



A brief introduction to chaotic strings

Stefan Groote

Institute of Physics, Tartu University

What is a chaotic string?



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- mathematical object with ...



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- mathematical object with ...
- ... 1 cyclic dimension i



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- mathematical object with ...
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- ... 1 fictitious time parameter n



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described by iteration

$$\Phi_{n+1}^i = (1 - a)T_N(\Phi_n^i) + s\frac{a}{2} \left(T_N^b(\Phi_n^{i-1}) + T_N^b(\Phi_n^{i+1}) \right)$$

with $T_N^0(\Phi) = \Phi$, $T_N^1(\Phi) = T_N(\Phi)$, where $T_N(\Phi)$ is N -th order Chebychev polynomial, $s = \pm 1$.



Chebyshev polynomials

$$T_1(\Phi) = \Phi$$



Chebyshev polynomials

$$T_2(\Phi) = 2\Phi^2 - 1$$



Chebyshev polynomials

$$T_3(\Phi) = 4\Phi^3 - 3\Phi \quad \dots$$



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- ... semi-conjugated to a Bernoulli shift with N symbols (shift of information)

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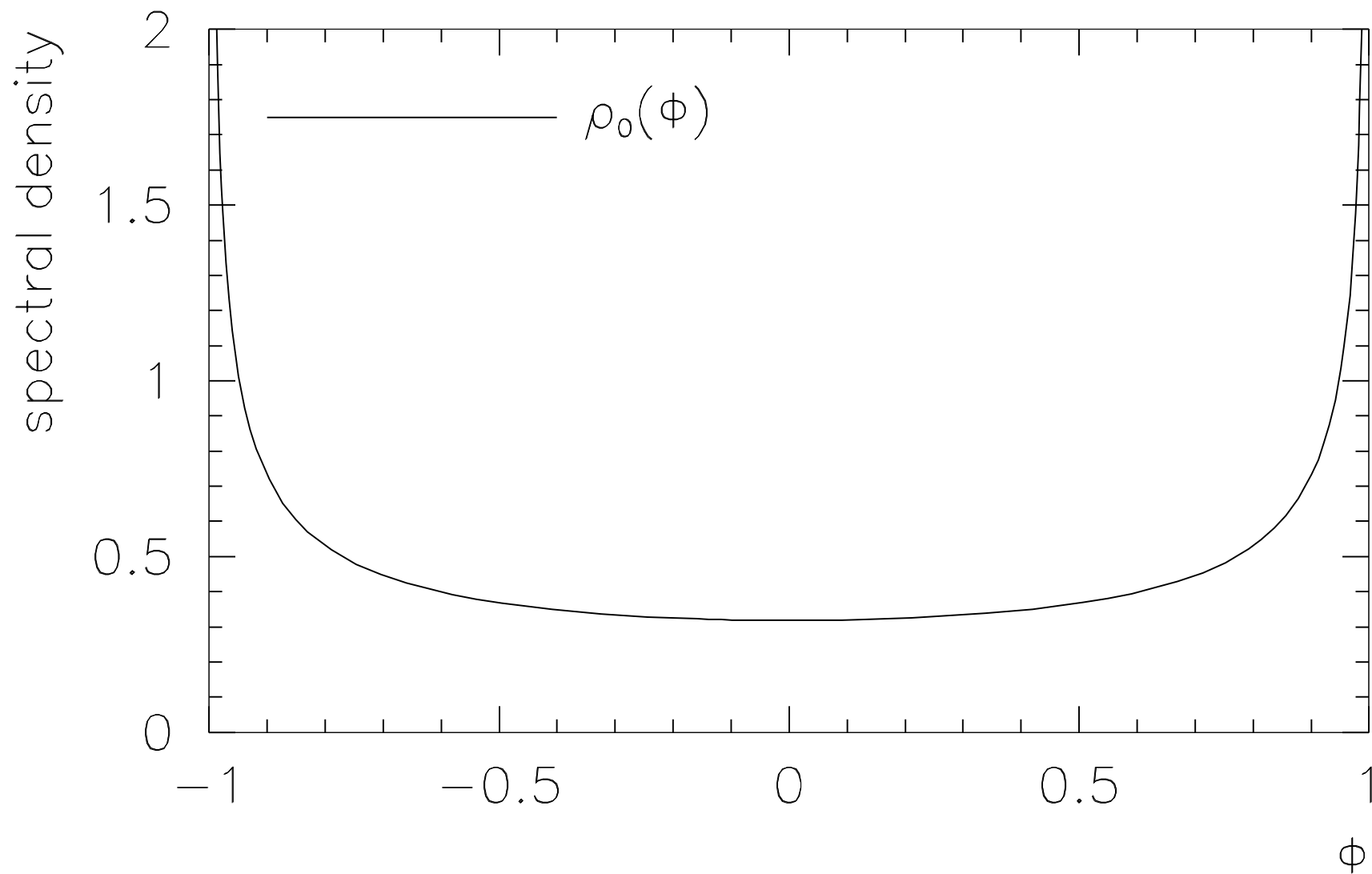
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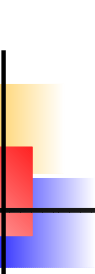
- ... semi-conjugated to a Bernoulli shift with N symbols (shift of information)
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Long term iteration leads to invariant density

$$\rho_0(\Phi) = \frac{1}{\pi \sqrt{1 - \Phi^2}}$$

Invariant density





Symmetry and classification

Laplacian coupling of the chaotic string

$$\Phi_{n+1}^i = (1 - a)T_N(\Phi_n^i) + s\frac{a}{2} \left(T_N^b(\Phi_n^{i-1}) + T_N^b(\Phi_n^{i+1}) \right)$$

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- NB string ($b = 0, s = +1, T_N^0(\Phi) = \Phi$)
diffusive backward coupling ($2B, 3B, \dots$)

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- NB string ($b = 0, s = +1, T_N^0(\Phi) = \Phi$)
diffusive backward coupling ($2B, 3B, \dots$)
- NA^-/NB^- string ($s = -1$, only for even N)
antidiffusive coupling ($2A^-, 2B^-, \dots$)

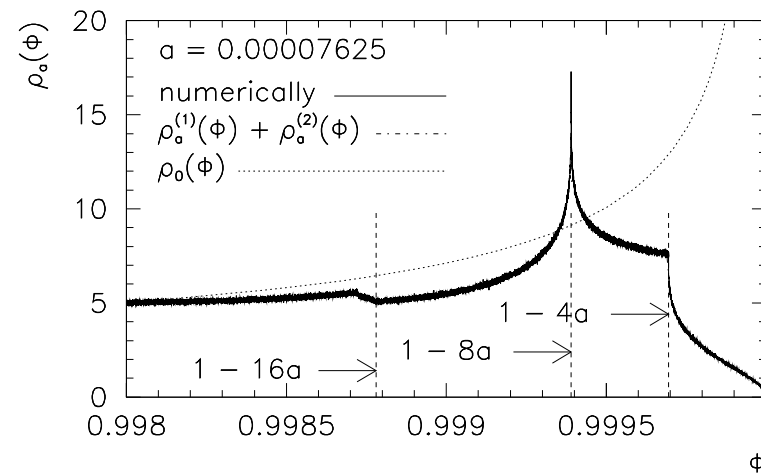
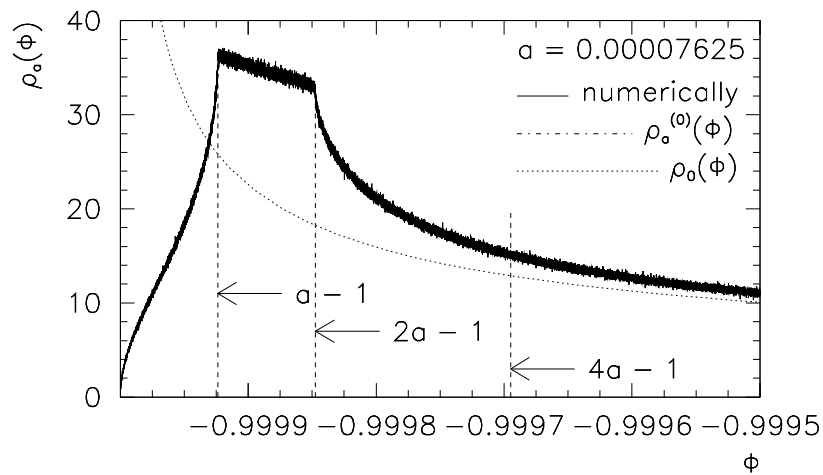


The coupling a

$a > 0$: deviation from $\rho_0(\Phi)$

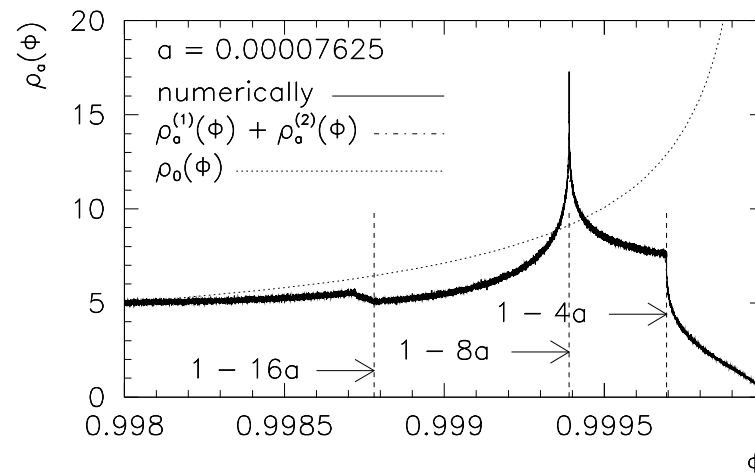
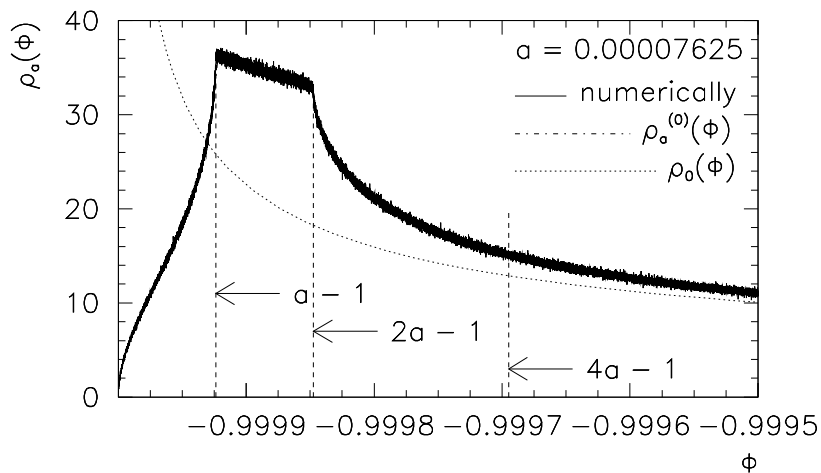
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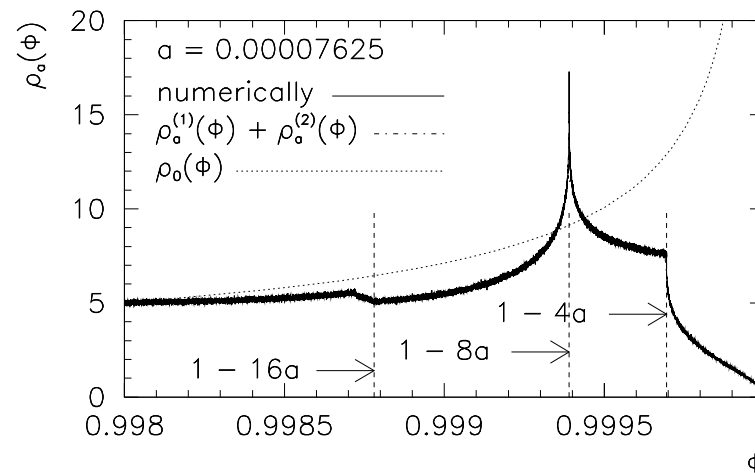
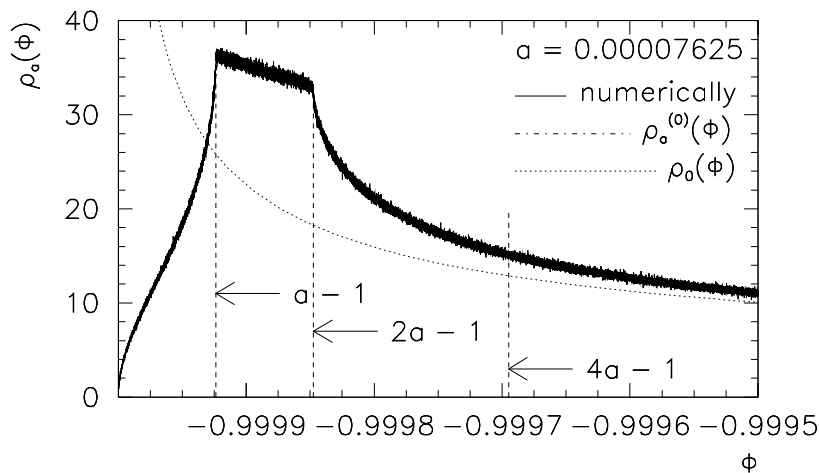


$\Rightarrow$ scaling behavior of observables $\langle f(\Phi) \rangle$

S. Groote, C. Beck, Phys. Rev. **E74** (2006) 046216; Dyn. Sys. **22(2)** (2007) 219

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What are observables on the chaotic string?

Self energy

Dynamics of $\Phi_{n+1} = \pm T_N(\Phi_n)$ described by

$$V_{\pm}^{(2)}(\Phi) = \pm \left(-\frac{2}{3}\Phi^3 + \Phi \right) + \frac{1}{2}V(\Phi)$$

$$V_{\pm}^{(3)}(\Phi) = \pm \left(-\Phi^4 + \frac{3}{2}\Phi^2 \right) + \frac{1}{2}V(\Phi)$$

$$V_{\pm}^{(4)}(\Phi) = \pm \left(-\frac{8}{5}\Phi^5 + \frac{8}{3}\Phi^3 - \Phi \right) + \frac{1}{2}V(\Phi) \quad \dots$$

$(V(\Phi) := \Phi^2 - \langle \Phi^2 \rangle)$ “potentials”

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Note that the minima of $V_{\pm}^{(N)}(\Phi)$ are larger than the minimum of $V(\Phi)$ for $N > 3$, indicating the significance of $N = 2$ and $N = 3$ strings for the following.

Interaction energy

Dynamics of

$$\Phi_{n+1}^i = \Phi_n^i + \frac{a}{2} (\pm \Phi_n^{i-1} - 2\Phi_n^i \pm \Phi_n^{i+1})$$

$(\Phi_n^i = T_N(\Phi_n^i))$ described by “interactions”

$$W_{\pm}(\Phi_n^i, \Phi_n^{i-1}) = \frac{1}{4} ((\Phi_n^i \pm \Phi_n^{i-1})^2 - \langle (\Phi_n^i)^2 \rangle - \langle (\Phi_n^{i-1})^2 \rangle)$$

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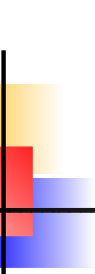
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Note that both observables are abstract quantities, as the string is not a physical but a mathematical object.



