On the intersection graph of modules and rings

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For a given module $M$, the intersection graph $G(M)$ of $M$ is defined [1, 2] as the simple undirected graph whose set of vertices consists of non-trivial submodules of $M$ and two distinct submodules $N_1, N_2$ are adjacent if and only if $N_1 \cap N_2 \neq 0$. For a given ring $R$ the intersection graph of $R$ is defined as $G(RR)$, where $RR$ denotes the left $R$-module $R$. One of the first remarkable results in the area was obtained in [3], where an isomorphism problem on the intersection graph of finite abelian groups (regarded as modules over the ring of integers) was solved. From this result it in particular follows that if $A, B$ are finite abelian $p$-groups for a prime $p$, then $A \simeq B$ if and only if $G(A) \simeq G(B)$. This result does not extend to all modules or even to all finite abelian groups but it shows that properties of the intersection graph of modules carry quite important information on modules and can be applied to study their structure. The aim of the talk is to present some results on the subject obtained in [1, 2, 4].

References


