Weak BCK*-algebras and orthosemilattices

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A dual BCK-algebra, or BCK*-algebra is a poset \((A, \leq)\) having the greatest element \(1\) and considered together with a binary operation \(\rightarrow\) such that, for all \(x, y, z \in A\),

1. \(x \leq y\) iff \(x \rightarrow y = 1\),
2. \(x \leq (x \rightarrow y) \rightarrow y\),
3. \(x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z)\).

A BCK*-algebra is commutative if, one more axiom

1. \(x \rightarrow (x \rightarrow y) = y \rightarrow (y \rightarrow x)\).

A weak [commutative] BCK*-algebra satisfies (0), (1), [(3)] and a weaker axiom

1. \((2^\sim)\) if \(x \leq y\), then \(y \rightarrow z \leq x \rightarrow z\).

In the talk, commutative weak BCK*-algebras will be shown to possess many structural properties of commutative BCK*-algebras. Moreover, it will be shown that the class of commutative weak BCK*-algebras is definitionally equivalent to that of orthosemilattices [2], known also as semilattices with sectionally antitone involutions [1].

References
