Workshop
“Algebra and its applications”

May 5-7, 2017

Taevaskoja Puhkekeskus
Taevaskoja village, Põlva county
Estonia

Abstracts
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Program

Friday, 05.05:

13:00 Lunch
13:50–14:00 Opening
14:00–18:00 Talks
   14:00 Sándor Radeleczki, *Congruence lattices of algebras on a finite set*
   15:00 Jouni Järvinen, *Representing regular Kleene algebras in terms of rough sets*
   16:00 Coffee break
   16:30 Mati Abel, *When the division topological algebra is trivial*
   17:00 Mart Abel, *Topological Segal algebras*
   17:30 Reyna María Pérez-Tiscareño, *About locally m-convex algebras with dense finitely generated ideals*
18:00 Dinner
19:00 Round Table

Saturday, 06.05:

8:00 Breakfast
9:00–12:15 Talks
   9:00 Kalle Kaarli, *On categorical equivalence of finite commutative rings*
   9:30 Peeter Puusemp, *Some results on endomorphism semigroups of groups*
   10:30 Coffee break
   10:45 Jānis Cīrulīs, *On focal Baer semigroups*
   11:45 Insa Cremer, *Rickart rings and skew nearlattices*
12:30 Lunch
13:30–15:45 Talks
   13:30 Tarmo Uustalu *Interaction morphisms and Turing computation*
   14:00 Niccolò Veltri, *Coherence for skew near-semiring categories*
   14:30 Coffee break
   14:45 Silvio Capobianco, *Cellular automata on Cayley graphs: an overview*
16:00-19:00 Free time
19:00 Dinner
19:30 Short Excursion
Sunday, 7.05:

8:00 Breakfast
9:00–12:15 Talks
   9:00 Peteris Daugulis, *Classifying nonequivalent presentations of finite groups*
      - some special cases and series
   9:55 Valdis Laan, *Fair semigroups*
   10:20 Lauri Tart, *On Morita equivalence of fair semigroups*
   10:45 Coffee break
   10:55 Laur Tooming, *Morita invariants of semirings*
   11:25 Ülo Reimaa, *Morita theorems for firm categorical semigroups*
11:55 Closing
12:00 Lunch
Segal algebras have been in use for more than 50 years, starting in the context of subalgebras of $L^1(G)$ with $G$ a locally compact group and settling in the context of Banach algebras about 20 years ago. During the last 5 years, the scope of Segal algebras has broadened to the classes of Fréchet or locally multiplicatively convex (topological) algebras. In this talk we offer an overview of the different classes of Segal algebras used before and propose a generalisation of the definition of Segal algebra to the class of arbitrary (real or complex) topological algebras, introduced in [1] and [2], with some recent results.

References


When the division topological algebra is trivial

Mati Abel
University of Tartu
Tartu, Estonia

Conditions, that a topological division algebra $A$ over $\mathbb{C}$ is trivial (that is, $A = \mathbb{C}e$, where $e$ is the unit element of $A$), are given. A new model of the micro world by this result is introduced.
A cellular automaton (CA) on a group $G$ is an endomorphism of the dynamical system $(S^G, \sigma)$ where $S$ is a finite set of states and $\sigma$ is the action of $G$ on the space $S^G$ of configurations (global states) by left multiplication. A famous theorem by Curtis, Lyndon and Hedlund characterizes CA as those transformations of $S^G$ obtainable by synchronous update of identical finite-state automata, whose input alphabet is the set of patterns (local states) on a neighborhood defined by finitely many fixed offsets.

In this expository talk we illustrate the basics of classical cellular automata theory and discuss how some of the most famous results depend on specific properties of the underlying groups. A special focus will be given to the Garden of Eden theorem, which states that, for CA on a specific class of groups, surjectivity is mutually exclusive with the ability to correct finitely many errors in finite time.

