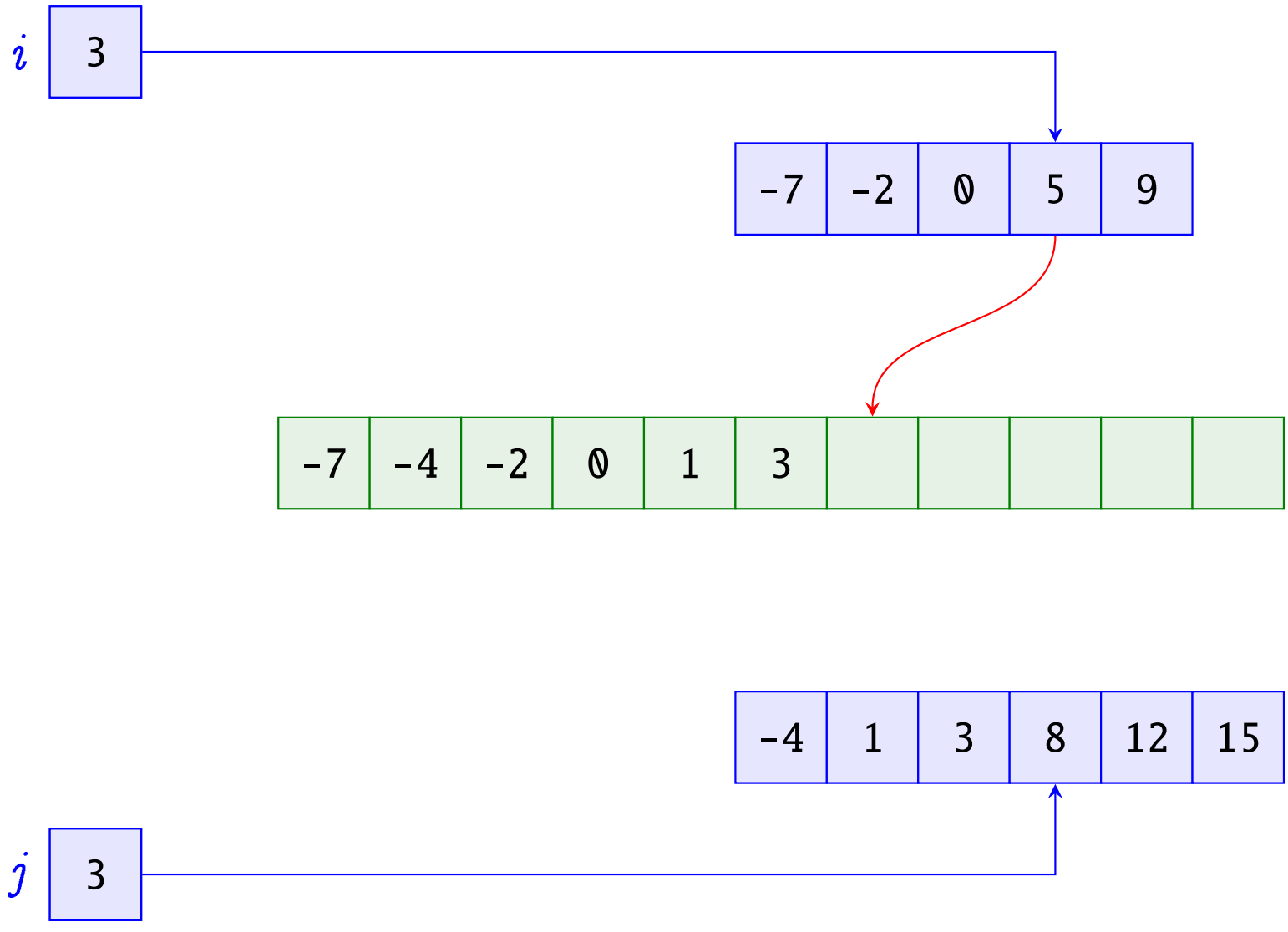

