G. Czédli's tolerance factor lattice construction and weak ordered relations

Gábor Czédli proved that the blocks of any compatible tolerance T of a lattice L can be ordered in such a way that they form a lattice L/T called the factor lattice of L modulo T. Here we show that the Dedekind-MacNeille completion of the lattice L/T is isomorphic to the concept lattice of the context (L, L, R), where R stands for the reflexive weak ordered relation $T \circ \leq$. Weak ordered relations constitute the generalization of the ordered relations introduced by S. Valentini. Reflexive weak ordered relations can be characterized as compatible reflexive relations $R \subseteq L^2$ satisfying $R = R \circ \leq$.