References


We aim to give some general information about Baer semigroups, to specify a subclass of them, and to show that several significant characteristics of star-ordered Rickart *-rings can be transferred to semigroups in this subclass. A focal Baer semigroup $S$ is a semigroup with 0 expanded by two unary idempotent-valued operations, $\overline{\cdot}$ and $\overline{\cdot}'$, such that the left (right) ideal generated by $x\overline{\cdot}$ (resp., $x\overline{\cdot}'$) is the left (resp., right) annihilator of $x$. $S$ is said to be equifocal if the ranges of both operations coincide and $p\overline{\cdot}=p\overline{\cdot}'$ for every $p$ from the common range $P$. Such a semigroup is shown to be $P$-semiabundant. If $S$ is also Lawson reduced, then $P$ is an orthomodular lattice (under the standard order of idempotents), and a modified version of Drazin’s star partial order turns out to be the natural order on $S$. Under this order, $S$ is a partial lower semilattice, and every initial segment of $S$ is order isomorphic to a subortholattice of $P$. 
Sussman and Subrahmanyam proved in [1] and [2] that a certain kind of reduced ring (called \textit{m-domain ring} in [2]) can be decomposed into a collection of disjoint subsets which are closed with respect to multiplication. In [3] it is shown that reduced Rickart rings and m-domain rings are the same thing. This talk is about the order structure of a reduced Rickart ring’s decomposition into disjoint semigroups.

Cīrulis proved in [4] that every right normal skew nearlattice can be regarded as a structure called a \textit{strong semilattice of semigroups}, and in [5] he shows that any reduced Rickart ring admits the structure of a right normal skew nearlattice. It turns out that this strong semilattice of semigroups arises from the semigroup decomposition of [2].

References


Classifying nonequivalent presentations of finite groups - some special cases and series

Peteris Daugulis
Daugavpils University
Daugavpils, Latvia

The problem of classifying equivalence classes of presentations up to isomorphism of Cayley graphs is considered in this talk. We give definitions and discuss computational results. We find all equivalence classes of presentations of dicyclic groups having two generators. An innovative technique for solving such problems is used. These results may be used in characterizing group structure and properties.

References


Representing regular Kleene algebras in terms of rough sets

Jouni Järvinen
Turku, Finland

This talk describes a joint work with Sándor Radeleczki [1].

A collection $\mathcal{H}$ of nonempty subsets of $U$ is called a covering of $U$ if $\bigcup \mathcal{H} = U$. A covering $\mathcal{H}$ is irredundant if $\mathcal{H} \setminus \{X\}$ is not a covering for any $X \in \mathcal{H}$. A tolerance is a reflexive and symmetric binary relation. Each covering $\mathcal{H}$ induces a tolerance $R_{\mathcal{H}} = \bigcup \{X^2 \mid X \in \mathcal{H}\}$.

For any binary relation $R$ on $U$, the lower approximation of a subset $X$ of $U$ is $X^\downarrow = \{x \in U \mid R(x) \subseteq X\}$ and $X$’s upper approximation is $X^\uparrow = \{x \in U \mid R(x) \cap X \neq \emptyset\}$, where $R(x) = \{y \in U \mid x Ry\}$. The set of rough sets is $\mathcal{RS} = \{(X^\downarrow, X^\uparrow) \mid X \subseteq U\}$.

A De Morgan algebra $(L, \lor, \land, \sim, 0, 1)$ is a bounded distributive lattice with an operation $\sim$ satisfying $\sim x = x$ and $x \leq y$ iff $\sim y \leq \sim x$. A Kleene algebra is a De Morgan algebra in which $x \land \sim x \leq y \lor \sim y$ holds.

A double pseudocomplemented lattice $(L, \lor, \land, *, +, 0, 1)$ is called regular if $x^* = y^*$ and $x^+ = y^+$ imply $x = y$. If a De Morgan algebra is such that its underlying lattice is pseudocomplemented, then it forms a double pseudocomplemented lattice where $x^+ = \sim(\sim x)^*$. We say that a De Morgan algebra (or a Kleene algebra) is regular if its underlying lattice is a regular double pseudocomplemented lattice. Note that a De Morgan algebra defined on an algebraic lattice is always a double pseudocomplemented lattice.