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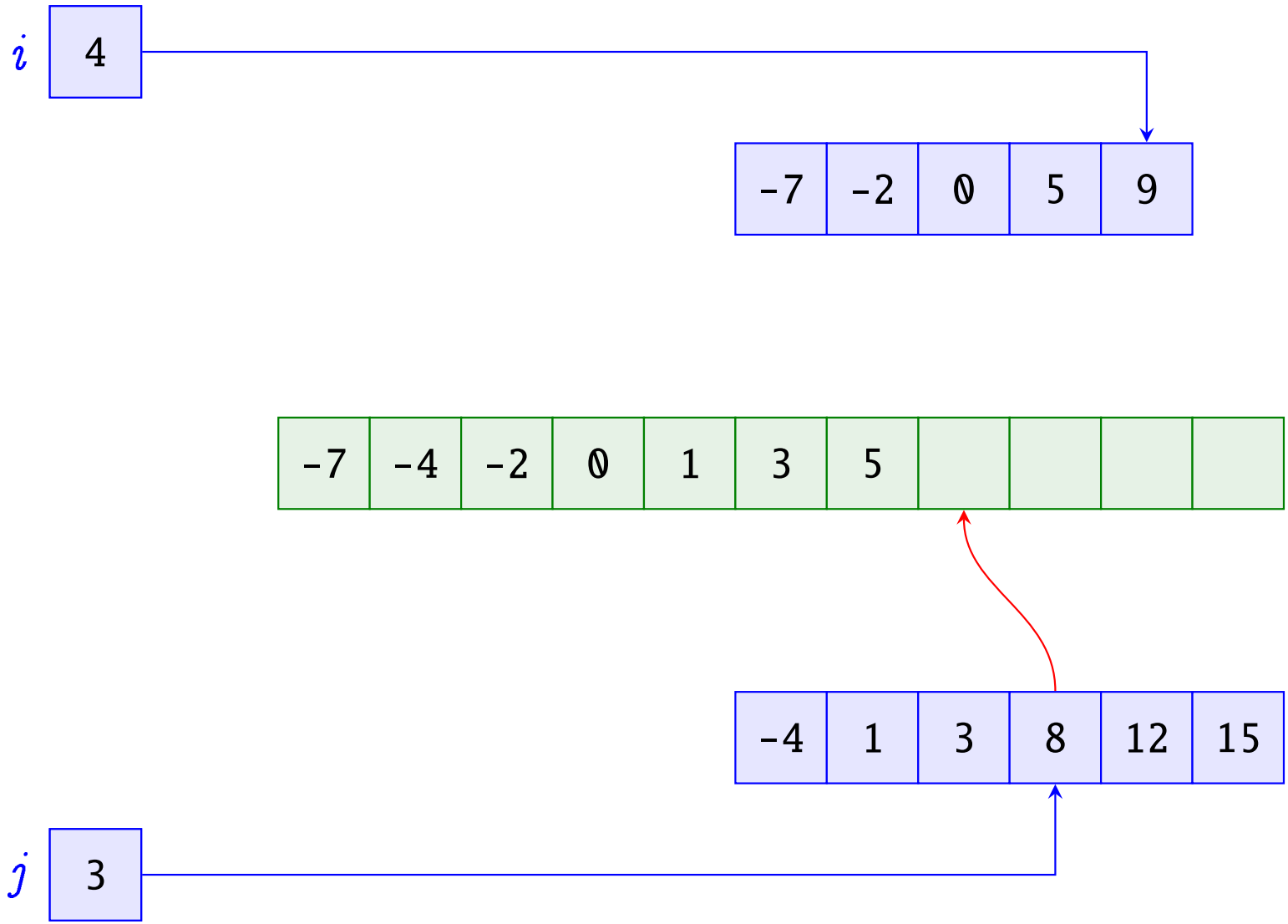

