CHAPTER 1

Curves

1.1 EVOLUTE OF A PLANE CURVE

1. Find the equation of the evolute of a curve. Draw the curve and its evolute.

    - $\xi(t) = (a \cosh t, b \sinh t)$ (hyperbola);
    - $y = x^2$ (parabola);
    - $y = x^{2k}$, where k is an integer greater than 1;
    - $y = \ln x$;
    - $y = \sin x$.

1.2 FRENET-SERRET (BARTELS) FORMULAE

1. Let $(\xi = \xi(s), I)$ be a unit-parametrized space curve and $\{T, N, B\}$ be a moving frame along this curve. Prove that the Frenet-Serret formulae can be written in the form

$$T' = D \times T,\ N' = D \times N,\ B' = D \times B. \tag{1.1}$$

   where the vector field D is called a *Darboux vector field* along a parametrized curve ξ. Find the Darboux vector field and explain its kinematic nature.

2. Find the curvature and the torsion of the following parametrized curves

    - $\xi(t) = (a \cosh t, a \sinh t, at)$,
    - $\xi(t) = (e^t, e^{-t}, \sqrt{2}t)$,
    - $\xi(t) = (2t,\ \ln t,\ t^2)$.

1.3 ANSWERS

1. $D = \tau T + \kappa B$ **2.** $\kappa = \tau = \frac{1}{2a\cosh^2 t}$, $\kappa = -\tau = {\frac{\sqrt{2}}{e^t+e^{-t}}}^2$, $\kappa = -\tau = \frac{2t}{(1+2t^2)^2}$