It is known that if $\mathcal{RS}$ is determined by a tolerance induced by an irredundant covering, then $\mathcal{RS}$ forms an algebraic lattice and determines a regular Kleene algebra. We show how any regular Kleene algebra defined on an algebraic lattice is isomorphic to a rough set Kleene algebra defined by a tolerance induced by an irredundant covering.

References

On categorical equivalence of finite commutative rings

Kalle Kaarli
University of Tartu
Tartu, Estonia

This is a joint research with Tamás Waldhauser (Szeged, Hungary).

The (universal) algebras $A$ and $B$ are said to be \textit{categorically equivalent} if there is an equivalence functor $F$ between varieties generated by them such that $F(A) = B$. The algebras $A$ and $B$ are said to be \textit{term equivalent} if they have the same universe and their clones of term functions coincide. It is easy to see that term equivalent algebras are categorically equivalent.

A well known result by C. Bergman and J. Berman says that for any primes $p$ and $q$ and any natural number $n$, the Galois fields $GF(p^n)$ and $GF(q^n)$ are categorically equivalent [1]. We proved in [2] that in the case of finite rings semisimplicity is a categorical property. Moreover, we showed that if two finite semisimple rings are categorically equivalent then this follows from the aforenamed result of C. Bergman and J. Berman.

On the other hand, in [2] we could not find non-trivial examples of categorically equivalent finite non-semisimple rings. We proved that if two finite non-semisimple rings of prime power characteristic are categorically equivalent then their characteristics coincide.

In the present work we have focused on the commutative case. Our basic results are the following.

1. Two categorically equivalent finite commutative rings of the same prime power characteristic have isomorphic additive groups. Thus, in particular, they have the same order.

2. For every odd prime $p$ there exist non-isomorphic but term equivalent rings of order $p^3$.

3. Let $R$ and $S$ be categorically equivalent finite commutative rings of prime characteristic $p$ such that $R$ is generated by one element over a subfield of $R$. Then $R$ and $S$ are isomorphic.

\textbf{Problem} Is it true that categorically equivalent finite commutative non-semisimple rings with the same universe are necessarily term equivalent?

\textbf{References}


Fair semigroups

Valdis Laan
University of Tartu
Tartu, Estonia

Fair semigroups are non-additive analogues of xst-rings, introduced by Xu, Shum and Turner-Smith [2].

If $S$ is a semigroup then a right $S$-act $A_S$ is called unitary if $A_S = A$. We say that a semigroup $S$ is a right fair semigroup (see [1]) if every subact of a unitary right $S$-act is unitary. One defines left fair semigroups dually. By a fair semigroup we mean a semigroup which is both left and right fair.

It turns out that a semigroup $S$ is right fair if and only if for every sequence $(s_i)_{i \in \mathbb{N}} \in S^{\mathbb{N}}$ of elements of $S$ there exist $n \in \mathbb{N}$ and $u \in S$ such that

$$s_n \ldots s_2 s_1 u = s_n \ldots s_2 s_1.$$

We will give a list of examples of fair semigroups and some basic facts about them.

This talk is based on joint research with László Márki.

References


About locally $m$-convex algebras with dense finitely generated ideals

Reyna María Pérez-Tiscareño
University of Tartu
Tartu, Estonia

It is known, as a consequence of a theorem of Richard Arens, that a commutative Fréchet locally $m$-convex algebra $E$ with unit, does not have dense finitely generated ideals. In this talk we shall see that this result can fail to hold if $E$ is not complete and metrizable.