Stable zeros and minima

Stochastic quantization of 1-dimensional Einstein-type field equations leads to

$$\dot{a} = a^2 \Gamma W(a) + \text{noise}, \quad \dot{a} = -\Gamma \frac{\partial V(a)}{\partial a} + \text{noise}$$

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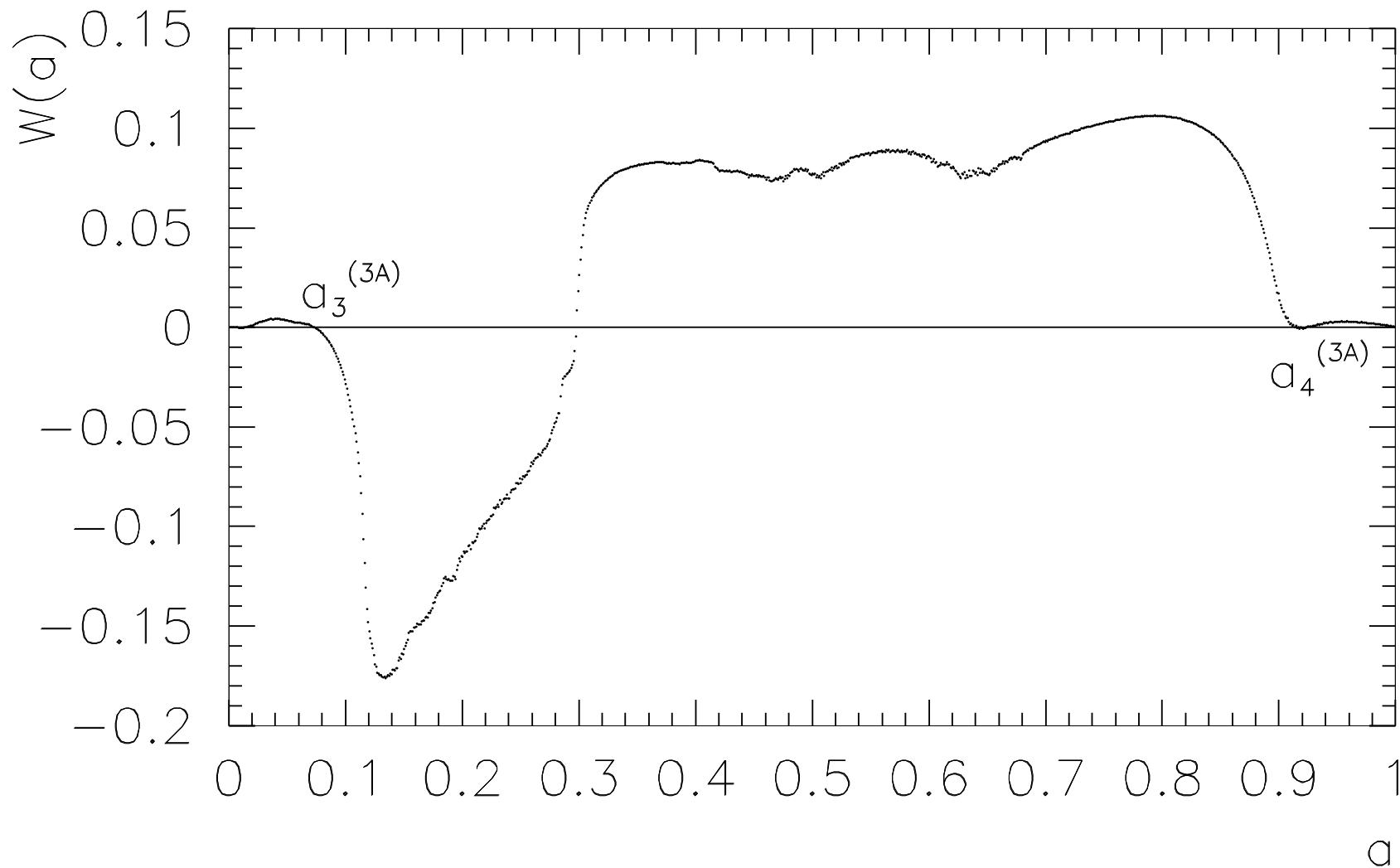
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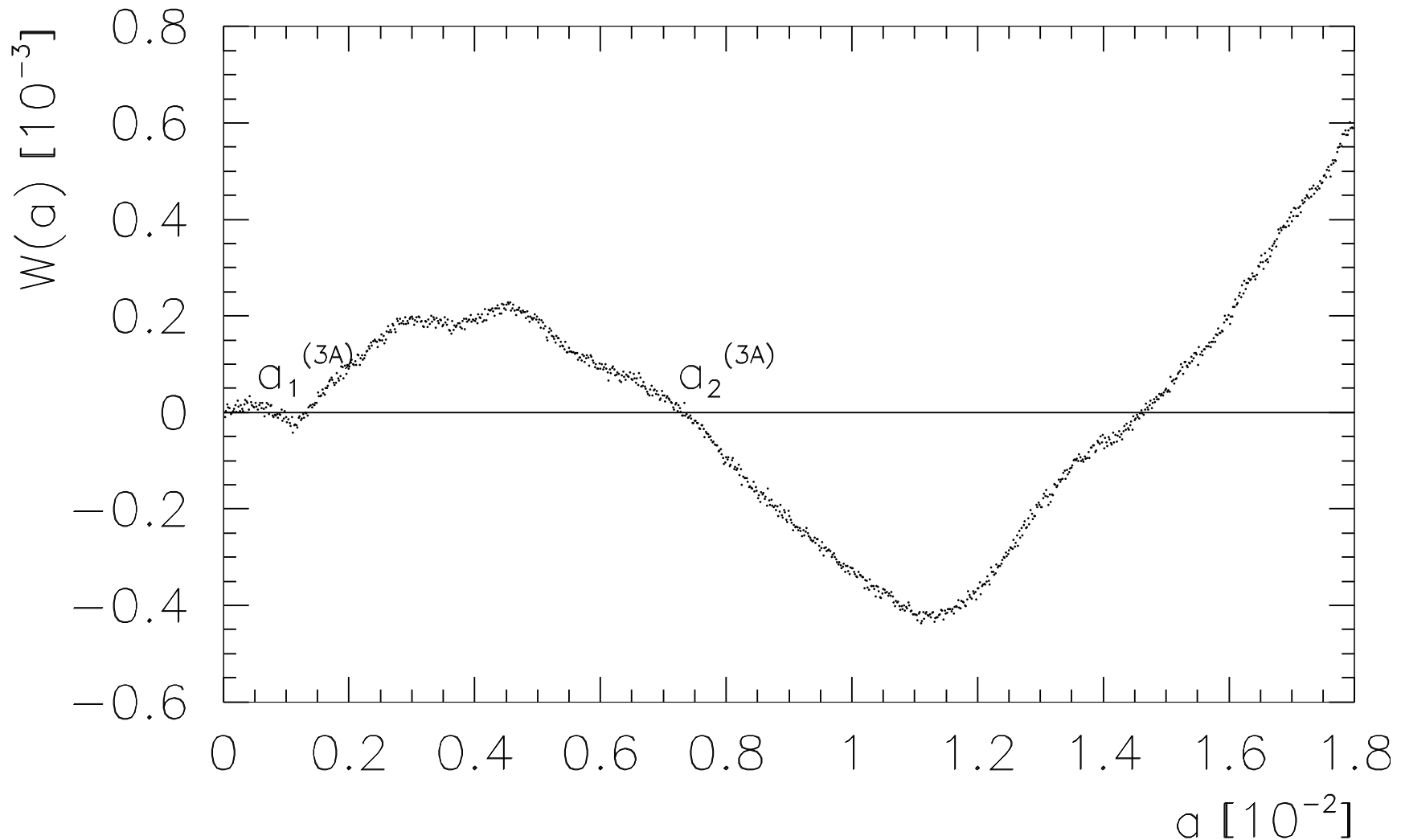
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that have something to do with “our physics”.

3A interaction energy



3A interaction energy



3A interaction energy

$$\begin{aligned}a_1^{(3A)} &= 0.0008164(8) \\ a_2^{(3A)} &= 0.0073038(17)\end{aligned}$$

First exciting observation:

$$a_2^{(3A)} \approx \alpha_{\text{el}} \approx \frac{1}{137}, \quad a_1^{(3A)} \approx \frac{1}{9} \alpha_{\text{el}}$$

It turns out that the zeros are equal to the electromagnetic coupling for a particle of charge $\pm e$ ($a_2^{(3A)}$, e.g. electron), or of charge $\pm e/3$ ($a_1^{(3A)}$, e.g. down-quark)

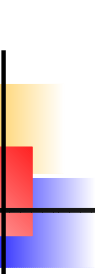
Running coupling

Due to the renormalization group equation the coupling runs with the energy. Using the 1-st order QED formula

$$\alpha_{\text{el}}(E) = \alpha_{\text{el}}(0) \left\{ 1 + \frac{6\alpha_{\text{el}}(0)}{\pi} \sum_i Q_i^2 \times \right. \\ \left. \times \int_0^1 x(1-x) \ln \left(1 + \frac{E^2}{m_i^2} x(1-x) \right) dx \right\}$$

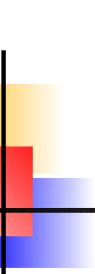
where the sum runs over all charged elementary particle flavors with charges Q_i and masses m_i , the best fit is obtained for

$$\alpha_{\text{el}}(3m_e) = 0.007303, \quad \frac{1}{9}\alpha_{\text{el}}(3m_d) = 0.0008165$$



Some words about masses

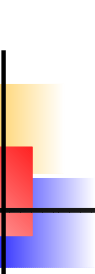
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$\overline{\text{MS}}$ mass $\bar{m}_q(\bar{m}_q)$ for heavy flavors

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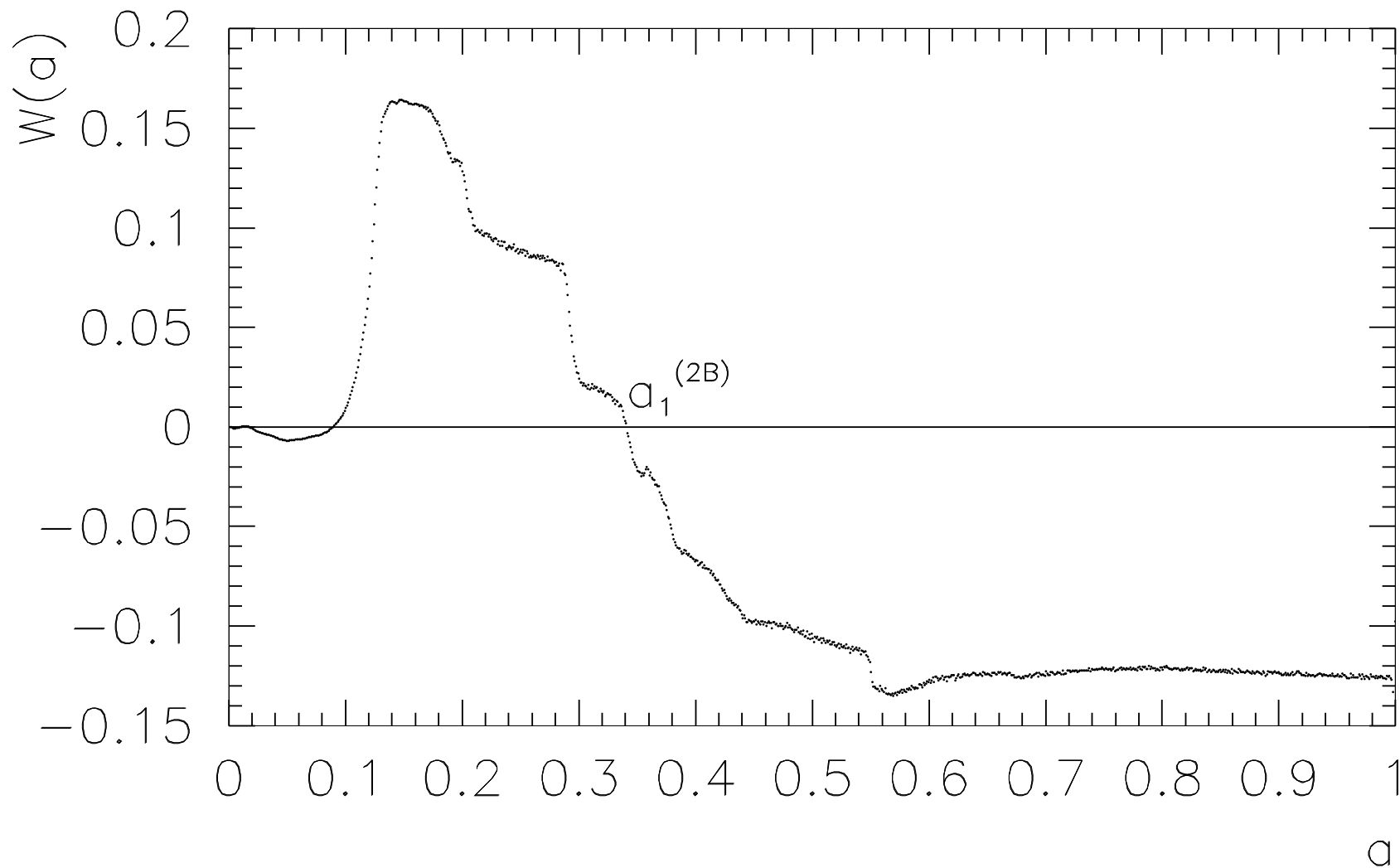
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Vice versa, we can use the zeros to determine masses:

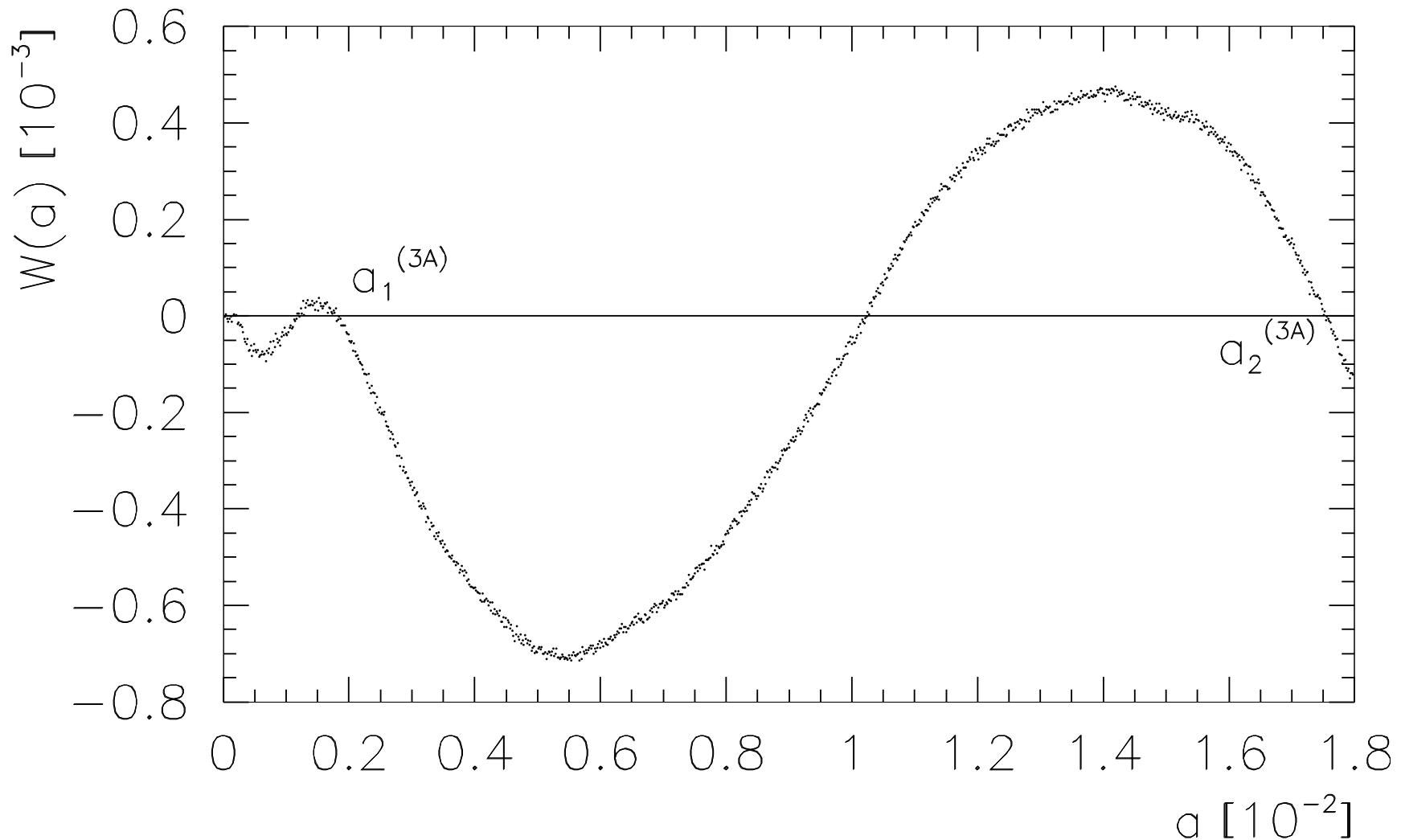
$$m_d = (8.7 \pm 2.1) \text{ MeV}$$

coincides with $\bar{m}_d(1 \text{ GeV})$ (PDG)