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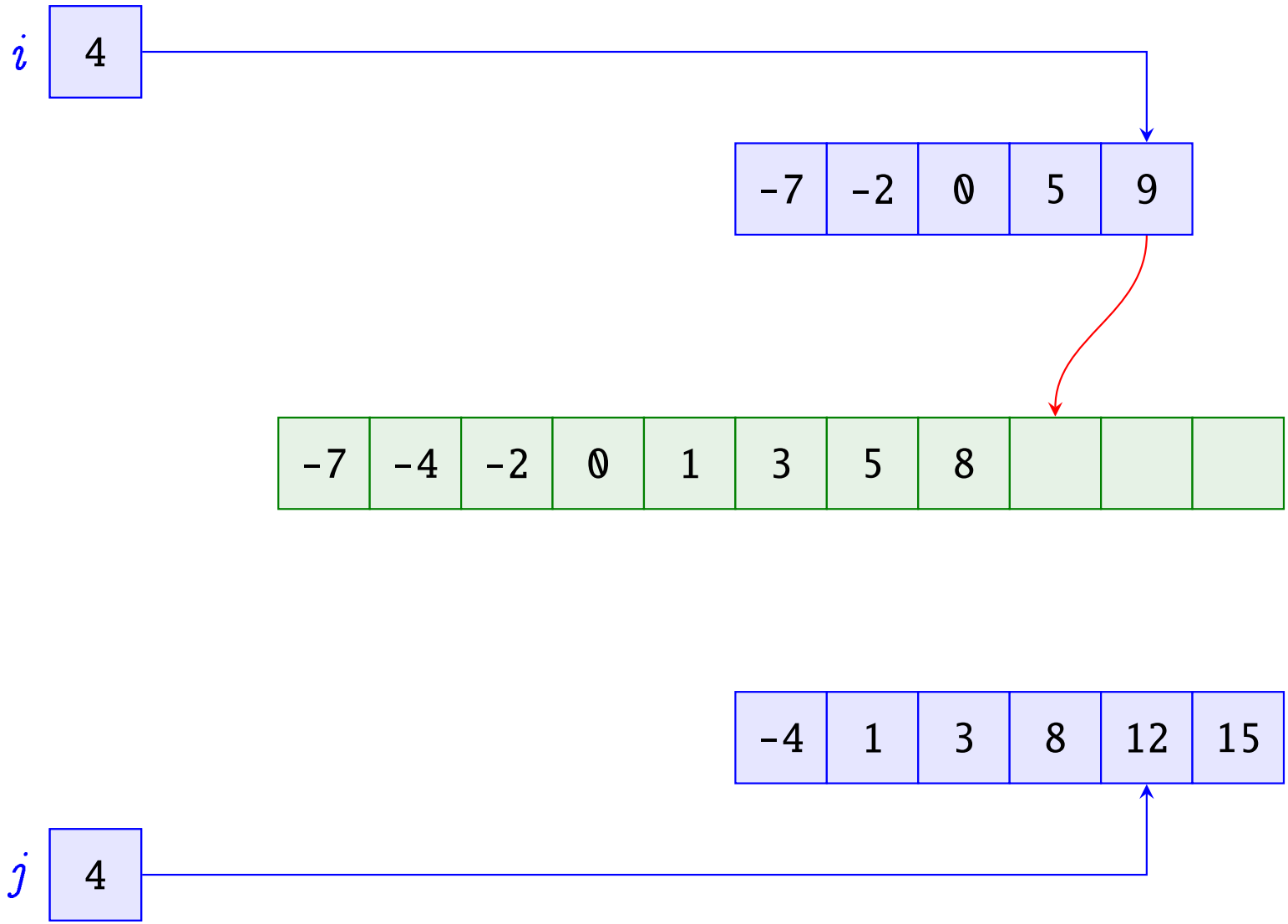