Some results on endomorphism semigroups of groups

Peeter Puusemp
Tallinn University of Technology
Tallinn, Estonia

I consider the following problems on endomorphism semigroups of groups:

- endomorphism semigroups of Abelian groups;
- some embedding theorems in connection with endomorphism semigroups of groups;
- endomorphism semigroups of wreath products of groups.
This is joint work with R. Pöschel and D. Jakubíková-Studenovská.

The congruence lattices of all algebras defined on a fixed finite set $A$ ordered by inclusion form a finite atomistic lattice $E$. We describe the atoms and coatoms of this lattice, based on some results of [1] and [2]. Using the description of some particular meet-irreducible elements we deduce several properties of the lattice $E$; in particular, we prove that $E$ is tolerance-simple whenever $|A| \geq 4$.

References


The Morita theory of rings can be thought of as the study of equivalences between categories of modules, or equivalently, as the study of invertible bimodules.

The the additive structure of rings is not necessary to get a nice theory of Morita equivalence. When we forget the additive structure, we get the Morita theory of monoids. Indeed, a good theory of Morita equivalence is known to exist for categorical monoids, meaning monoids on objects $M$ of some category $C$, with multiplication maps $M \otimes M \to M$, where $- \otimes -$ is a (tensor) product functor on $C$. The special case of rings arises when $- \otimes -$ is the tensor product functor on the category of abelian groups.

We outline how to get a similar theory of Morita equivalence for firm categorical semigroups, which are categorical semigroups that can act as a unit with respect to the tensor product of bimodules. We will illustrate the results with examples from topology.
On Morita equivalence of fair semigroups

Lauri Tart
University of Tartu
Tartu, Estonia

Let $S$ be a semigroup. A right $S$-act $A_S$ is called unitary if $AS = A$. A semigroup $S$ is called a right fair semigroup (see [1]) if every subact of a unitary right $S$-act is unitary. Left fair semigroups are defined dually and a fair semigroup is one that is both left and right fair. Every fair semigroup $S$ contains a subsemigroup $S'$ with weak local units, called its unitary part.

Semigroups $S$ and $T$ are called strongly Morita equivalent if there are biacts $SP_T$ and $TQ_S$ and morphisms $\theta : S(P \otimes Q)_S \to SS_S$ and $\phi : T(Q \otimes P)_T \to TT_T$ with certain ‘nice’ properties.

A right $S$-act $A_S$ is firm if the mapping $\mu_A : A \otimes S S \to A, \ a \otimes s \mapsto as$ is bijective. An act $A_S$ over a fair semigroup $S$ is called strongly firm if the mapping $\mu_A : A \otimes S S' \to A, \ a \otimes z \mapsto az$ is bijective.

In [1], fair semigroups were called right Morita equivalent if their categories of firm acts are equivalent. They were able to show that right fair semigroups $S$ and $T$, where $S'$ and $T'$ have common weak right local units, are right Morita equivalent if and only if $S'$ and $T'$ are strongly Morita equivalent.

We call fair semigroups $S$ and $T$ right Morita equivalent if their categories of strongly firm acts are equivalent. Mirroring the approach of [2], we aim to show that right fair semigroups without any additional conditions are right Morita equivalent in this new sense if and only if $S'$ and $T'$ are strongly Morita equivalent. We have some promising partial results, but it is too early to tell whether this approach works without imposing any extra assumptions altogether.

This talk is based on joint research with Valdis Laan.

References


A *semiring* is an algebraic structure similar to a ring but without additive inverses. There is a zero element, but there need not be a unity. Structures analogous to modules over rings are called *semimodules*.

A semiring $S$ has *weak local units* if for every $s \in S$, $s \in sS$ and $s \in Ss$, and *common joint weak local units* if for all $s, s' \in S$ there exist $u, v \in S$ such that $s = usv$ and $s' = us'v$. A left semimodule $SM$ over $S$ is *unitary* if $SM = M$.

A *quantale* is a complete lattice with an additional binary operation (multiplication) which is distributive over joins of any cardinality. The ideals of a semiring (defined, together with the product of two ideals, as in ring theory) form a quantale.