3B interaction energy



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we extend the previous results to

$$a_1^{(3B)} = \alpha_{\text{el}}(3m_{d_L}) + \alpha_{\text{weak}}^{u_R}(3m_{u_R})$$

$$a_2^{(3B)} = \alpha_{\text{el}}(3m_{e_R}) + \alpha_{\text{weak}}^{\nu_L}(3m_{\nu_{eL}})$$

Electroweak coupling

We used

$$\alpha_{\text{weak}}^f = \alpha_{\text{el}} \frac{(T_f^3 - Q_f \sin^2 \theta_W)^2}{\sin^2 \theta_W \cos^2 \theta_W}$$

where T_f^3 is the weak isospin of the fermion and Q_f its charge (in units of e), and the fact that left handed and right handed particles cannot interact.

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$$\alpha_{\text{el}}(3m_{d_L}) + \alpha_{\text{weak}}^{u_R}(3m_{u_R}) = 0.001801$$

$$\alpha_{\text{el}}(3m_{e_R}) + \alpha_{\text{weak}}^{\nu_L}(3m_{\nu_{eL}}) = 0.01755$$

which is in perfect agreement with the zeros.

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Our mass estimate:

$$m_u = (7.1 \pm 3.0) \text{ MeV}$$

Electroweak parameters

Vice versa, knowing $\alpha_{\text{el}}(E = 3m_e)$, we determine

$$\alpha_{\text{el}}(0) = \alpha_{\text{el}}(E) \left\{ 1 - \frac{2\alpha_{\text{el}}(E)}{\pi} \sum_i Q_i^2 \times \right. \\ \left. \times \int_0^1 d(1-x) \ln \left(1 + \frac{E^2}{m_i^2} x(1-x) \right) dx \right\}$$

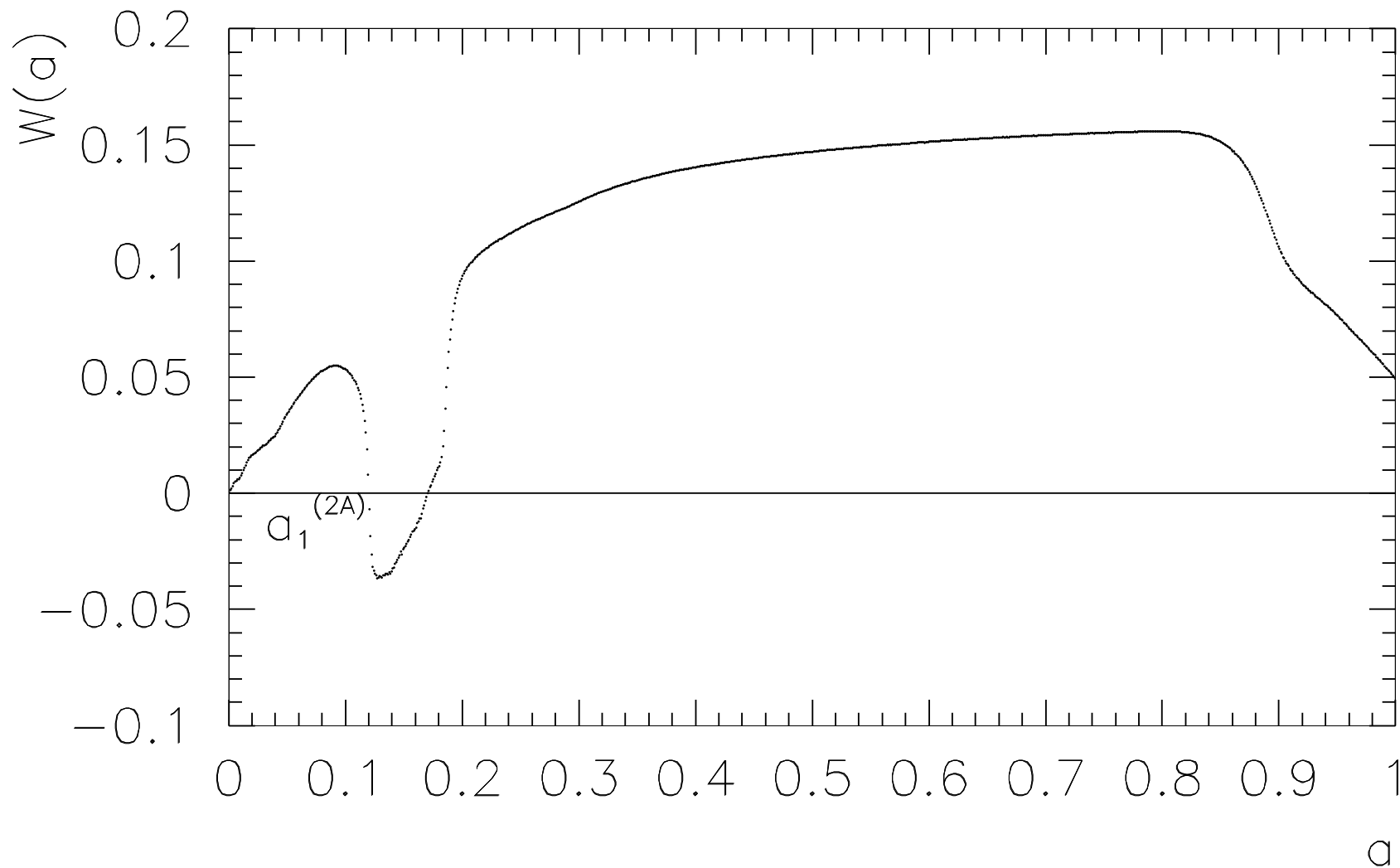
as $\alpha_{\text{el}}(0) = 1/137.03(3)$, to be compared with $1/137.036$.

In the same manner,

$$\sin^2 \theta_W = \frac{1}{2} \left(1 - \sqrt{1 - \frac{\alpha_{\text{el}}(0)}{a_2^{(3B)} - a_2^{(3A)}}} \right) = 0.23177(7)$$

perfectly agrees with measurements in (PDG)

2A interaction energy



2A interaction energy

The only zero

$$a_1^{(2A)} = 0.120093(3)$$

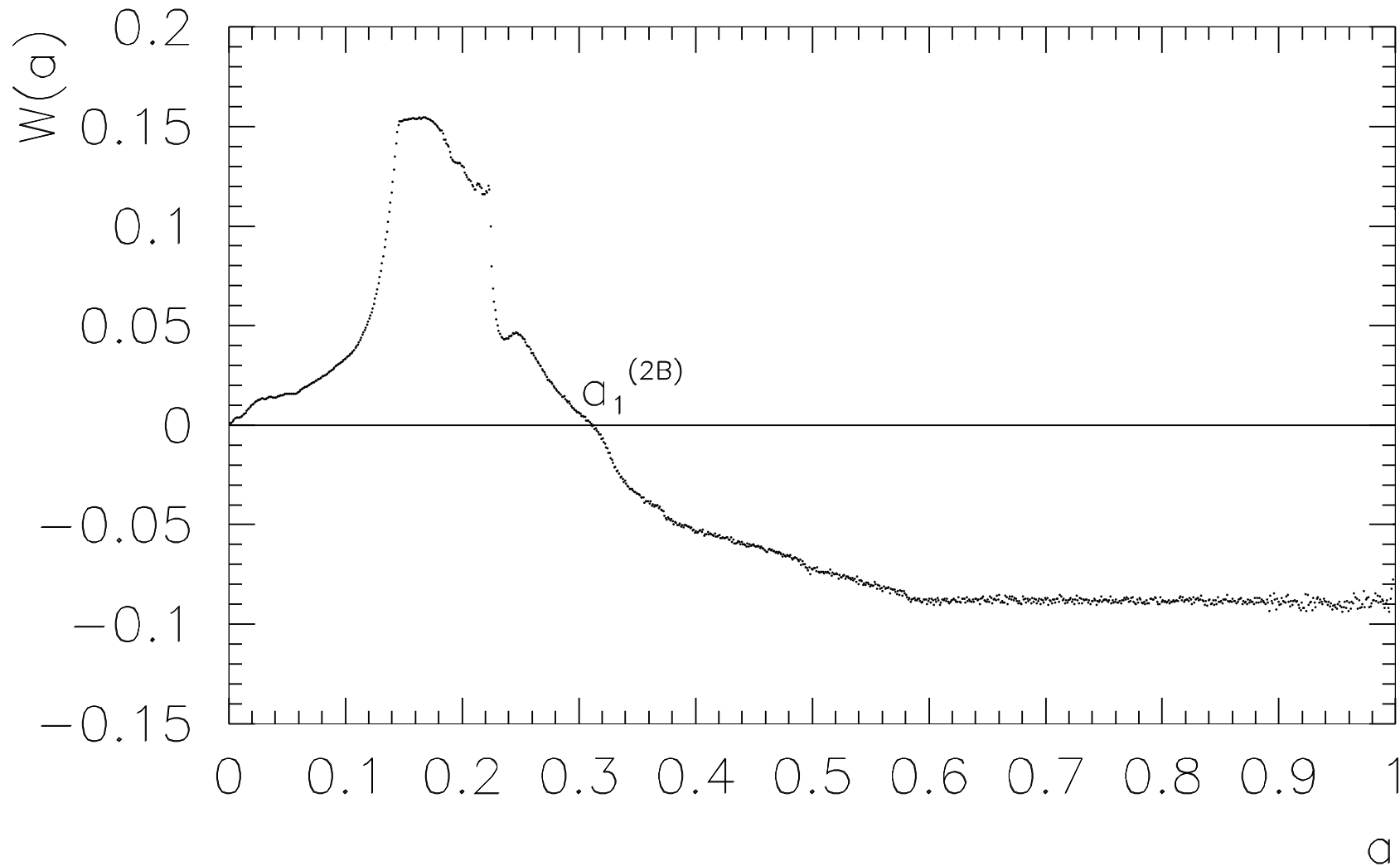
numerically seems to coincide with the strong coupling constant α_s at the W or Z boson mass scale ($\alpha_s(m_Z) = 0.1184(7)$ (PDG)). Assuming that

$$a_1^{(2A)} = \alpha_s(m_W)$$

with $m_W = 80.35(6)$ GeV, we can use the 3-rd order formula for α_s to calculate ($m_Z = 91.188(2)$ GeV)

$$\alpha_s(m_Z) = 0.117804(12)$$

2B interaction energy



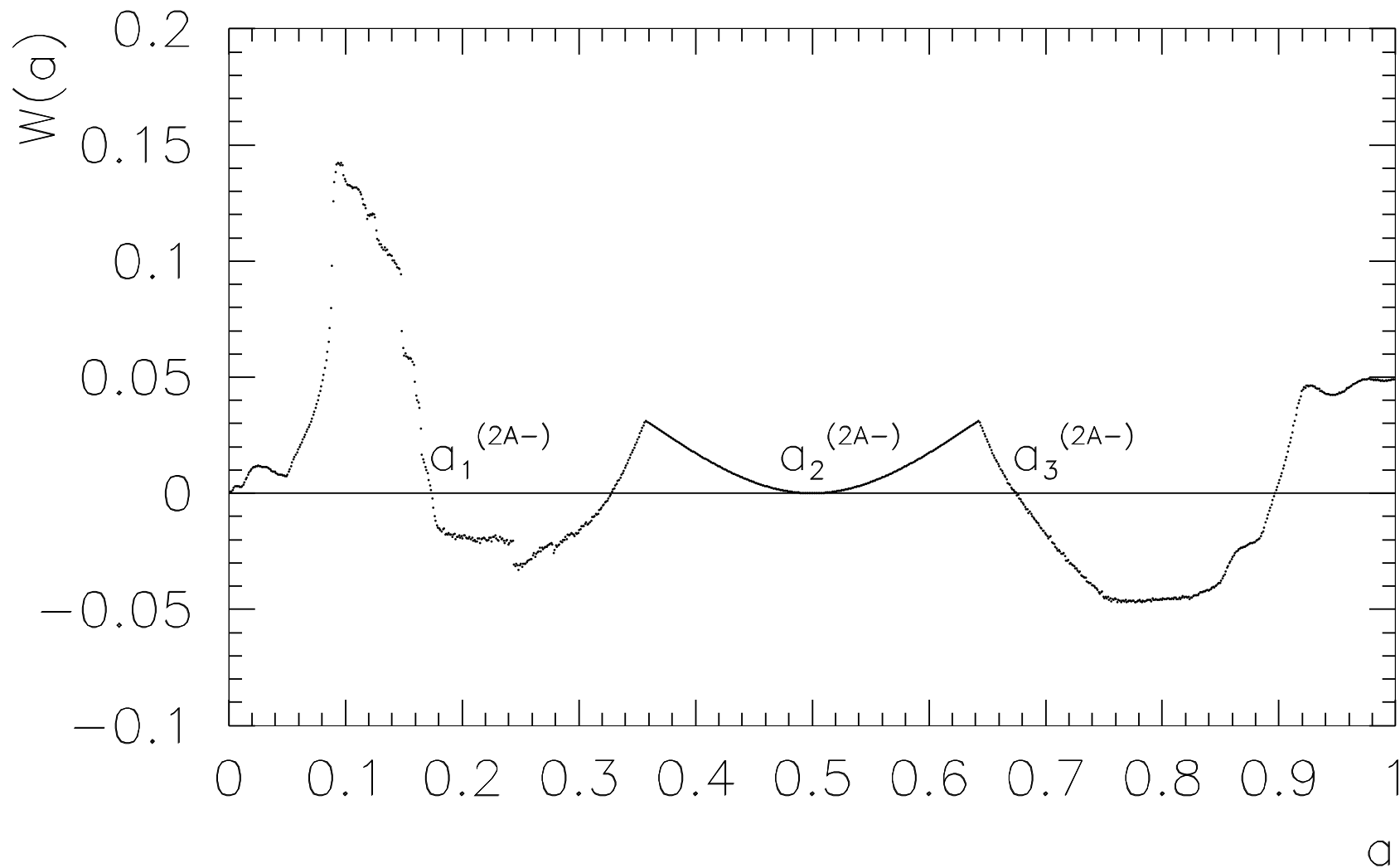
2B interaction energy

$$a_1^{(2B)} = 0.3145(1)$$

lightest glueball with quantum numbers $J^{PC} = 0^{++}$.