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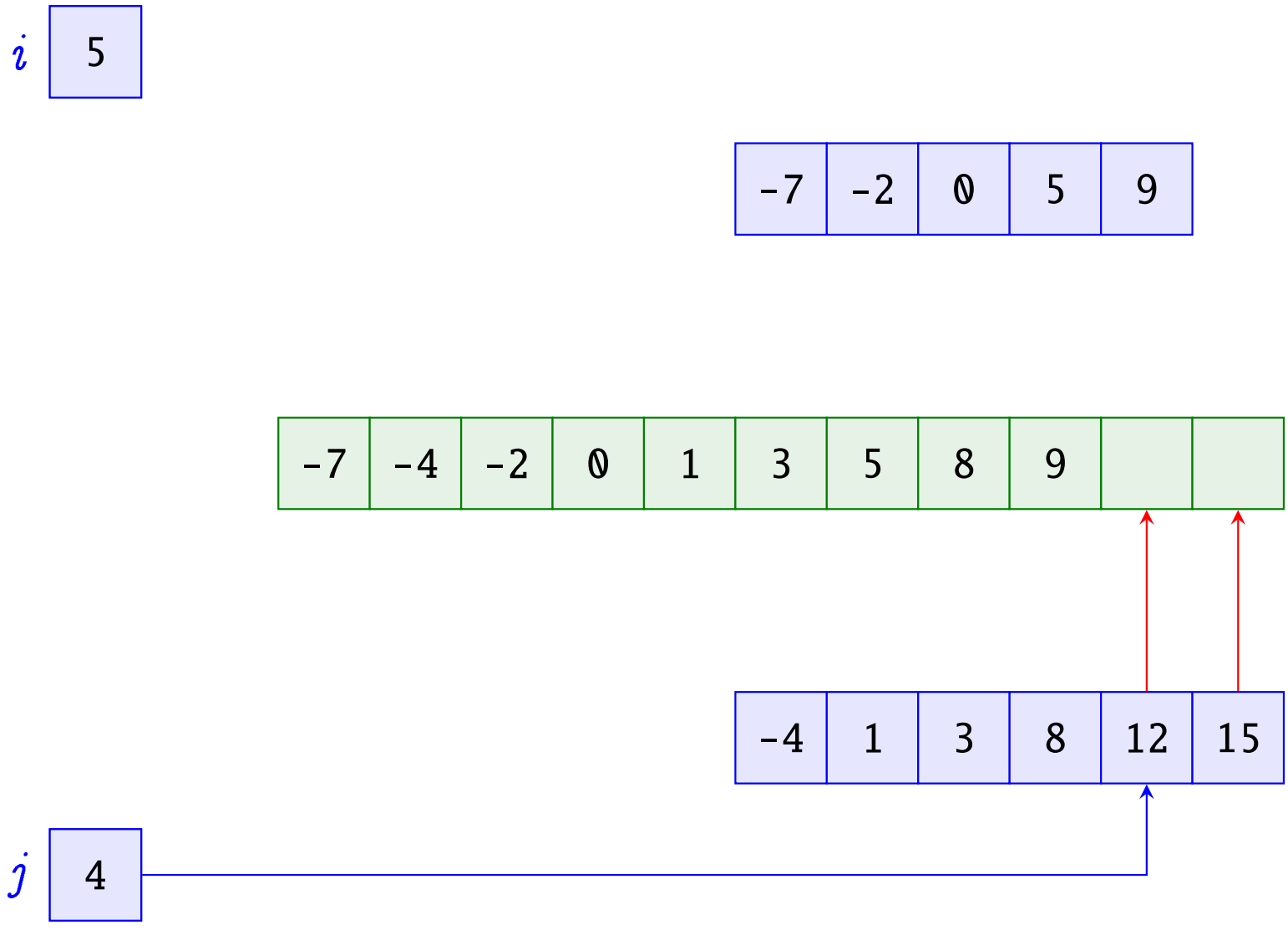

