Congruences of Morita equivalent categories

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Categories $\mathcal{A}$ and $\mathcal{B}$ are called **Morita equivalent** ([2]) if the functor categories $\text{Fun}(\mathcal{A}, \text{Set})$ and $\text{Fun}(\mathcal{B}, \text{Set})$ are equivalent.

**Definition 1** (See [1], p. 52) A **congruence** on a category $\mathcal{A}$ is a family $\rho = (\rho_{A, B})_{(A, B) \in \mathcal{A}^2}$ of equivalence relations $\rho_{A, B}$ on morphism sets $\mathcal{A}(A, B)$ that are compatible with the composition of morphisms.

**Theorem 1** If $\mathcal{A}$ and $\mathcal{B}$ are Morita equivalent small categories then there is an isomorphism $\Pi : \text{Con}(\mathcal{A}) \to \text{Con}(\mathcal{B})$ between their congruence lattices. Moreover, if $\rho \in \text{Con}(\mathcal{A})$ then $\mathcal{A}/\rho$ is Morita equivalent to $\mathcal{B}/\Pi(\rho)$.

**References**