At $m_{gg^{0++}} = (1.74 \pm 0.07) \text{ GeV}$ we obtain $\alpha_s = 0.32 \pm 0.05$.

$2A^-$ interaction energy



$2A^-$ interaction energy

The first stable zero is

$$a_1^{(2A^-)} = 0.1757(1)$$

lightest glueball with quantum numbers $J^{PC} = 2^{++}$.

$$a_1^{(2A^-)} = \alpha_s(E) \text{ with } E = m_{gg^{2++}} + 2m_q$$

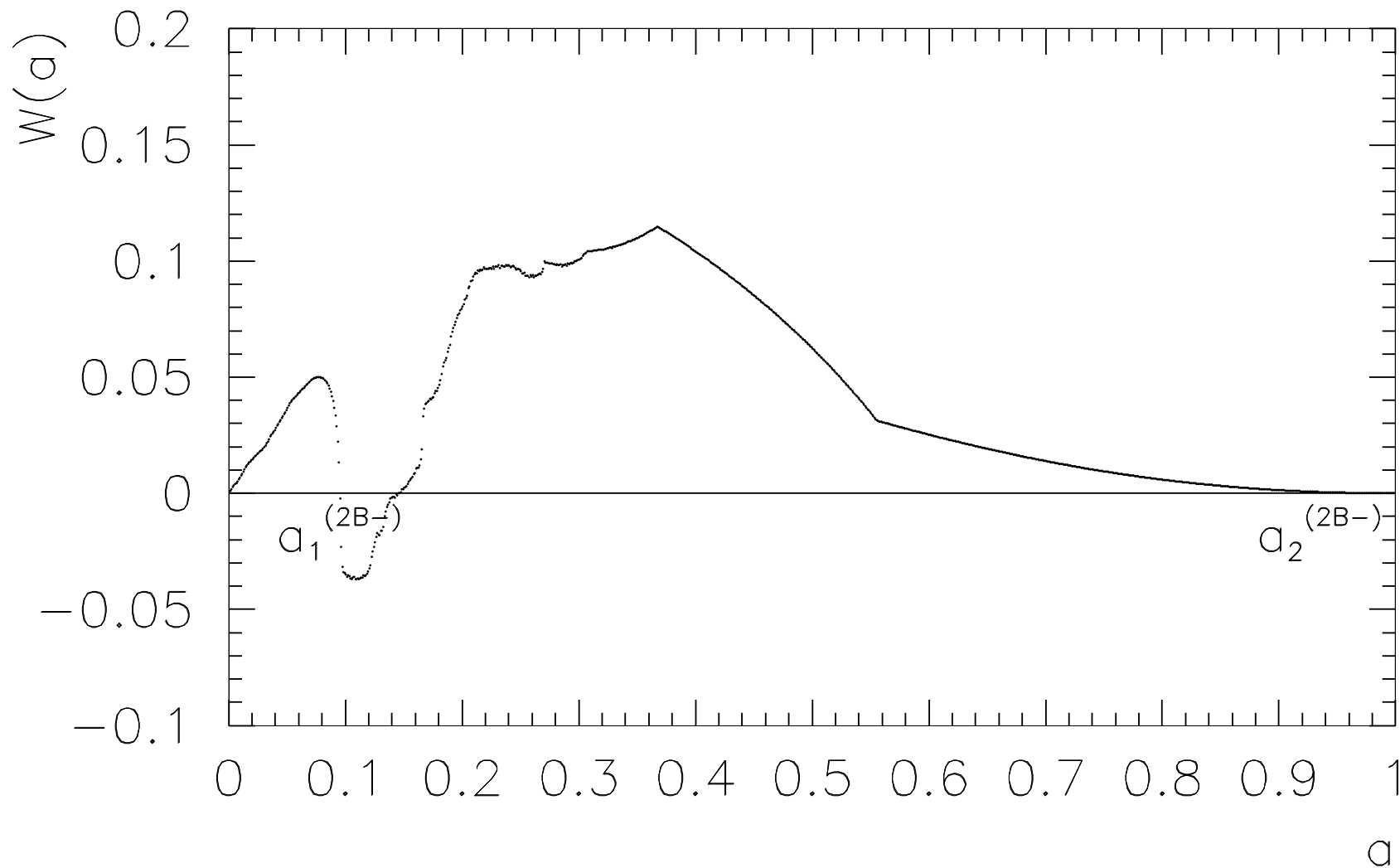
can be fulfilled by

$$m_{gg^{2++}} = (2.23 \pm 0.31) \text{ GeV},$$

$$m_q = m_b = 4.23(1) \text{ GeV (derived later)}$$

The second stable zero will be considered soon.

$2B^-$ interaction energy



$2B^-$ interaction energy

The first stable zero at

$$a_1^{(2B^-)} = 0.095370(1)$$

represents possibly the lightest Higgs, because

$$a_1^{(2B^-)} = \alpha_s(E) \text{ with } E = m_H + 2m_q$$

can be fulfilled by

$$m_H = 154.4(5) \text{ GeV},$$

$$m_q = m_t = 164.5(2) \text{ GeV (derived later)}$$

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high precision estimate for the Higgs mass!

Feynman webs

Motivation for the energy point E can be found in so-called Feynman webs, i.e. networks consisting of Fermion loops and intermediate boson connections. Energy is given similar to the harmonic oscillator by

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This rule is found by “trial and error”

Feynman webs

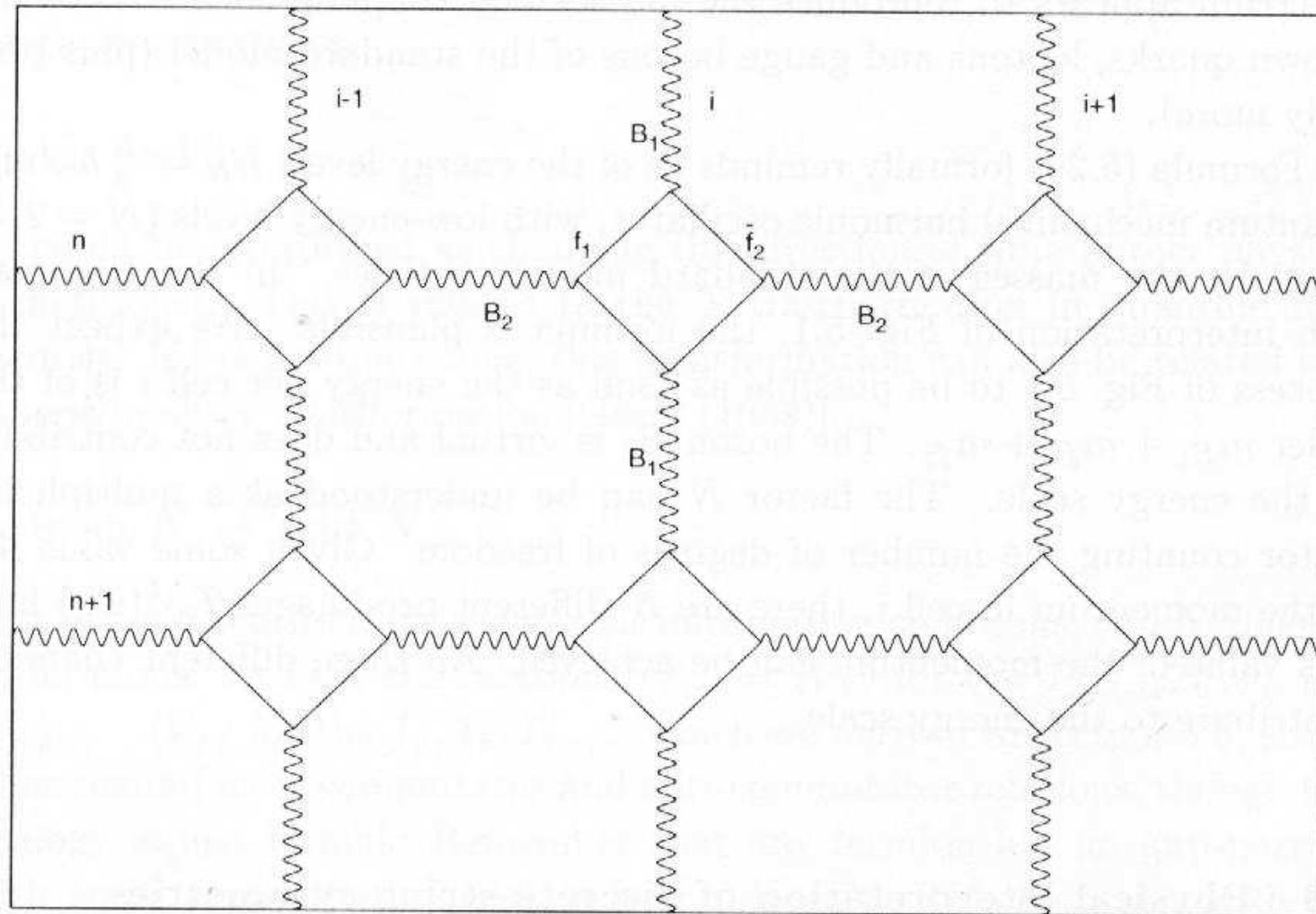


Fig. 5.1 Feynman web interpretation of the coupled map dynamics.

$2A^- / B^-$ interaction energies

The second stable zeros are

$$a_2^{(2A^-)} = \frac{1}{2} \quad a_2^{(2B^-)} = 1$$

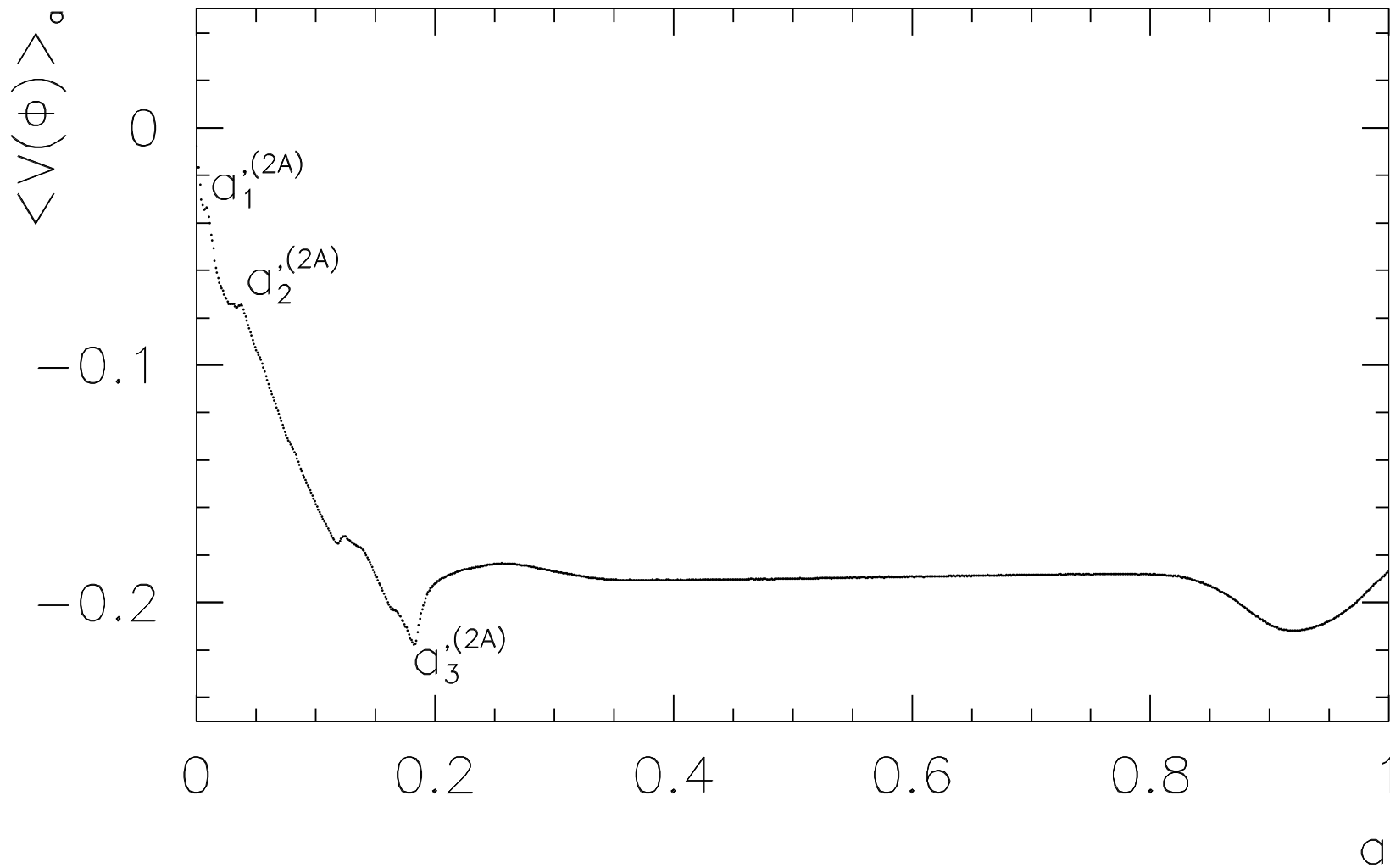
- At these points chaotic fluctuations cease to exist
 $\Rightarrow$ classical theory arises
- dimensionless gravitational coupling strength

$$\alpha_G = \frac{1}{2} \left(\frac{E}{m_{Pl}} \right)^2$$

at the Planck scale $E = m_{Pl}$ represented by one
resp. two particles $\bar{W}$ of that mass (see later)