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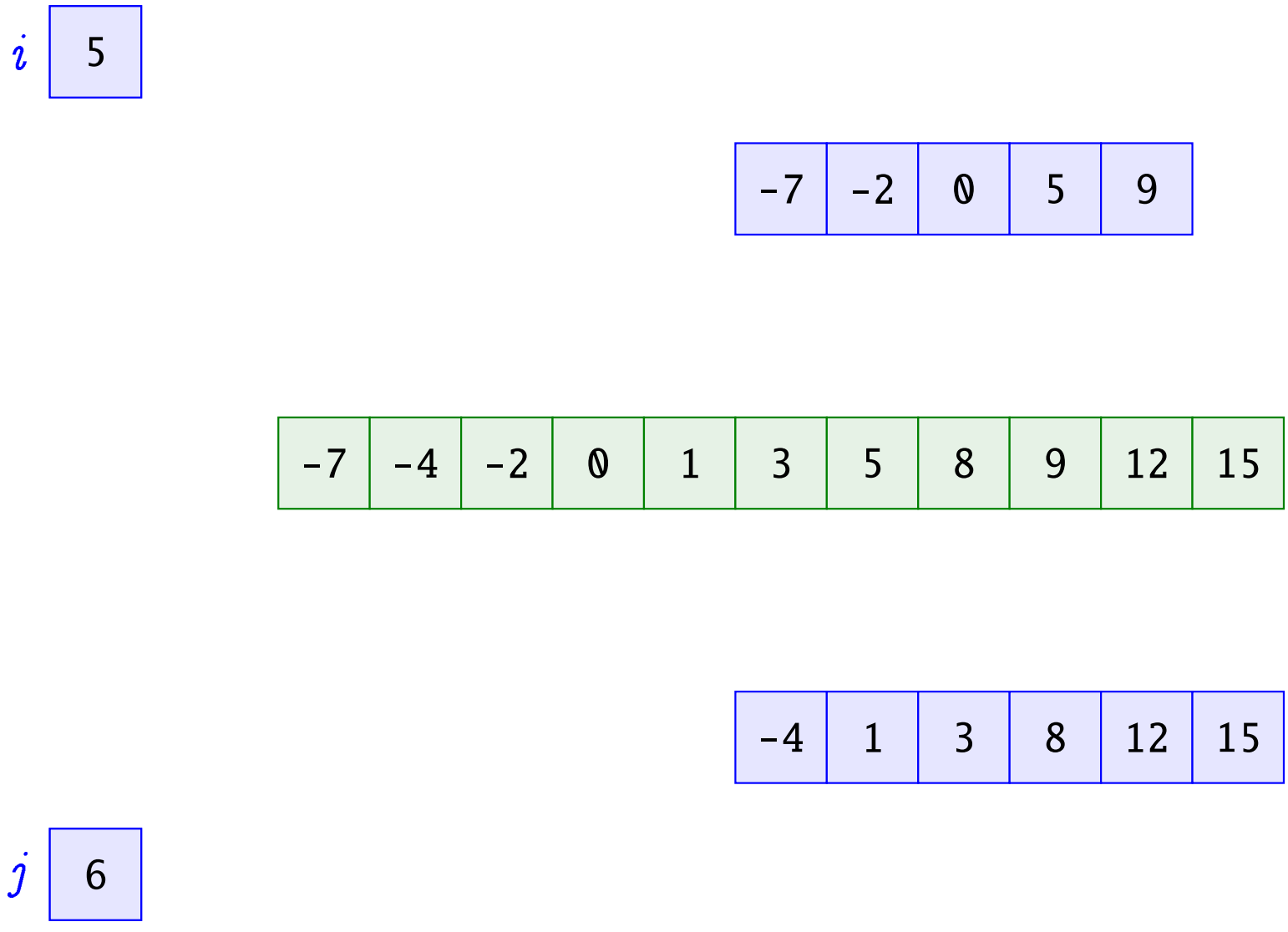

Ühildusmeetodil sorteerimine

- Kahe järjestatud massiivi ühildamine on lineaarne.
- Igal rekusiooni väljakutsepuu tasemel kõik vastavad alammassiivid lõikumatud.
- Seega on ka igal tasemel tehtav töö lineaarne.
- Kuna massiivid tükeldatakse alati pooleks, siis on väljakutsepuu sügavus $\log n$.
- Seega on ühildusmeetodi keerukus $\mathcal{O}(n \cdot \log(n))$.

Suur tänu osalemast

ja

kohtumiseni!