The concept of Morita equivalence is known classically from ring theory. It has been studied for semirings with a unit in [1]. We consider Morita equivalence on semirings with weak local units, using as basis the work of [2] on *semigroups* with weak local units.

We define *strong Morita equivalence* of two semirings $S$ and $T$ as the existence of a Morita context $(S, T, SP_T, TQ_S, \theta, \phi)$ (defined as in ring and semigroup theory), where the bisemimodules $SP_T$ and $TQ_S$ are unitary (from both sides), and the mappings $\theta$ and $\phi$ are surjective. We show that:

- two strongly Morita equivalent semigroups with weak local units have isomorphic quantales of ideals, with finitely generated ideals mapping to finitely generated ideals;
- two strongly Morita equivalent semigroups with common joint weak local units have isomorphic congruence lattices.

**References**


I will introduce interaction morphisms as a means to specify how an effectful (e.g., non-deterministic, interactive I/O or stateful) computation is to be run on an abstract state machine. An interaction morphism is given by a monad $T = (T, \eta, \mu)$ and a comonad $D = (D, \varepsilon, \delta)$ on a Cartesian category with a family of maps $\psi_{X,Y} : TX \times DY \rightarrow X \times Y$ natural in $X$ and $Y$ and agreeing suitably with $\eta, \varepsilon, \mu, \delta$. Intuitively, $\psi_{X,Y}$ takes a computation and a behavior from an initial state and sends them into a return value and a final state. Interaction morphisms enjoy neat properties: they are the same as monoids in a certain monoidal category; interaction morphisms of $T$ and $D$ are in a bijective correspondence with carrier-preserving functors between the categories of coalgebras of $D$ and stateful runners of $T$ (monad morphisms from $T$ to state monads); they are also in a bijective correspondence with monad morphisms from $T$ to a monad induced in a certain way by $D$.

I will illustrate interaction morphisms on the example of Turing computation, i.e., computations interacting with a readable, writeable, walkable bi-infinite tape storing symbols from a finite alphabet. Describing the monad and comonad concretely is an instructive exercise in this case.

The work on interaction morphisms continues my earlier work [2] on stateful runners and is joint with Shin-ya Katsumata (Kyoto University). Turing computation was studied in a related setting by Goncharov et al. [1].

References


Coherence for skew near-semiring categories

Niccolò Veltri
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A near-semiring category is a category with two monoidal structures, \((Z, \oplus)\) and \((I, \otimes)\), where \(\otimes\) distributes over \(Z\) and \(\oplus\) from the right. Distributivity from the left is not required and neither is commutativity of \(\oplus\). In a skew near-semiring category, the unitors, associators and distributors are just natural transformations rather than isomorphisms.

Uustalu [2] showed that skew monoidal categories satisfy a coherence theorem similar to MacLane’s for monoidal categories. The free skew monoidal category \(\text{Tm}_X\) over a set of objects \(X\) has the free strict monoidal category \(\text{Nf}_X\) over \(X\) as a reflective subcategory of \(\text{Tm}_X\). The category \(\text{Nf}_X\) is discrete and its objects are lists over \(X\). From this it follows that there exists exactly one map between an object and its normal form.

We extend that theorem to skew near-semiring categories. Here the objects of \(\text{Nf}_X\) are given by the grammar \(\text{Nf}_X := Z \mid (X \otimes \text{Nf}_X) \oplus \text{Nf}_X \mid I \oplus \text{Nf}_X\).

Rivas et al. [1] used near-semiring objects in near-semiring categories as an axiomatization of Haskell type-classes MonadPlus and Alternative. Relevant for them were categories of endofunctors with composition or Day convolution multiplicative structure and product additive structure. Switching to general functor categories skews the multiplicative structure.

This talk is based on joint work with Mauro Jaskelioff, Exequiel Rivas and Tarmo Uustalu.

References