- only attractive if Γ changes sign

2A self energy



2A self energy

$$a_3'^{(2A)} = 0.1848(1) \leftrightarrow \alpha_s(2m_b) = 0.1848$$

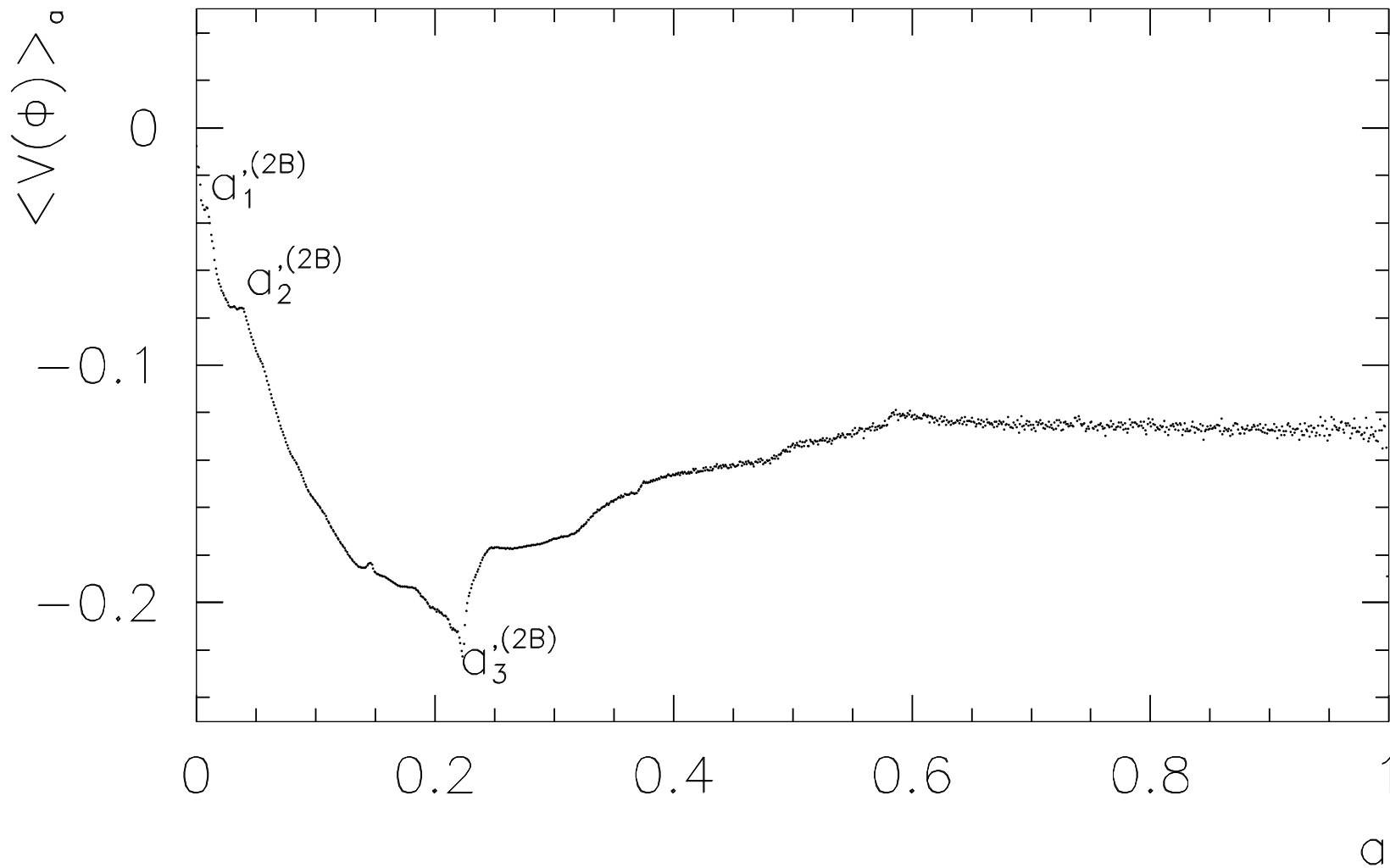
obtained for $m_b = 4.23 \text{ GeV}$

$$a_2'^{(2A)} = 0.03369(2) \leftrightarrow \alpha_2(m_Z + 2m_b) = 0.03369(1)$$

with $\alpha_2(E)$ the $SU(2)$ part of the electroweak coupling

$$a_1'^{(2A)} = 0.00755(3) \leftrightarrow \alpha_{\text{el}}(2m_\tau)$$

2B self energy



2B self energy

$$a_3'^{(2B)} = 0.2235(2) \leftrightarrow \alpha_s(2m_b) + \alpha_s(2m_d)$$

$$a_2'^{(2B)} = 0.03440(2) \leftrightarrow \alpha_{\text{Yu}}(m_H + 2m_t)$$

where the Yukawa coupling is given by

$$\alpha_{\text{Yu}}^f = \frac{\alpha_2}{4} \left(\frac{m_f}{m_W} \right)^2$$

With $m_H = 154 \text{ GeV}$ and $m_W = 80.35(6) \text{ GeV}$ we obtain

$$m_t = 164.5(2) \text{ GeV} \quad \Leftrightarrow \quad M_t = 174.4(3) \text{ GeV}$$

NA/B self energies

For small couplings a we enter the scaling region

$$V(a) - V(0) = \sqrt{a} f^{(N)}(\ln a)$$

$(f^{(N)}(\ln a))$ periodic function of $\ln a$ with period N^2)

NA/B self energies

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Consequences:

- determination only modulo N^2

NA/B self energies

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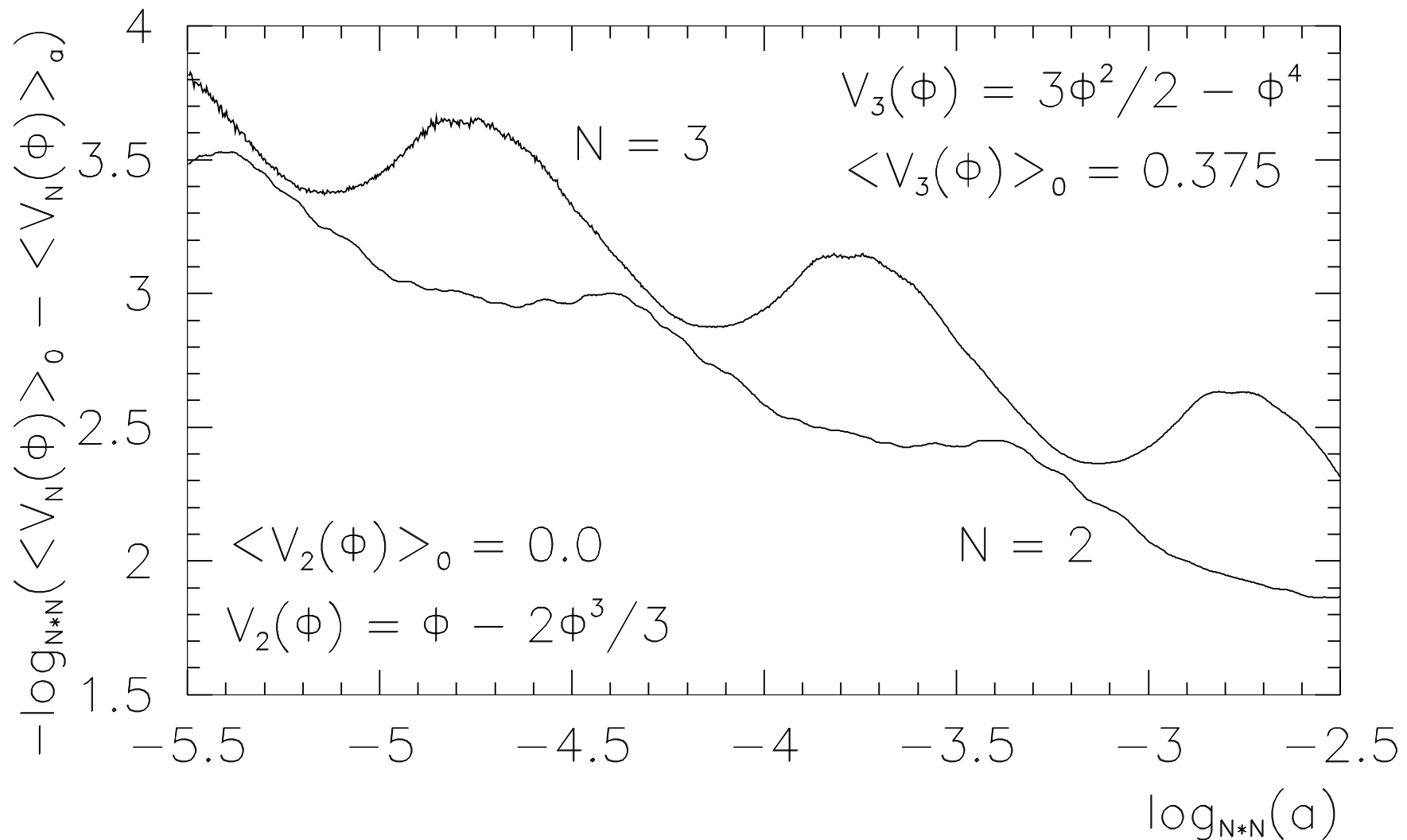
($f^{(N)}(\ln a)$ periodic function of $\ln a$ with period N^2)

- minima of $V(a)$ repeat periodically
- results for NA and NB strings coincide

Consequences:

- determination only modulo N^2
- determination possible even for very small a

NA/B self energies



3A/B self energy

$$a_1'^{(3A)} = 0.000246(2) \quad \alpha_{\text{weak}}^{d_R}(3m_d) = 0.000246$$

$$a_2'^{(3A)} = 0.00102(1) \quad \alpha_{\text{weak}}^{c_R}(3m_c) = 0.001013$$

$$a_3'^{(3A)} = 0.00220(1) \quad \alpha_{\text{weak}}^{e_R}(3m_e) = 0.00220$$

$$a_4'^{(3A)} = 0.00965(1) \quad \alpha_{\text{weak}}^{\nu_R}(3m_\nu)$$

$$a_5'^{(3A)} = 0.02145(3) \quad \alpha_s(3m_Q)$$

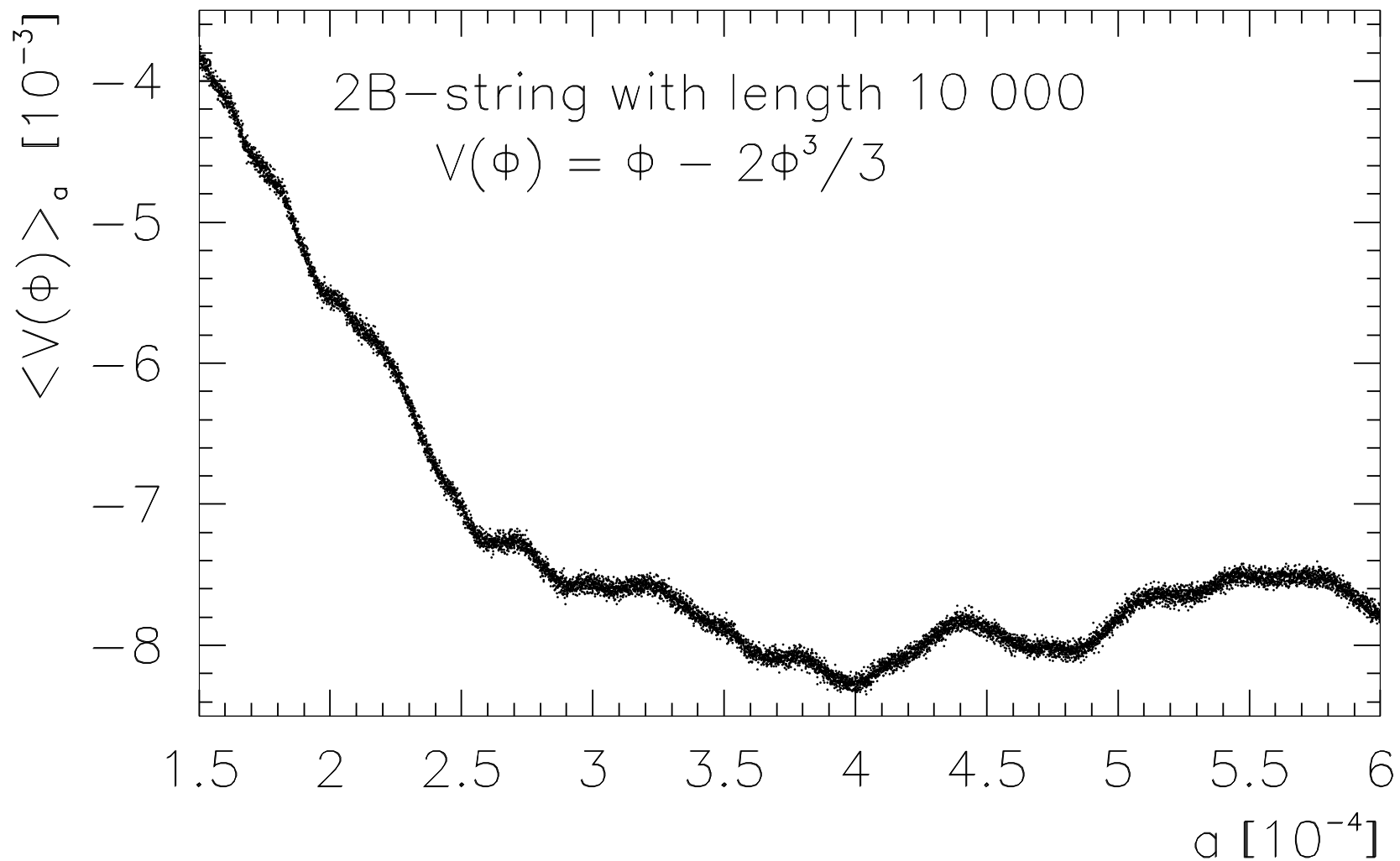
$$a_6'^{(3A)} = 0.0953(1) \quad \alpha_s(3m_t) = 0.0952(3)$$

$$a_7'^{(3A)} = 0.1677(5) \quad \alpha_s(3m_b) = 0.1684(4)$$

$$a_8'^{(3A)} = 0.2327(5) \quad \alpha_s(3m_c) = 0.2323(20)$$

scaling region: determination of charges

2A/B self energy



2A/B self energy

f	$\alpha_2(m_H + 2m_f)$	k	$b_k \times 10^4$	p	m_f [GeV]
t	0.03284	(5)	344.0(2)	0	164.5(2)
b	0.03340	6	3.68(3)	2	4.22(2)
τ	0.03342	2	2.63(2)	3	1.782(7)
c	0.03342	10	5.25(2)	4	1.259(4)

$$b_k = \alpha_{Y_u} = \frac{1}{2} \alpha_2(m_H + 2m_f) \left(\frac{m_f}{m_W} \right)^2 4^p$$

2A/B self energy

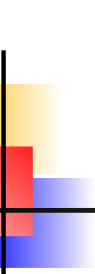
f	k	$b_k \times 10^4$	p	m_f [MeV]
μ	1	1.99(1)	61	105.6(3)
e	4	3.06(1)	69	0.5117(8)
d	7	3.99(1)	65	9.35(1)
u	8	4.69(1)	66	5.07(1)
s	9	4.82(1)	61	164.4(2)

$$b_k = \alpha_G = \frac{1}{2} \left(\frac{m_f}{m_{\text{Pl}}} \right)^2 4^p$$

2A/B self energy

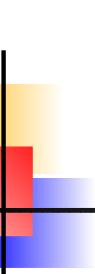
Prediction of neutrino masses
due to symmetry considerations

f	k	$b_k \times 10^4$	p	m_f [eV]
ν_1	3	2.91(1)	104	$1.452(3) \times 10^{-5}$
ν_2	0	1.31(5)	99	$3.218(4) \times 10^{-4}$
			96	$2.574(3) \times 10^{-3}$
ν_3	1	1.99(1)	92	$4.92(1) \times 10^{-2}$



$2A/B$ self energy

This string (region) currently under consideration



$2A/B$ self energy

This string (region) currently under consideration

- pattern does not appear for $\dim > 1$

M. Maher, C. Beck, *Chaos, Solitons and Fractals* **37** (2008) 9

$2A/B$ self energy

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- pattern does not appear for $\dim > 1$

M. Maher, C. Beck, *Chaos, Solitons and Fractals* **37** (2008) 9

- pattern appears also for strings of length 3

S. Groote, H. Veermäe, *Chaos, Solitons and Fractals* **41(5)** (2009) 2354

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- Relation to renormalisation and bubble diagrams

S. Groote, H. Veermäe, C. Beck, *Chaos, Solitons and Fractals* **53** (2013) 18

$2A/B$ self energy

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M. Maher, C. Beck, *Chaos, Solitons and Fractals* **37** (2008) 9

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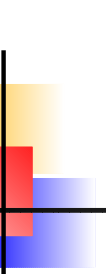
S. Groote, H. Veermäe, *Chaos, Solitons and Fractals* **41(5)** (2009) 2354

- Relation to renormalisation and bubble diagrams

S. Groote, H. Veermäe, C. Beck, *Chaos, Solitons and Fractals* **53** (2013) 18

- the origin of the pattern (fine structure)
is due to effects in the generating partition

S. Groote, C. Beck, current research



Hadron states

Minima of the total vacuum energy

$$H_{\pm}(a) = V(a) \pm aW(a)$$

describes hadronic states

C. Beck, "Spatio-Temporal Chaos and Vacuum Fluctuations of Quantized Fields", World Scientific 2002, Advanced Series in Nonlinear Dynamics, Vol. 21

3A vacuum energy $H_-(a)$

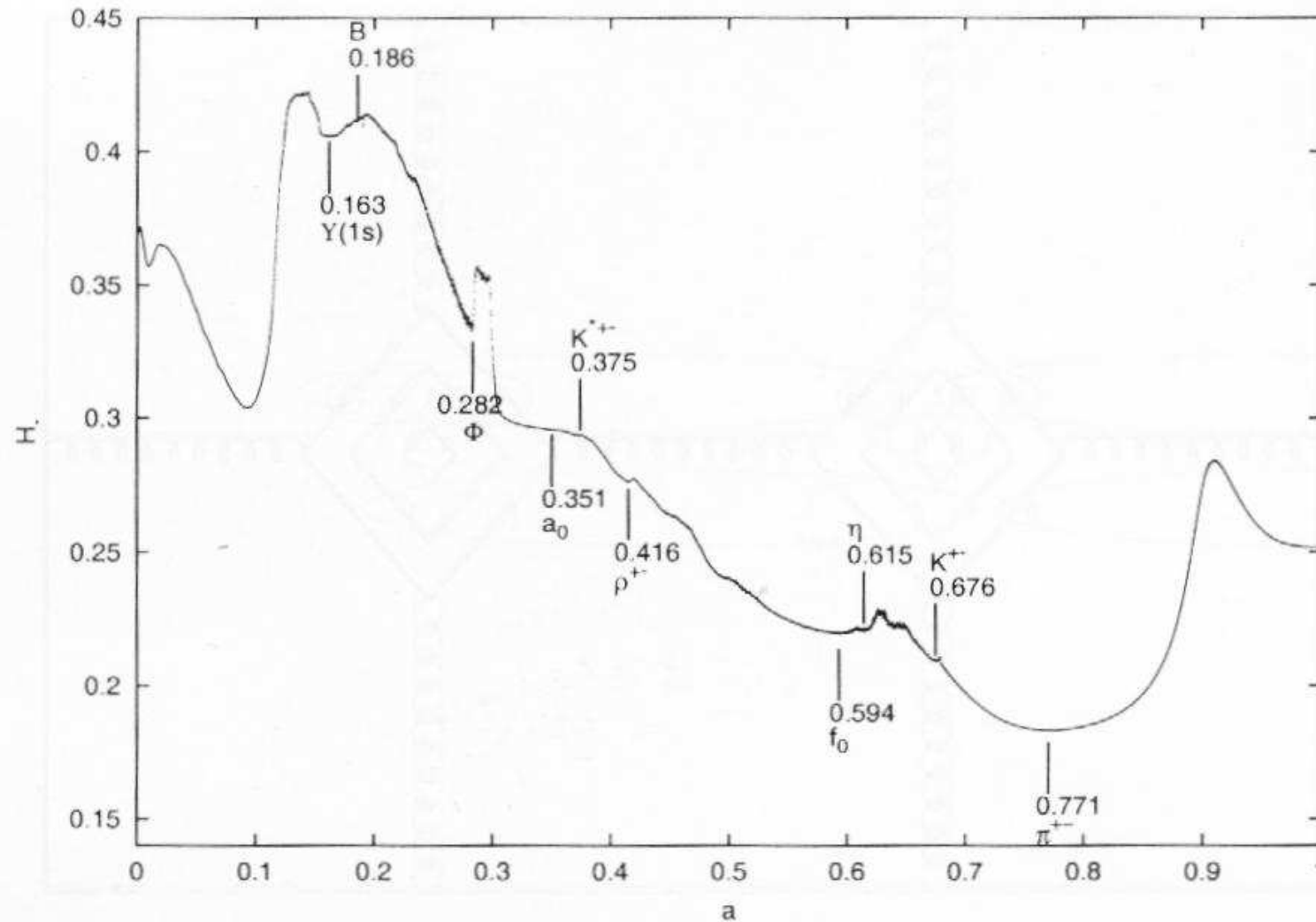


Fig. 9.5 Total vacuum energy $H_-(a)$ of the 3A string.

$3B$ vacuum energy $H_-(a)$

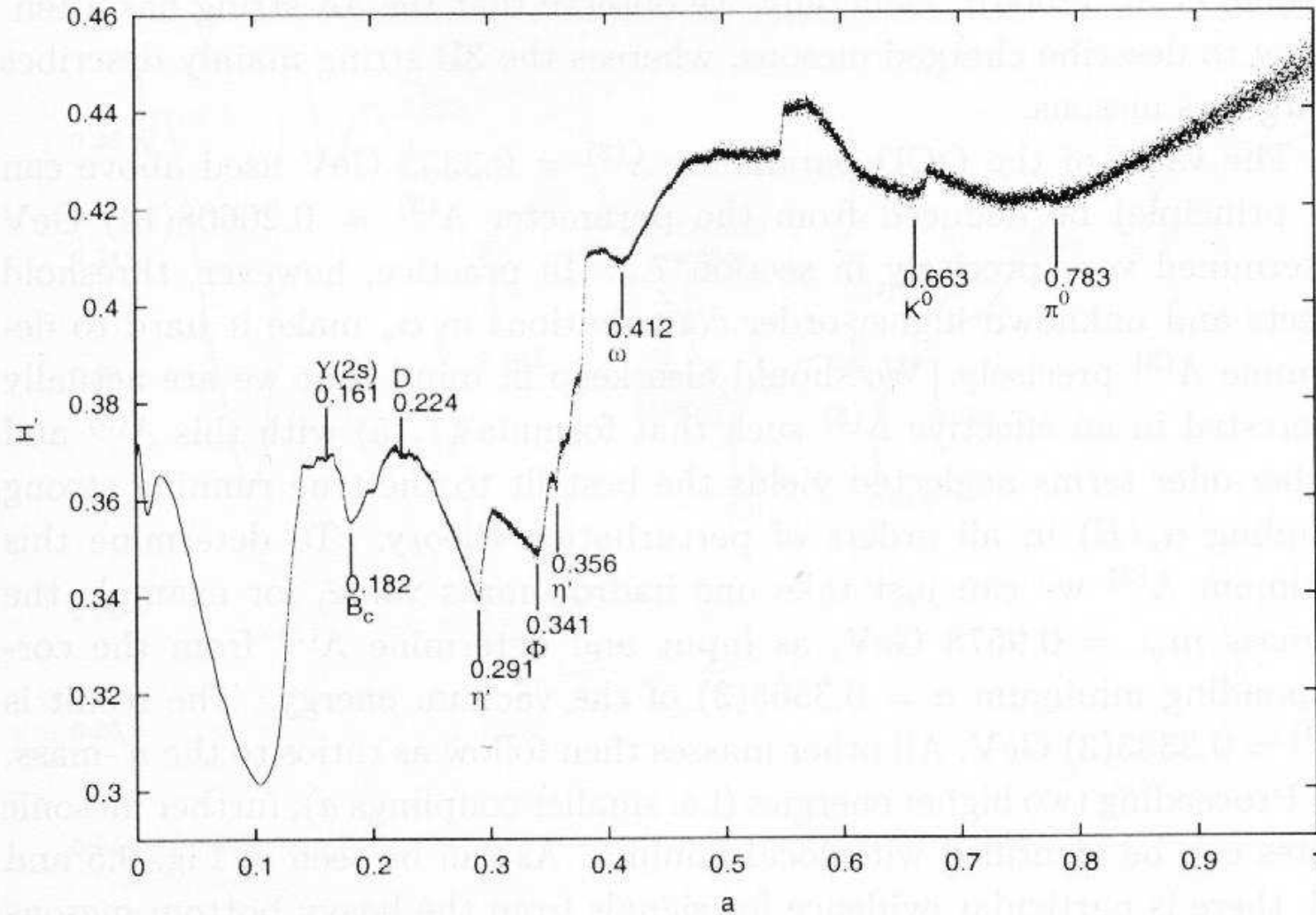


Fig. 9.6 Total vacuum energy $H_-(a)$ of the 3B string.

3A vacuum energy $H_+(a)$

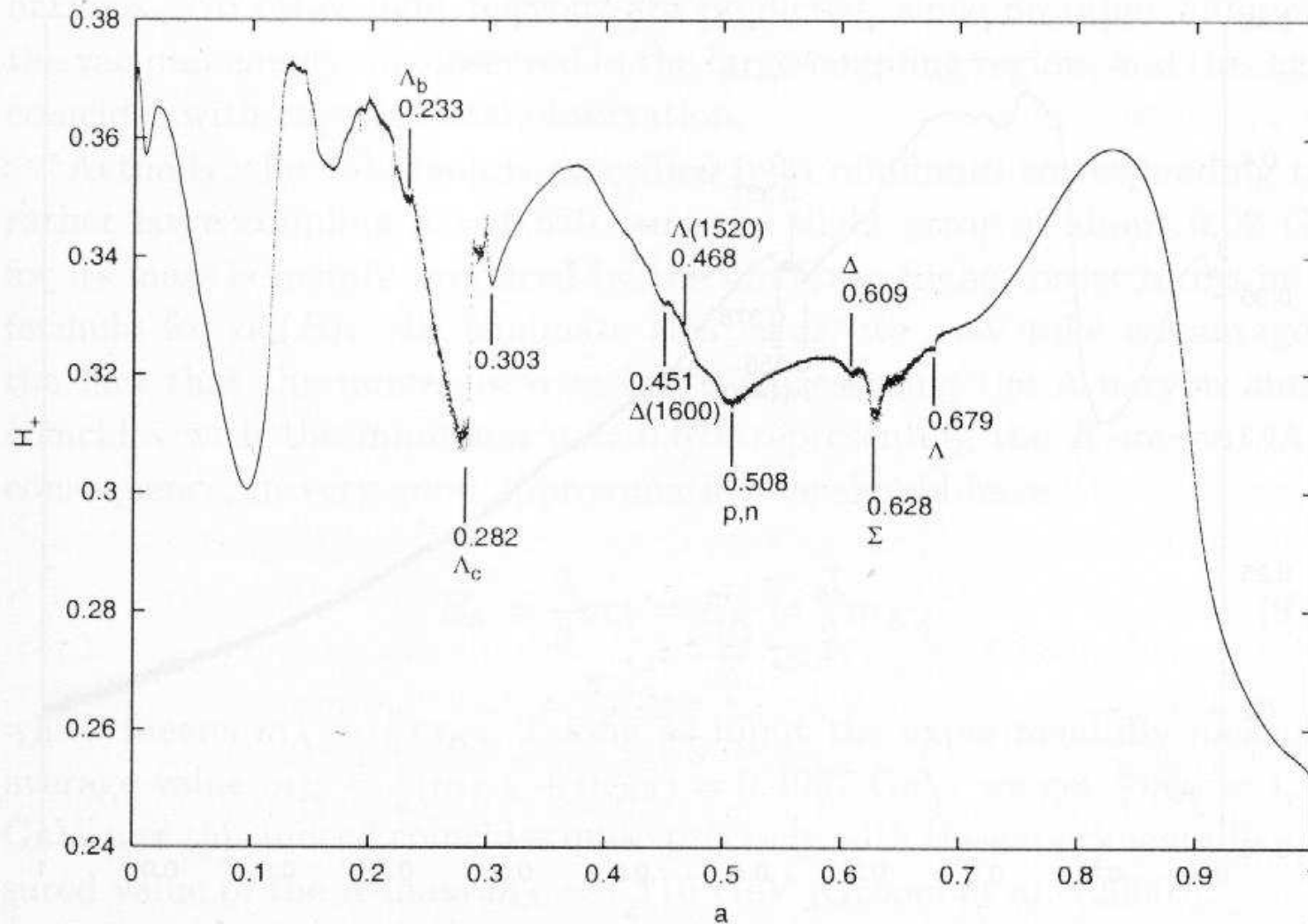


Fig. 9.7 Total vacuum energy H_+ of the 3A string.

3B vacuum energy $H_+(a)$

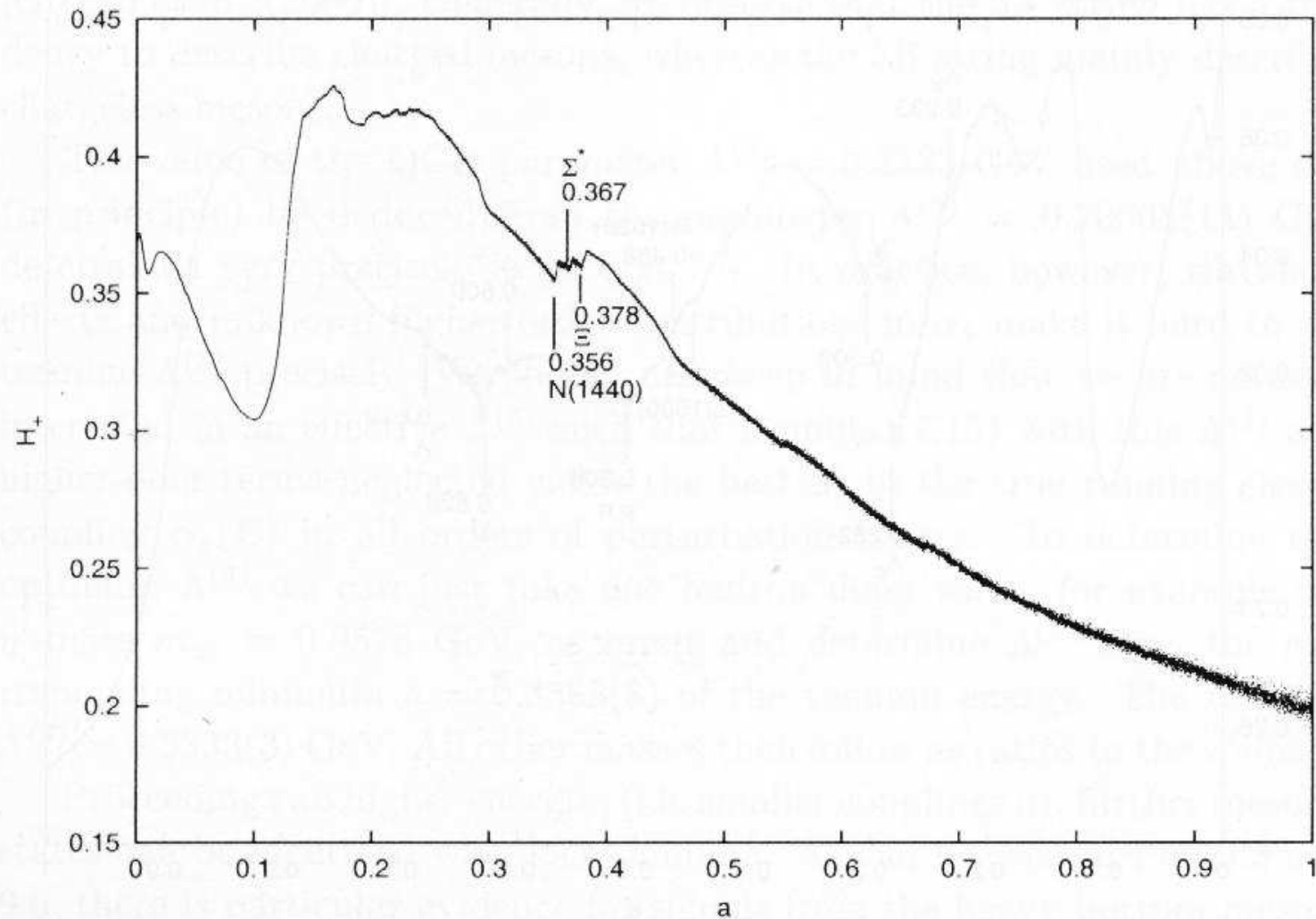


Fig. 9.8 Total vacuum energy H_+ of the 3B string.

Dark matter

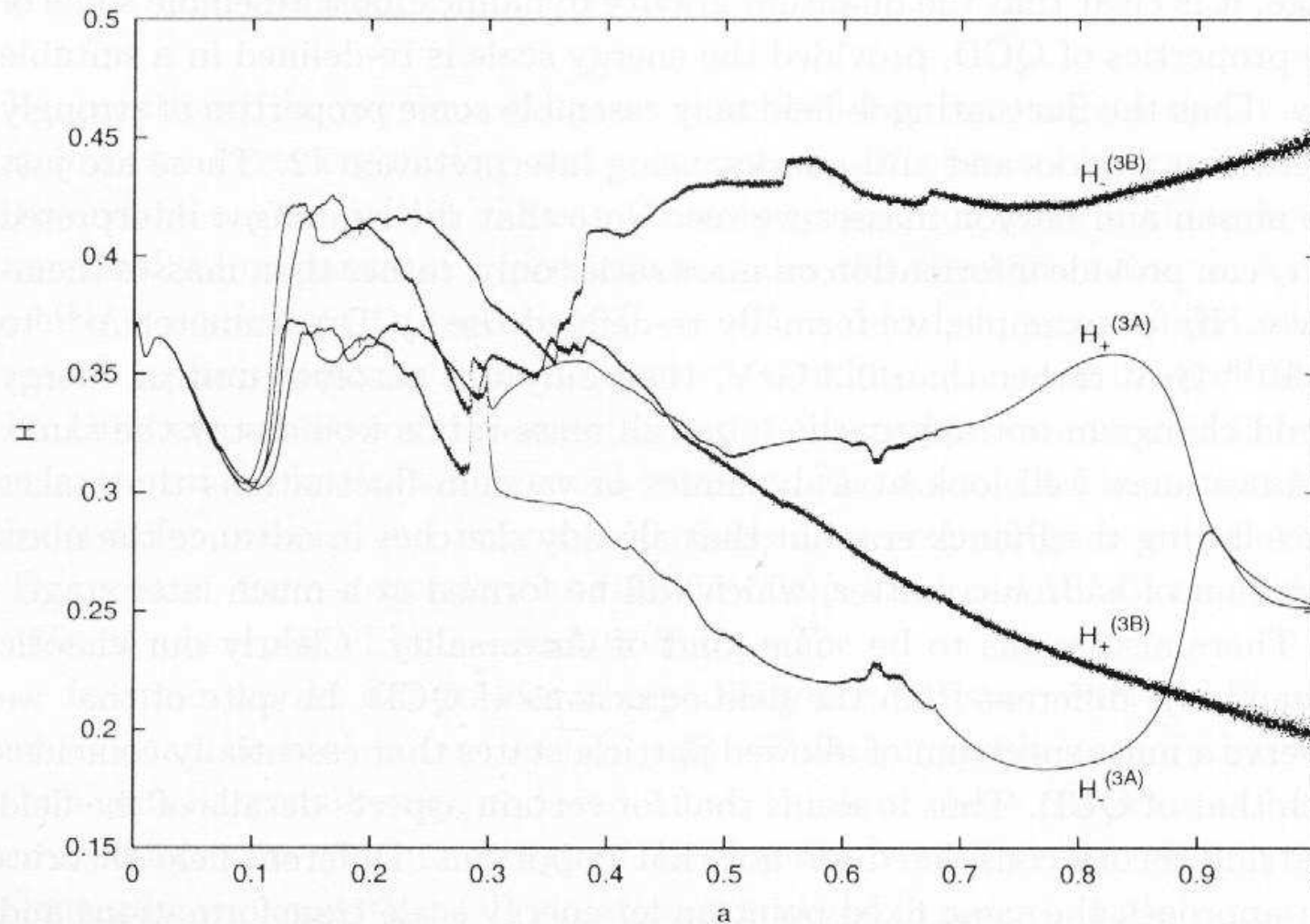


Fig. 9.9 Total vacuum energies H_+ and H_- for the 3A, 3B dynamics. The figure, which looks a bit like a 'broken violin', is the union of the data of Fig. 9.5-9.8.

Dark matter

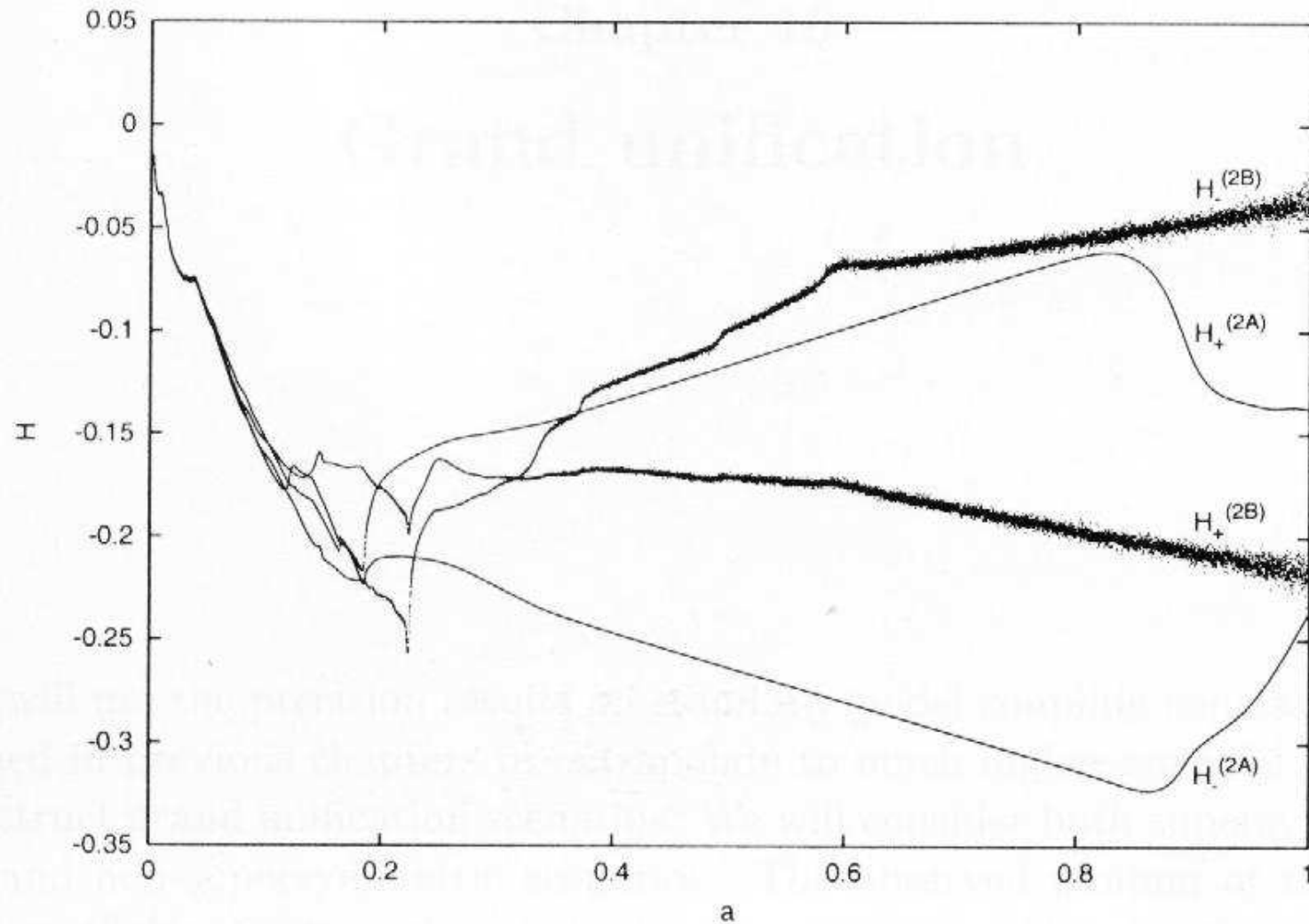


Fig. 9.10 Total vacuum energies H_+ and H_- for the 2A, 2B dynamics.

Dark matter

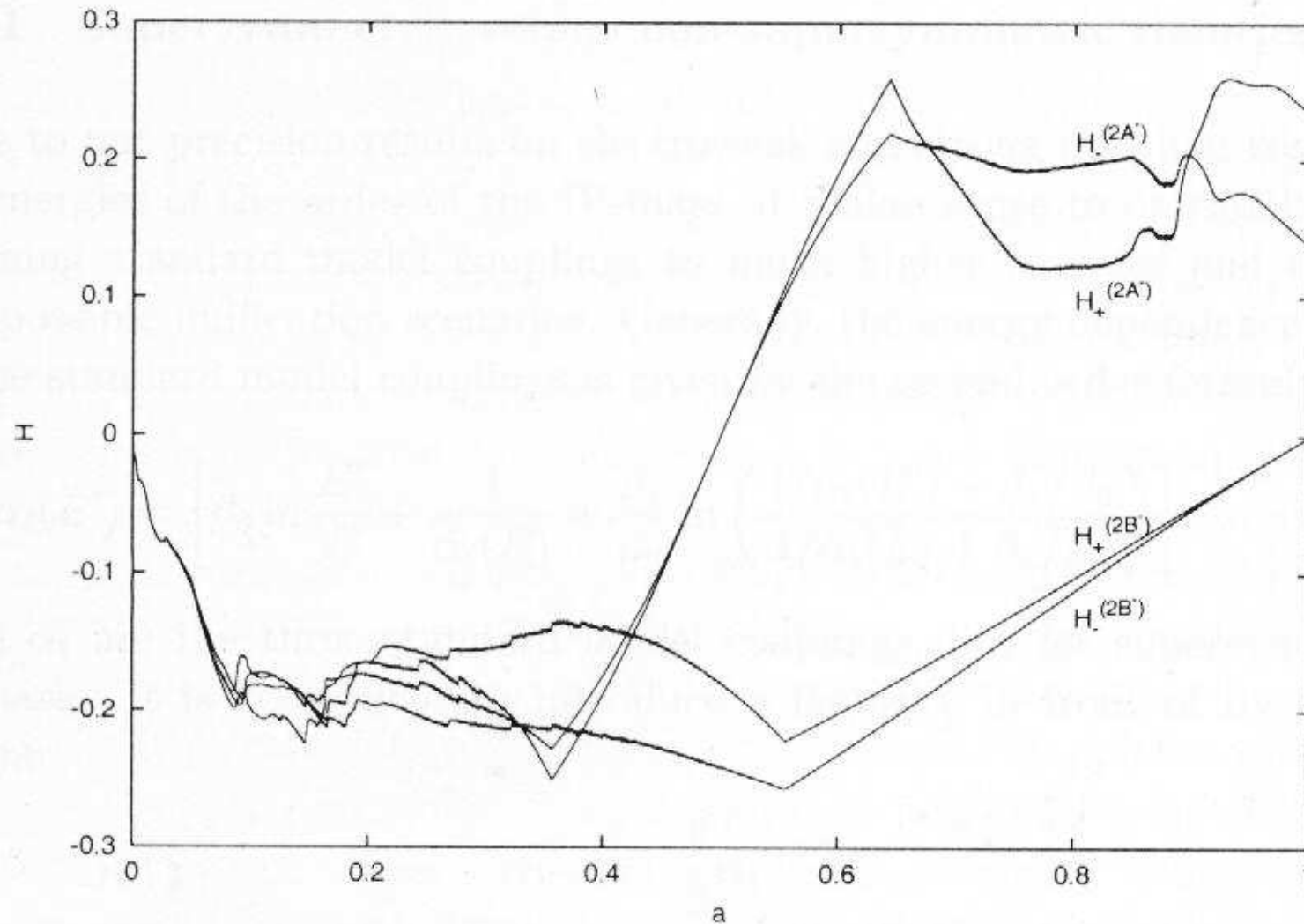


Fig. 9.11 Total vacuum energies H_+ and H_- for the $2A^-$, $2B^-$ dynamics.

Grand unification scenario

Energy dependence of the three SM couplings given by the second-order formula

$$\frac{1}{\alpha_i(E')} = \beta_0 \ln \left(\frac{E'}{E} \right) + \frac{1}{\alpha_i(E)} + \frac{\beta_1}{\beta_0} \ln \left(\frac{1/\alpha_i(E') + \beta_1/\beta_0}{1/\alpha_i(E) + \beta_1/\beta_0} \right)$$

$$\beta_0 = \frac{-1}{2\pi} \left(b_i + \sum_{j \neq i} \frac{b_{ij}}{4\pi} \alpha_j(E) \right), \quad \beta_1 = \frac{-b_{ii}}{8\pi^2}$$

$$(b_i) = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_F \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_H \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$(b_{ij}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_F \begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix} + N_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

U. Amaldi, W. de Boer, H. Fürstenau, Phys. Lett. **B260** (1991) 447

Grand unification scenario

Three Standard Model (SM) couplings

$$\begin{aligned}\alpha_1 &= \frac{g_1^2}{4\pi}, & g_1 &= \frac{e}{\cos \theta_W} & U(1) \\ \alpha_2 &= \frac{g_2^2}{4\pi}, & g_2 &= \frac{e}{\sin \theta_W} & SU(2) \\ \alpha_3 &= \frac{g_s^2}{4\pi} = \alpha_s & & & SU(3)\end{aligned}$$

N_F : number of active families

N_H : number of active Higgs bosons (SM: 0 or 1)

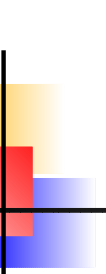
For a quick and systematical overview see e.g.

Gordon Kane, "Modern Elementary Particle Physics", Addison-Wesley 1993



Grand unification scenario

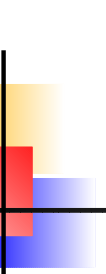
Non-supersymmetric scenario:



Grand unification scenario

Non-supersymmetric scenario:

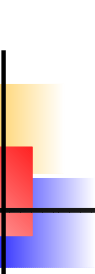
- symmetry breaking at $E_{\text{weak}} = m_H = 154.4 \text{ GeV}$



Grand unification scenario

Non-supersymmetric scenario:

- symmetry breaking at $E_{\text{weak}} = m_H = 154.4 \text{ GeV}$
- from this point onwards $N_H = 1$



Grand unification scenario

Non-supersymmetric scenario:

- symmetry breaking at $E_{\text{weak}} = m_H = 154.4 \text{ GeV}$
- from this point onwards $N_H = 1$
- at $M_t = 174.3(3) \text{ GeV}$ we have $\alpha_1(M_t) = 0.01023(1)$,
 $\alpha_2(M_t) = 0.03337(1)$ and $\alpha_3(M_t) = 0.10734(3)$

Grand unification scenario

Non-supersymmetric scenario:

- symmetry breaking at $E_{\text{weak}} = m_H = 154.4 \text{ GeV}$
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 $\alpha_2(M_t) = 0.03337(1)$ and $\alpha_3(M_t) = 0.10734(3)$
- from this point onwards $N_F = 3$

Grand unification scenario

Non-supersymmetric scenario:

- symmetry breaking at $E_{\text{weak}} = m_H = 154.4 \text{ GeV}$
- from this point onwards $N_H = 1$
- at $M_t = 174.3(3) \text{ GeV}$ we have $\alpha_1(M_t) = 0.01023(1)$,
 $\alpha_2(M_t) = 0.03337(1)$ and $\alpha_3(M_t) = 0.10734(3)$
- from this point onwards $N_F = 3$
- couplings α_2 and α_3 meet at

$$m_{\text{GUT}} = m_Q = 1.73(2) \times 10^{16} \text{ GeV}$$

with $\alpha_2(m_Q) = \alpha_3(m_Q) = 0.02192(1)$,
 $\alpha_1(m_Q) = 0.01616(1)$

Grand unification scenario

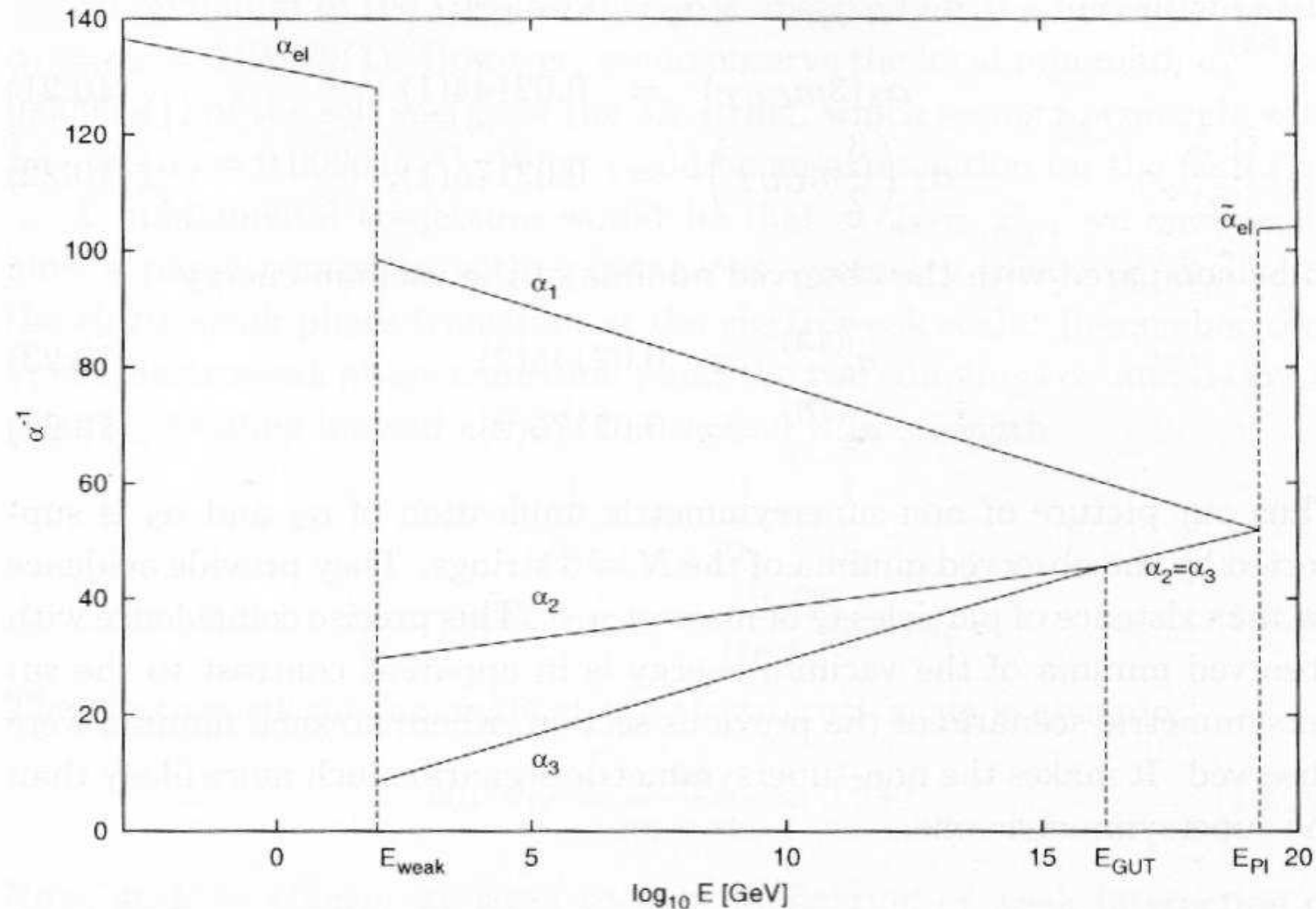
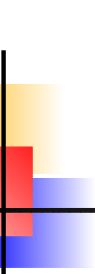


Fig. 10.1 Grand unification scenario based on non-supersymmetric beta functions.



Grand unification scenario

Unification of α_2 and α_3 at m_Q accompanied by the creation of an exotic quark-like particle Q of mass m_Q

Grand unification scenario

Unification of α_2 and α_3 at m_Q accompanied by the creation of an exotic quark-like particle Q of mass m_Q

- minimum $a_5'^{(3A)} = 0.02145(3) = \alpha_s(3m_Q)$ fits,
provided that we continue at m_Q with $N_F = 4$

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- further evolution with $N_F = 4$, $N_H = 1$ and $\alpha_2 = 0$

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provided that we continue at m_Q with $N_F = 4$
- further evolution with $N_F = 4$, $N_H = 1$ and $\alpha_2 = 0$
- α_1 and α_3 meet at $E = 1.73(2) \times 10^{19}$ GeV where
 $\alpha_1 = \alpha_3 = 0.01929(1)$

Grand unification scenario

Unification of α_2 and α_3 at m_Q accompanied by the creation of an exotic quark-like particle Q of mass m_Q

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- α_1 and α_3 meet at $E = 1.73(2) \times 10^{19}$ GeV where
 $\alpha_1 = \alpha_3 = 0.01929(1)$
- remarkably, $E = \sqrt{2}m_{\text{Pl}} =: E_{\text{Pl}}!$

Grand unification scenario

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 $\alpha_1 = \alpha_3 = 0.01929(1)$
- remarkably, $E = \sqrt{2}m_{\text{Pl}} =: E_{\text{Pl}}!$
- observed minimum of self energy of the $3A$ string
at $a_4'^{(3A)} = 0.00965(1)$, while $\frac{1}{2}\alpha_1 = \frac{1}{2}\alpha_3 = 0.009645(5)$

Grand unification scenario

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- α_1 and α_3 meet at $E = 1.73(2) \times 10^{19}$ GeV where
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at $a_4'^{(3A)} = 0.00965(1)$, while $\frac{1}{2}\alpha_1 = \frac{1}{2}\alpha_3 = 0.009645(5)$

Why $\frac{1}{2}\alpha_1$?

Grand unification scenario

compare electroweak phase transition point

$$\frac{1}{\alpha_{\text{el}}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \Leftrightarrow \alpha_{\text{el}} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$$

with

$$\frac{1}{\tilde{\alpha}_{\text{el}}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_3} \Leftrightarrow \tilde{\alpha}_{\text{el}} = \frac{\alpha_1}{2}$$

Grand unification scenario

compare electroweak phase transition point

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with

$$\frac{1}{\tilde{\alpha}_{\text{el}}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_3} \Leftrightarrow \tilde{\alpha}_{\text{el}} = \frac{\alpha_1}{2}$$

- at this point $m_L = \sqrt{2}m_{\text{Pl}}$

$$\alpha_G(m_L) = \frac{1}{2} \left(\frac{\sqrt{2}m_{\text{Pl}}}{m_{\text{Pl}}} \right)^2 = 1$$

Grand unification scenario

compare electroweak phase transition point

$$\frac{1}{\alpha_{\text{el}}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \Leftrightarrow \alpha_{\text{el}} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$$

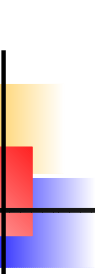
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- at this point $m_L = \sqrt{2}m_{\text{Pl}}$

$$\alpha_G(m_L) = \frac{1}{2} \left(\frac{\sqrt{2}m_{\text{Pl}}}{m_{\text{Pl}}} \right)^2 = 1$$

- largest coupling where dynamics is well defined



Grand unification scenario

Unification of α_1 and α_3 at m_L accompanied by the creation of an exotic lepton-like particle L of mass m_L

Grand unification scenario

Unification of α_1 and α_3 at m_L accompanied by the creation of an exotic lepton-like particle L of mass m_L

- minimum $a_4'^{(3A)} = 0.00965(1) = \tilde{\alpha}_{\text{el}}(3m_L)$ fits,
provided that we continue at m_L with $\sin^2 \theta_W = \frac{1}{2}$

Grand unification scenario

Unification of α_1 and α_3 at m_L accompanied by the creation of an exotic lepton-like particle L of mass m_L

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provided that we continue at m_L with $\sin^2 \theta_W = \frac{1}{2}$

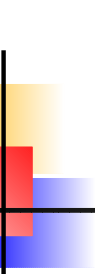
Because of

$$\alpha_{\text{weak}} = \alpha_{\text{el}} \frac{(T_3 - Q \sin^2 \theta_W)^2}{\sin^2 \theta_W \cos^2 \theta_W}$$

for $\sin^2 \theta_W = \frac{1}{2}$ we have the effective replacements

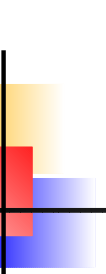
$$\begin{array}{llll} \nu_L \rightarrow e, & e_L \rightarrow \nu, & \nu_R \rightarrow \nu, & e_R \rightarrow e, \\ u_L \rightarrow d, & d_L \rightarrow u, & u_R \rightarrow u, & d_R \rightarrow d \end{array}$$

with opposite charges.



Grand unification scenario

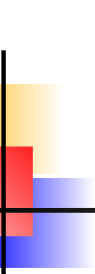
From m_L onwards almost every interaction strength seems to reduce to the unified coupling $\tilde{\alpha}_{\text{el}}$.



Grand unification scenario

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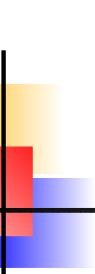
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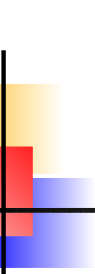
- Duality between theories described by α_G and

$$\tilde{\alpha}_G = \frac{1}{2} \left(\frac{\tilde{E}}{m_{\text{Pl}}} \right)^2 = 2 \left(\frac{m_{\text{Pl}}}{E} \right)^2 = \frac{1}{\alpha_G}$$



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- the four space-time dimensions emerge

$N = 1$ strings

Four $N = 1$ strings exist (named by $+1^+$, $+1^-$, -1^+ , -1^-)

$$N = +1, s = +1 \quad \Phi_{n+1}^i = +(1 - a)\Phi_n^i + \frac{a}{2}(\Phi_n^{i-1} + \Phi_n^{i+1})$$

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Note: strings are not ergodic, i.e. their behavior is not independent of the initial conditions which can be given by the $N = 2$ and 3 strings

$N = 1$ strings

$N = 1$ string excitations decay rapidly but leave a significant sign pattern (up: n incr., right: i incr.)

$+1^+ :$

+	+	+	+
+	+	+	+
+	+	+	+
+	+	+	+

$-1^+ :$

+	+	+	+
-	-	-	-
+	+	+	+
-	-	-	-

$+1^- :$

+	-	+	-
+	-	+	-
+	-	+	-
+	-	+	-

$-1^- :$

+	-	+	-
-	+	-	+
+	-	+	-
-	+	-	+

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+	-	+	-
+	-	+	-

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-	-	-	-
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-	-	-	-

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+	-	+	-
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Note special shape of $+1^+$ string pattern (time!)

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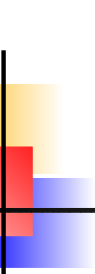
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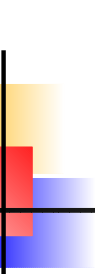
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- particles flow into ordinary space-time
- with the emergence of space-time the particles “decide” whether interact gravitationally (dark matter) or by gauge coupling $\tilde{\alpha}_{el} = 0.00965$

$$\frac{m_{\text{bright}}}{m_{\text{dark}}} = \frac{1}{2} \sqrt{\frac{\tilde{\alpha}_{el}}{\alpha_G}} = 0.049$$



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Dark energy

Parisi-Wu approach for stochastic quantization
applied to self-interacting scalar field ϕ
of the Robertson-Walker metric

$$\frac{\partial}{\partial s}\phi = \ddot{\phi} + 3H\dot{\phi} + V'(\phi) + L(s, t)$$

$L(s, t)$: Gaussian white noise

H : Hubble constant

V : potential under consideration

Discretization $s = n\tau, t = i\delta, \phi_n^i = p_{\max}\Phi_n^i$ under the
assumption that $a := 2\tau/\delta^2$ remains finite leads to

$$\Phi_{n+1}^i = (1-a)T(\Phi_n^i) + \frac{3}{2}a\delta H(\Phi_n^i - \Phi_n^{i-1}) + \frac{a}{2}(\Phi_n^{i-1} + \Phi_n^{i+1}) + \tau \times \text{noise}$$

Dark energy

Connection between $V'(\phi)$ and $T(\Phi)$:

$$V'(\phi) = \frac{1-a}{\tau} \{-\phi + p_{\max} T(\Phi)\}$$

or integrated over ϕ

$$V(\phi) = \frac{1-a}{\tau} p_{\max}^2 \left\{ -\frac{1}{2} \Phi^2 + \int T(\Phi) d\Phi \right\} + C$$

For $T(\Phi) = T_{-3}(\Phi) = -4\Phi^3 + 3\Phi$ and a suitable choice for C we obtain

$$\langle V_{-}^{(3)}(\phi) \rangle = \frac{3p_{\max}^2}{8\tau}$$

where $\langle A \rangle = \int A(\Phi) \rho_0(\Phi) d\Phi$.

Dark energy

Using Heisenberg's uncertainty principle $\Delta E \Delta t = \frac{\hbar}{2}$ for $\Delta t = t_0 = (4.32 \pm 0.06) \times 10^{17} s$ (age of the universe) and $\Delta E = p_{\max}$, we obtain

$$p_{\max} = \frac{\hbar}{2t_0} = (7.62 \pm 0.08) \times 10^{-43} \text{ GeV}$$

Choosing $\tau = \kappa m_{\text{Pl}}^{-2}$ where κ is dimensionless number of order 1, we obtain

$$\langle V_-^{(3)}(\phi) \rangle = \frac{3p_{\max}^2}{8\tau} = (3.19 \pm 0.05) \times 10^{-47} \kappa^{-1} \text{ GeV}^4$$

Current observational estimate of dark energy density is $\rho_\phi = (2.9 \pm 0.3) \times 10^{-47} \text{ GeV}^4$ ($\Rightarrow \kappa = 1.10 \pm 0.10$)

To conclude by philosophy

